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AN INTRODUCTORY BOOK FOR ENGINEERS AND STUDENTS

BY

W. PERREN MAYCOCK, M.I.E.E.

LATE TECHNICAL EDITOR TO THE WESTINGHOUSE COMPANIES PUBLISHING DEPARTMENT IN EUROPE

AN ENLARGEMENT OF, AND AN IMPROVEMENT UPON THE AUTHOR'S FORMER WORK "THE ALTERNATING-CURRENT CIRCUIT AND MOTOR"

SECOND EDITION

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PREFACE.

THIS book is virtually a revised and extended edition of my earlier work, *The Alternating-Current Circuit and Motor*. But as the original matter has been most thoroughly overhauled, and its scope enlarged to embrace Alternators and Transformers, the book has been given a new title.

I have had throughout the most able and painstaking assistance of Mr EDWARD HUGHES, B.Sc. Eng. (Lond.), of the Heriot-Watt College, Edinburgh; and it is hoped that our combined efforts have resulted in providing a good and easy introduction to the subject. Should there be any serious "kinks" that we have failed to straighten out, I shall be both sorry and glad to hear about them.

The scope and object of the book are explained in the Introduction on p. 1, so that little need be said here on these points. It may be mentioned, however, that the reader must have some elementary knowledge of the continuous-current side of electrical engineering, such as is given—for instance—in Vol. I. of my *Electric Lighting and Power Distribution* (7th Ed.).

The recent work of Dr C. V. Drysdale, considered in conjunction with the earlier work of Maxwell and Sir Oliver Lodge, seems to prove that mechanical analogies are by no means to be despised in approaching the subject of alternating currents. Consequently, the analogies of this description given in the first two editions of the *Alternating-Current Circuit and Motor*, which were founded on some introduced by Forbes, have been extended and improved in the present volume.

As regards the numerous technical terms peculiar to the

subject, care has been taken, as far as possible, to secure agreement with the recommendations of the International Electrotechnical Commission.

Most of the matter in §§ 45, 49, 50, 52, and 112 to 114 has been taken from my *Electric Circuit Theory and Calculations*, and in connection with this book the help of Mr W. H. Bray must be mentioned.

The collections of City Guilds' and A.M.I.E.E. Exam. Questions at the end of each Chapter will be found useful: though it has been impossible, of course, to cover thoroughly the very wide field which they embrace. The numerical answers given at the ends of all questions of this nature are entirely due to Mr Hughes.

The final preparation for the blockmakers of nearly all the new diagrams has been the work of Mr Harold Bishop.

A Key or Set of fully-worked Answers to all the numerical questions herein, and to kindred ones in the 1915 Exams., will be published should any demand arise.

With further reference to the earlier book on which the present volume is based, the assistance I received from Mr C. H. Yeaman (on the 1st Edition), and from Mr William Cramp and Mr Llew. R. Lester (on the 2nd Edition), is here once more acknowledged.

W. PERREN MAYCOCK.

PREFACE TO SECOND EDITION.

A FEW corrections have been made, and three Appendices have been inserted at the end, at the kind suggestion of Mr E. Hughes.

W. P. M.

WEST NORWOOD, LONDON, S.E.27.
July, 1917.

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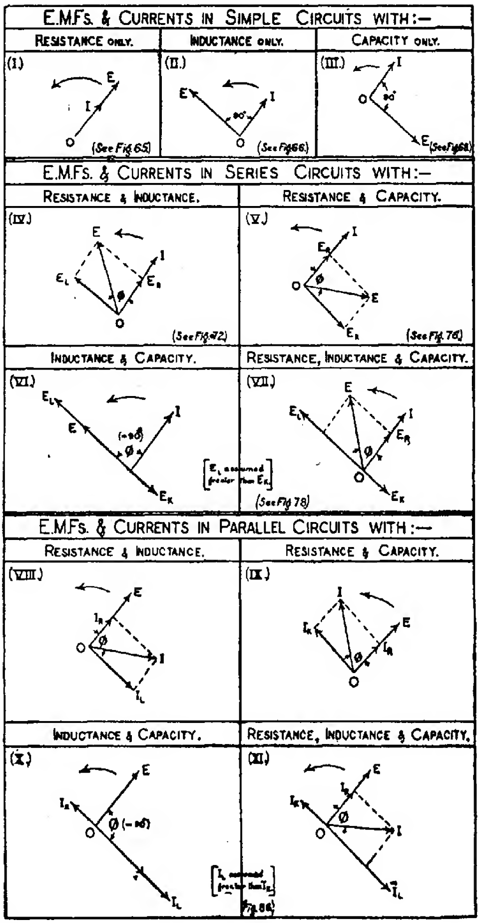

ALTERNATING-CURRENT WORK.

CHAPTER I.

INTRODUCTION.

CONCERNING THIS BOOK.

THIS Book deals with the Elementary Theory of Alternating Currents of Electricity, with their Generation and Application for Motive Power, and with Transformers and certain other Accessory Apparatus.

As regards Lighting and Heating, except in the case of arc lamps, alternating currents are applied in much the same way as continuous currents; so that practically nothing is said about these applications herein.

The treatment of the subject has been made as simple as possible, the book being written for elementary students, and for those numerous older readers to whom some knowledge of alternating currents is necessary, and who possess only the rudiments of mathematical knowledge. The Calculus has not been allowed to show its fearsome face within these pages, and the very little Trigonometry that has been used is briefly explained.

Good mathematical knowledge is an accomplishment worthy of great respect, and is of decided advantage to an engineer, provided he does not neglect other aspects of his work. But those who—through lack of tuition or sheer mental inability—are unable to rise to such heights, may comfort themselves with the reflection that there is plenty of room for distinction in other ways. It is easy to imagine numerous cases (a breakdown of plant for instance) where the highly-mathematical man would only be able to look on, while the practical man was righting things. Then there is the indispensable "commercial engineer," who often does not possess very special knowledge of either theory or practice, but whose duties would be quite beyond some theorists and practicians.

Any reader who, on glancing through these pages, becomes at all overawed on seeing some of the formulæ and vector diagrams therein, should understand that a little application will smooth out all difficulties with these things. It is certain that the subject could not have been treated satisfactorily in a more elementary way.

Some knowledge of alternating currents is of course indispensable to nearly all electrical engineers, as well as to many other kinds of engineer: and the alternating - current side of electrical engineering becomes increasingly important every year. In short, most large schemes of electrical distribution and application are only practicable with this kind of current. Thus this volume really opens the gate of the field of what might be termed "alternating-current engineering": and the theoretical first portion of the book, though it may not appear so attractive to some readers as the latter portion, deals with the foundations of the whole subject.

CONCERNING ELECTRICAL ENGINEERING.

I. **PRELIMINARY.** — Comparatively few people understand what is covered by the term *Electrical Engineering*, and it is both instructive and encouraging to get some slight idea of its scope and possibilities. Briefly, Electrical Engineering comprises the generation, distribution, utilization, and control of electrical power for lighting, railways, tramways, the driving of machinery of every conceivable description, telegraphy, telephony, heating, chemical work, and various other purposes; together with the theoretical principles underlying these matters. In short, Electrical Engineering enters into all other branches of engineering, and into many other kinds of work: and most people derive benefit from it in their everyday life.

The subject of Electrical Engineering is complicated: *firstly*, because there are so many main methods of arranging generating stations; *secondly*, because there are so many methods of distribution; *thirdly*, because there are so many applications and so many details underlying each and every application; *fourthly*, because there are generally several ways of effecting any given application; and *fifthly*, because there are often many different makes and patterns of apparatus for the same purpose.

The scope of Electrical Engineering is so universal that it is impossible for any individual electrical engineer to be even partially conversant with all its branches. Very few, in fact, could give a detailed list of the branches off-hand. Thus it is that electrical engineers could be divided into a number of different classes more or less distinct; and thus it is that practically every electrical student eventually has to choose

one or more branches in which to specialize. In very few cases is specialization unnecessary. Some lecturers or teachers, for instance, must have a superficial knowledge of the more prominent branches of electrical work; and so also must certain classes of consulting engineers, manufacturers, and contractors, who wish to deal with varied kinds of such work. But it is essential for these general consulting, manufacturing, and contracting men to call in the help of specialists if any given work of importance is to be done really well.

Another reason for the necessity for specialization is that the electrical engineer must be also something of a mechanical engineer; and according to the branch he takes up, must also know something of steam or water engineering, of physics, or of chemistry. In any case, good mathematical knowledge is useful, if not essential.

Yet another reason for specializing is that hardly any branch of Electrical Engineering can yet be said to be stereotyped, many being still more or less in the development stage. The electrical engineer, in fact, always has something new to learn.

In some branches of electrical work, specialization is carried to such a degree that — taking any one branch—some men invent or design, others manufacture, others install or lay-down the plant, and others manage or "run" the plant; *plant* meaning any collection or system of electrical machinery, appliances, and accessories for some given purpose or purposes.

II. **THE ELECTRICAL SYSTEM.** — By *electrical system* is meant here any self-contained arrangement of "apparatus" for generating, distributing, and utilizing electrical energy.

Every complete electrical system may be said

roughly to consist of three parts, as shown in Fig. 1, viz.:—

(*a.*) The *Generation* portion, G,

(*b.*) The *Distribution* portions, C, C, C, and

(*c.*) The *Application* portions, A, A, A.

In other words, electrical power is generated at a generating station G; distributed by means of copper cables, C, C, C; and applied or utilized in numerous buildings, works, streets, or other places, A, A, A.

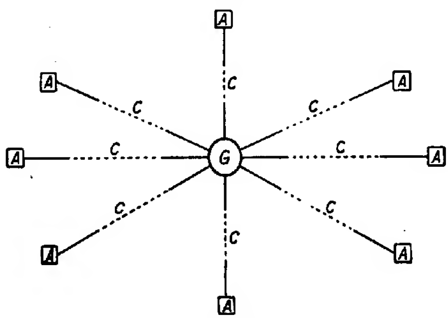

Fig. 1.—The Electrical System.

If the continuous or direct current mainly dealt with in Vol. I.* were the only kind of electrical current, the study of electrical systems would be a comparatively simple matter; but there are, in addition, three kinds of alternating current, viz., the single-phase, the two-phase, and the three-phase. The two-phase current, however, is comparatively little used.

It is this availability of at least three different kinds of current, each of which has its special advantages,

* The Author's *Electric Lighting and Power Distribution*, Vol. I. Seventh Edition.

which enables electrical power to be employed for so many different purposes. Thus the single-phase current is suitable for numerous applications where the continuous current would be useless; and the three-phase current can cope with conditions under which neither the single-phase nor the continuous current would be a success. On the other hand, for some purposes, the continuous current is the only kind that can be utilized.

The details of the electrical system, which is shown very roughly and diagrammatically in Fig. 1, differ according to the kind of current to be used. The generators vary, the arrangement of the copper distributing cables varies, and the methods of application vary—especially in the case of motive power.

The generating and distributing systems for any one kind of current also vary according to the area or distance over which the power is to be supplied, and according to the uses to which it is to be put: and when a generating station supplies two (or all three) kinds of current, the arrangements become somewhat complex.

III. **THE ELECTRICAL SYSTEM** (*cont.*).—Put in a brief and simple manner, but in a more complete way than on p. 5, an electrical system (Fig. 1) consists of:—

(*a.*) Steam-, water-, gas-, or oil-driven electric generators at the generating station *G*.

(*b.*) Insulated copper cables *C*, *C*, *C*, leading the current from the generators to the various places where it is to be used.

(*c.*) Lamps, electric motors, heaters, and various other apparatus or appliances, connected to the cables at the points of application *A*, *A*, *A*.

If this were all, things would be fairly simple, even with three different kinds of current. But the electrical power has to be *controlled*, *regulated*, and *transformed* or *converted*; and the methods of control, regulation, and transformation or conversion constitute very important and interesting portions of the subject.

Finally, there is the all-important matter of *insulation*. The paths of the current within the generator, in the cables which lead it to the lamps, motors, heaters, and so forth; and (in most cases) the paths back again to the generator, must all be insulated; *i.e.*, surrounded with something which will not allow electricity to flow through it. The greater the pressure at which the electricity is generated, distributed, and used, the better must be the insulation; and the kind of insulation to be employed in any given part of the circuit will also depend upon the conditions and surroundings of that part.

The distribution of gas, water, air, or mechanical power is a much simpler matter than the distribution of electrical power. But the latter possesses the *enormous advantages* that it may be carried in far greater quantities and to far greater distances; that bends, inclines, levels, atmospheric pressures, and temperatures make no difference; that the uses are so diverse; and that the methods of control are so much more elastic and convenient. The cost of electric light is now less than that of gas light; and its inherent advantages are so undeniably great that it would be waste of time to dwell on them. When electrical power from an outside source is available at a reasonable price; it may be accepted at once that the adaptability, convenience, cleanliness, quietness, and compactness of the electric motor can never be approached by the steam-, gas-, or oil-engine. When there is no external

supply, it frequently pays to have a private electric-generating plant.

The transmission of water, air, or mechanical power in any quantity, and over any distance worth mentioning, cannot be treated as serious propositions. But even if this were not so, and even assuming that there were efficient motors for converting the water or air power into mechanical power; the fact that these three forms of power are not directly convertible into light or heat, at once shows that they cannot be of general utility.

IV. **POWER, AND ITS CONVERSION AND TRANSMISSION.**—In everyday life, the main uses of energy and power,* apart from that derived from air, food, and water for sustaining life, are in lighting, heating, and motive power: and it is with the supply of these three latter forms of power that electrical engineering has most to do.

The present prime sources of industrial energy are coal, water, and oil; and we will now see how ultimate forms of power are derived from them.

If we have coal and inert water (*i.e.*, water with little or no pressure behind it), as at (*a*) in Fig. 2, we can generate steam and get electrical power therefrom by means of engines and electric generators. This electrical power may then be distributed in all directions and over very great areas or distances; and—except for mechanical power, where an electric motor is necessary for the conversion—it may be directly applied to the numerous uses shown on the right. This list of uses, by-the-bye, could be detailed to a surprising extent. In this way, electrical power could

* The student should remember the difference between power and energy. *Energy* is the *capacity for doing work;* and *power* is the *rate* at which that work is done.

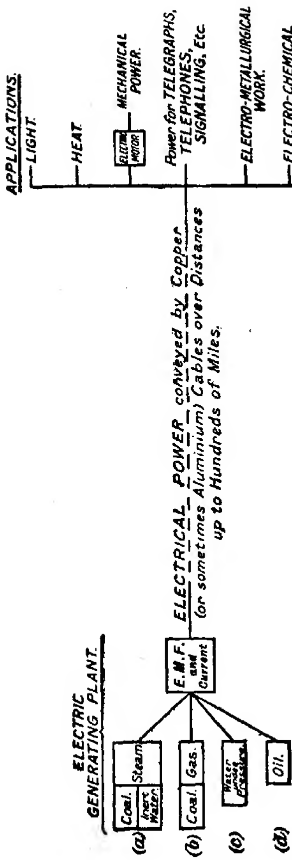

Fig. 2.—Electrical Power. Its Generation, Distribution, and Uses.

easily be generated at the pit's mouth, and distributed hundreds of miles to cities, towns, and villages.

Otherwise, as at (*b*) in Fig. 2, the coal may be converted into gas, and gas-engines used to drive the electric generators. For working on a large scale, however, present types of gas-engine are not so satisfactory as steam-engines (non-reciprocating or reciprocating).

A dream of the future is that gas may be generated in huge volume within the pit itself, and that non-reciprocating gas-engines (*i.e.*, gas-turbines) may be evolved. Another and vaguer dream is that electrical power may some day be generated direct from coal.

It is quite possible that, in course of time, after our coal supply has been exhausted, the sun's energy, at present used in its stored-up form in coal, may be directly utilized. If such a thing ever happened, the extensive deserts of Africa might eventually become the industrial centres of the world!

If we have water power in the shape of a waterfall or swiftly-flowing river, the water power is converted into electrical power by means of water-turbine-driven generators ((*c*) in Fig. 2).

Another dream of the future is that the sea-tides may ultimately be harnessed to this work.*

Electrical power may be derived from oil by means of oil-engines and generators ((*d*) in Fig. 2).

* It is not at all out of place to mention these imaginations here; for some electrical dreams have a way of materializing in due course. Many electrical applications of to-day were looked upon as impossible (even by some electrical engineers) when first mooted. The proper attitude of an electrical engineer towards a reasonable idea or a "dream," is to think that there *may* be something in it. He should assume that very many wonderful things remain to be accomplished, and he should know that there are many workers quietly seeking to unravel such problems

V. **POWER, AND ITS CONVERSION AND TRANSMISSION** (*cont.*).—If we leave electricity out of account, it is surprising how comparatively little can be done with coal, oil, or water.

Take coal to start with, and compare the Applications in Fig. 3 with those in Fig. 2.

The distribution of gas power through pipes can only be effected over very limited distances compared with those with which an electrical system can cope. And the direct employment of coal by individual users

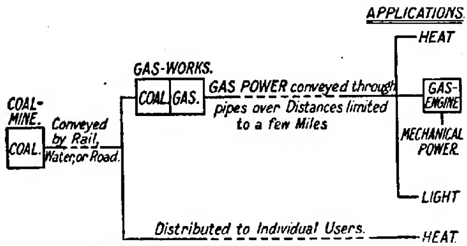

Fig. 3.—Coal Power and its Uses. (Electricity excluded.)

entails its distribution by water, rail, and road by very laborious and expensive methods; and its consumption in a very inefficient manner. The general applications of gas power are few compared with those of electricity; and it is only in its heating application that it can be said seriously to rival electricity. The presence of gas lighting and gas-engines in places where electric power is available, may generally be taken as sure evidence of the apathy, stupidity, or ignorance of those in charge. Occasionally, of course, the expense of changing-over from one to the other stands in the way of progress.

In populated districts there is but one serious use

for oil fuel, viz., for conversion into mechanical power. Only in isolated places, and when neither electricity or gas are available, do its uses for lighting and heating become valuable (Fig 4). But even here electricity is not to be ousted, for the best way of getting light from oil—if cleanliness, convenience, safety, and comfort are valued—is to interpose an oil-engine and an electric generator.

If someone would only invent a secondary battery light enough for automobiles, small boats, and aeroplanes; oil and the oil-engine would have a formidable rival to contend with in the accumulator and electric

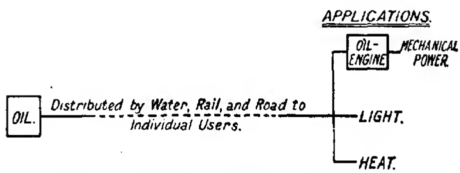

Fig. 4.—Oil Power and its Uses. (Electricity excluded.)

motor; and the current would also be available for *safe* lighting and heating. With large boats, and with railway locomotives for certain purposes, there are advantages in using an oil-engine to drive an electric generator, the current from which propels the boat (or loco.) through an electric motor. An interesting example of the former application is given at the end of this book.

Water, as water simply, is one of the primary necessities of existence. As a source of power, however, it is negligible, except in great bulk and under great pressure for driving electric generators. The harnessing of the power of any ordinary small waterfall or stream is only worth serious consideration in isolated places

where neither electricity, gas, coal, or oil are available. Water motors for connecting to the mains are little more than expensive toys. Lastly, neither heat nor light can be directly derived from water (Fig. 5).

Wind power is too intermittent in character to be of serious account, though a few very small installations exist in which windmills are used to drive dynamos and charge accumulators for electrical work.

VI. **ELECTRICAL APPLICATIONS.** — A comparison of Fig 2 with Figs. 3 and 4 will show that the uses of electricity are immensely more important and varied than those of gas or oil.

Probably very few readers of this volume have ever

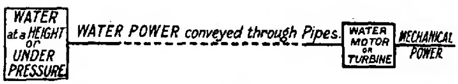

Fig. 5.—Water Power and its Uses. (Electricity excluded.)

been in a large electric generating station; nor will many have any conception of the vast system of "mains" radiating therefrom. And although nearly all may be familiar with electric light, electric trams, and electric railways; few of them can have much knowledge of the numerous other applications of electricity, especially those relating to the multitudinous uses of "stationary" electric motors. These uses may be said to constitute about one-half of the third "application" in Fig. 2; yet they extend to every conceivable kind of machine, however large or small; ranging, for example, from the "winders" which draw up men or coal from the depths of a mine (Fig. 256), to a dentist's drill.

The foregoing should serve to impress the student with the importance of the subject of Electrical

Engineering; and this impression will go far towards carrying him enthusiastically through his work, some of the details of which may sometimes seem to have little connection with great matters. A piece of copper wire, a lump of iron, and a chunk of rubber would not convey much to the ordinary observer, but to the thinking electrical engineer they would symbolize a very great deal.

VII. **THE ELECTRICAL SYSTEM** (*cont.*).—Reference was made in Sec. III. to the control, regulation, and transformation or conversion of electrical power; and we will now get some preliminary ideas about these matters, with the help of Fig. 6.

A generating station always contains a number of generators, *G, G, G;* these and the distributing mains or cables, *C, C, C*, being interconnected at one or more *switchboards S*. The latter bear various apparatus for switching individual generators on or off, for enabling their currents to be indicated and their e.m.fs. regulated, and for permitting the same being done with the currents and pressures supplied to the various cables. There are also automatic devices for cutting off faulty generators or cables.

In large stations, the main switches, etc., are generally situated above or below the "operating platform," and are actuated therefrom by mechanical or electro-mechanical means. Certain apparatus is also often installed for increasing the electrical pressure before it is delivered to the cables. At the points of application *A, A, A* (Fig. 6), there may be apparatus for reducing the pressure, or for converting or changing the *character* of the current, say from alternating to continuous.

Most ordinary application - circuits are fed at pressures ranging from 100 to 500 volts. But some large alternating - current motors run directly off

voltages ranging as high as 6000 volts. Continuous-current traction circuits, *i.e.*, for tramways or railways, are usually fed at from 500 to 600 volts.

Power is proportional to the voltage × current, and the size of a cable depends mainly on the current it has to carry. Now as it is necessary to keep the weight, bulk, and cost of cables as low as possible, the current

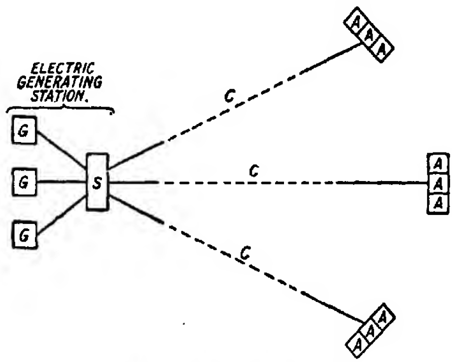

Fig. 6.—The Electrical System.

for a given amount of power must also be kept low; and this can only be done by employing a high voltage.

In most large supply systems it will be found that the generators deliver the current at a high pressure; that this pressure is often further increased before the supply is delivered to the cables or "lines"; and that at the other ends of the latter, the pressure has to be reduced to that necessary for applications.

Fig. 7, for example, illustrates a case in which the electrical power is generated at 6000 volts, transformed up to 20,000 volts, transmitted at that pressure, and

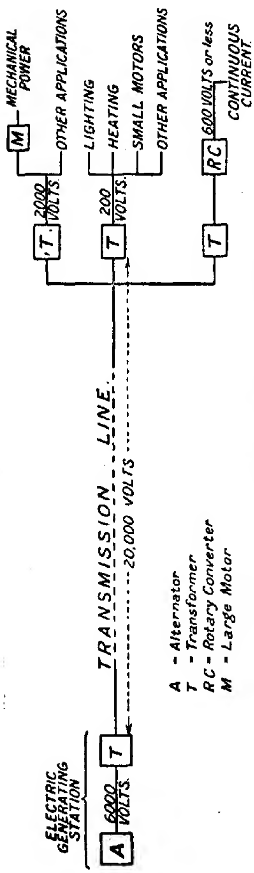

Fig. 7.—High-Voltage Alternating-Current System.

transformed down again at the places of application to, say, 2000 volts for large motors, and to about 200 volts for small motors and for lighting, heating, and miscellaneous circuits.

In order to simplify the diagram no switchboard has been shown; and only one generator, main transformer, and line.

In this way, the voltage employed for transmission may be made almost any desirable value. In fact, there are systems in America where the transmission pressure is as high as 150,000 volts, and still higher pressures are contemplated! It is thus possible to transmit enormous electrical powers over hundreds of miles, through comparatively small "lines," and at comparatively low costs.

For working at

pressures such as those just mentioned, the alternating-current system is the only one that can be employed. The reasons for this are that continuous-current generators or motors cannot be worked at very high voltages, and that these voltages cannot be transformed up or down very easily or efficiently with continuous-current apparatus. Nevertheless, if continuous current is absolutely essential at any application point or points, it may be obtained through the medium either of a *rotary converter* or of a *motor-generator*; which may be roughly described as combinations of an alternating-current motor and a continuous-current generator.

The importance of alternating-current engineering having been demonstrated, let us now turn to the very beginnings and elements of the subject, as set forth in the following pages.

CHAPTER II.

GENERAL PRINCIPLES.

1. **THEORY OF ELECTRICITY.**—To start with, it is necessary to adopt some theory of electricity. Now there have been many theories for electrical action, but it is still rather difficult to pick out any one and say that that is the right one. Nevertheless, the *electronic theory*, in which electricity is looked upon as a movement of electrified particles or *electrons*, is seemingly the most plausible. It would, however, be extremely difficult, if not impossible, to explain the phenomena with which we are about to deal, in the light of the electronic or any other advanced theory, in a book of this elementary character. Moreover, it should be remembered that we are here concerned not so much with what is vaguely called "electricity," as with certain of its effects. Hence we must choose some *simple practical theory*, at the same time remembering that it is adopted to facilitate explanations, and keeping our minds ready for the reception of a better one at a future time.

The "simple practical theory" advocated by the Author is that known as the *surplus and deficit theory;* and it was first fully treated and extended by him in a series of articles in the now defunct *Electrical Engineer*, which articles were subsequently embodied in the Author's *First Book of Electricity and Magnetism.**

It must suffice here to indicate the mere outlines of this theory. All things, conductors and insulators

* Fourth Edition.

alike, are supposed to be imbued with electricity normally distributed—*i.e.* at even pressure or potential. Electrification is the act of heaping up electricity on one body or bodies, leaving a corresponding deficit on another body or bodies. The former is or are then said to be positively electrified, and the latter negatively electrified. A battery, dynamo, alternator, or transformer, is simply an electric pump, whose electromotive (electro-pumping) force alters, or tends to alter, the even distribution of electricity in the circuit; so that these apparatus must be looked upon as generators of electromotive force, *not* of electricity. The current set up depends upon the "output capacity" of the apparatus. The term *electromotive force* will be constantly cropping up; and it is generally abbreviated to *E.M.F.* or *e.m.f.*

In all cases where an uneven distribution of electricity exists, there will be a tendency for it to flow so as to regain a general level or distribution. When there is such a tendency, there is said to be a difference of pressure or potential, or a potential difference (abbreviated *P.D.* or *p.d.*).

2. **ALTERNATING CURRENT.** — The simplest kind of current is that derived from a battery. This is a steady *continuous* or *direct* current, and its principal properties are presumed to be well known to the reader. A well-designed dynamo gives a current which is similar to that of a battery; and the laws which apply to the current from a battery may be applied equally well to that from a dynamo.

If a *reversing switch* R,* inserted in the circuit of

* The construction and action of this form of reversing switch are as follows:—Pivoted at p, and provided with a handle h, are mounted the U-shaped piece of metal + +, and the straight piece —, to which the + and — poles of the battery are respectively connected. When the switch-handle is in the position shown, the metal tongues TT', con-

a battery or a dynamo, as shown in Fig. 8, be operated at regular intervals, alternating e.m.fs. will be *impressed* on the outer circuit *C*, and an alternating current will be set up therein, as conveniently represented by the double-headed arrows ↔ ↔.

Supposing the circuit *C* had no inductance (§ 9) or other disturbing effect, the current or rate of flow of electricity in it would always be the same, but would

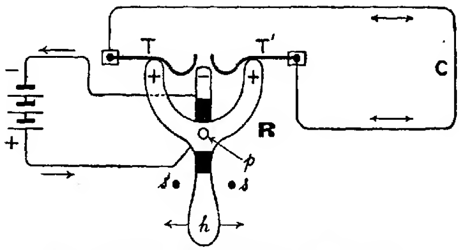

Fig. 8.—Battery and Reversing Switch for setting-up an Alternating Current.

be reversed in direction at regular intervals, as shown by the "curve" in Fig. 10.

The explanation of this curve is as follows:—Time is represented along the horizontal or base line (say, in one-second intervals), starting from the left. Current in one direction is represented by the short portions of horizontal line, *ab*, *ef*, etc., above the base line, and current in the other direction by similar short portions,

nected with the extremities of the outer circuit *C*, rest on + +, and no current flows from the battery. If the switch-handle be moved to the right, the right-hand leg of the U-piece remains in contact with *T′*, and the straight piece touches *T*, a current consequently flowing round *C* in a counter-clockwise or left-handed direction. If the switch-handle be moved to the left, the left-hand leg of the U-piece is in contact with *T*, and the straight piece with *T′*, and a current flows round *C* in a clockwise direction. Thus, if *h* be constantly moved to and fro, an alternating current will be set up in *C*. The movement of *h* is limited by the stops *s*, *s*. The actual apparatus is shown in Fig. 9.

cd, *gh*, below it. The distances of these "current lines" above and below the base line correspond with the "strength" of the current, which in this case is supposed to be 10 amperes.

It is usual to style currents in one direction + (positive), and those in the opposite direction − (negative); but these terms are confusing to the beginner, who would probably assume that a "+ current" was different in its properties from a "− current." We shall therefore refer to them as right- and left-hand currents respectively, these terms well conveying the idea that they flow in opposite directions round the circuit.

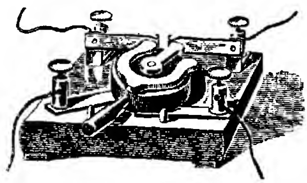

Fig. 9.—Reversing Switch. (*Griffin.*)

Suppose at the time of commencing the curve, a "right-hand current" was flowing, and that its value was 10 amperes; and suppose also that the direction was reversed every second. Our curve would then start at the point *a*, and would run in a horizontal direction for one second—*i.e.*, from *a* to *b*—when it would suddenly drop to *c*, the current having been reversed. The "left-hand current," *c d*, would continue for one second, as shown, and would then immediately change to the "right-hand current," *e f*. During the fourth second the current would be "left-handed," *g h*, during the fifth second "right-handed," *i j*, and so on. The current is therefore said to be alternating, since at one instant it is flowing in one direction, whilst at another instant it is flowing in the opposite direction.

It will be seen later that the current obtained from an alternator does not keep at a steady value and reverse instantaneously, like that depicted in Fig. 10.

3. **ALTERNATING CURRENT** (*cont.*).—A steady continuous or direct current* may be likened to a steady flow of water in one direction through a pipe. An alternating current may then be compared with the movement of water in the pipe when the direction of flow is changed more or less rapidly.

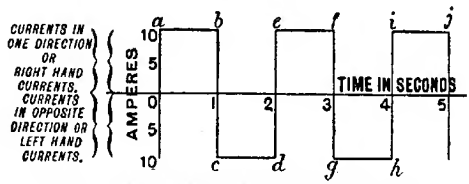

Fig. 10.—Graph of the Alternating Current set up by the Apparatus in Fig. 8.

Fig. 11 shows a pipe bent round so as to form a complete circuit, which includes a pump P; the whole being filled with water. The water stands for electricity, the pipe for the conductor, and P for the dynamo or alternator—according to its method of working. P is supposed to be actuated by a pulley or handle. If P be rotated continuously in one direction, its action is analogous to that of a battery or dynamo, the water in the pipe (electricity in the conductor) being set flowing in one direction. If P be rotated first in one direction and then in the other, at regular and rapid intervals, it represents the action of an alternator,

* The current from a battery or dynamo is called by some people—*continuous current*, and by others—*direct current*. It has been decided by International Authorities that the former is the better term to use.

for there will be a rapid to-and-fro flow of water in the pipe (electricity in the conductor).

Now electricity—like water—may, for the purposes of this argument, be assumed to be incompressible; so that with a given flow (current), the number of gills of water or coulombs of electricity passing any point, *a*, in the pipe or circuit, is the same as the number passing any point, *b*. Thus, let the shaded part, *C*, represent one gill of water or one coulomb of electricity. When *C* moves in either direction, the water or electricity in front and behind it, *i.e.* all round the circuit, moves at exactly the same rate, even when the size of the pipe or conductor varies at different parts of the circuit. In other words, the flow of electricity, in coulombs per second (amperes), is the same at all parts of a closed series circuit.* When the circuit is not of this description, *i.e.*, when it has branches, the current may vary in different parts.

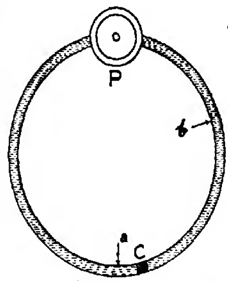

Fig. 11.—Hydraulic Analogy of an Alternating-Current Circuit.

The hydraulic analogy of an alternating-current circuit is often illustrated as in Fig. 12; the pulley, *p*, representing the rotating part of the alternator; the force of the pump piston, *P*, the electromotive force; and the up-and-down movement of the piston, the reversals in the direction of this force. Good as this analogy is in some respects, it is rather a faulty one, inasmuch as there is no actual passage of water through the pump; and the student might from this infer that

* Provided it has negligible capacity (§ 13, etc.).

there was no passage of electricity through the alternator. But we assume that the electricity flows through the alternator, or dynamo, or battery, just as it does through the other parts of the circuit.

An alternating current might be described as a "continual oscillation" of electricity in the circuit, just as the movement of the balance-wheel of a watch is a "continual oscillation." It must be borne in mind, however, that the use of the term "electrical oscillation" is more particularly applied to the movement of electricity when a condenser is discharged, this being a rapid to-and-fro movement *in an incomplete circuit*, which dies away to nothing (§ 15). This movement is similar to that of the prong of a tuning-fork, or of one end of a compass needle coming to rest in a strong magnetic field. The term "oscillation" should therefore be confined to the case of condensers, to prevent confusion.

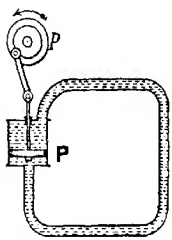

Fig. 12.—Hydraulic Analogy of an Alternating-Current Circuit.

4. **VARIATION OF THE E.M.F. INDUCED IN A COIL.**—The e.m.f. of an alternator is continually altering in value, as well as in direction, *i.e.*, it is in the form of waves. The current set up by an alternator is consequently also wavy; that is to say, it does not suddenly change from the full value in one direction to the full value in the other direction like the current represented in Fig. 10. The shape of these e.m.f. and current waves depends upon a number of conditions, but chiefly on the distribution of the magnetic field in the air-gap of the alternator, and the amount of inductance and capacity in the circuit.

To show approximately what an alternating current obtained from an alternator is like, one may draw a picture (in the form of a curve) of the changes which take place in the strength and direction of the impressed e.m.f. which sets up the current; and this will enable us to see what is meant by the *sine curve* or *sine wave*,—terms frequently used in speaking of alternating e.m.fs. and currents.

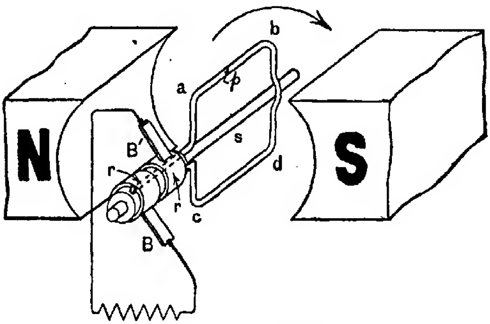

Fig. 13.—Simple Alternator.

When a simple coil of wire is rotated in a magnetic field, it has an alternating e.m.f. induced in it. A simple 2-pole field and coil are shown in Fig. 13, the ends of the coil *a b c d* being joined-up to collector rings *r r*, against which press the brushes *B B′*. The coil and rings are supposed to be mounted on—but insulated from—the shaft *s*.

Consider what happens to the top half, *p*, of the coil when the latter is evenly rotated on its shaft, *s*, in the direction shown by the curved arrow. Now *p* will, if viewed sideways from one of the pole faces, *N* or *S*, have an up-and-down motion; and its apparent

velocity will be variable during any one complete revolution of the coil; but the changes that take place will be repeated over and over again at regular intervals. This will be more clearly understood from Figs. 14 and 15. Fig. 14 represents the circular path traversed by p when the coil is looked at from the front end, only one pole, N, being shown: and as we suppose that the coil is being turned with uniform

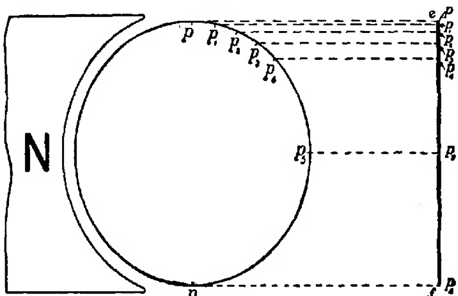

Fig. 14.—Induction of E.M.F. (View from end of Coil.)
Fig. 15.—Induction of E.M.F. (View from side of Coil.)

velocity, the actual rate of progress of p round its circular path will also be uniform. But if we look at p from one of the sides of the coil, it will appear to travel up and down in a straight line, $e\ f$ (Fig. 15); and its rate of motion in an actual up and down direction will not be uniform. When p has travelled round 10° from its topmost position, *i.e.* from p to p_1 (Fig. 14), its actual progress in a downward direction will be represented by the distance $p\ p_1$ in Fig. 15, which is relatively much less than the circumferential distance $p\ p_1$ in the first figure. Another 10° travel is from p_1 to p_2 (Fig. 14), from p_2 to p_3, from p_3 to p_4, and so on:

and as these distances are traversed in equal times, the apparent velocity of p, as viewed in Fig. 15, will at first be very slow, but will gradually increase until it reaches the 90° position, p_5. From p_5 to p_6 its apparent velocity will gradually decrease. The same thing will be observed when the coil is making its second half-turn, *i.e.*, when p is travelling from p_6 back again to its topmost position.

Now, the e.m.f. induced at p, or rather in the side $a\ b$ of the coil (Fig. 13), depends upon the rate at which it cuts the horizontal magnetic lines between N and S: and supposing this magnetic field to be uniform, the e.m.f. will depend upon the rate of motion of $a\ b$ in an actual up or down direction, as viewed in Fig. 15. It follows therefore that the e.m.f. in p (Fig. 13) will vary just as the rate of its travel along the assumed path $e\ f$ (Fig. 15) varies. It will change from zero to a maximum during the first quarter-turn of the coil, from maximum to zero during the second quarter-turn, from zero to maximum—in the reverse direction—during the third quarter-turn, and from maximum to zero during the last quarter-turn: by which time it will have made one complete revolution.

The other half, $c\ d$, of the coil (Fig. 13) will be acted upon in a precisely similar manner.

Motion of the kind described in connection with Fig. 15 is called *harmonic*, and obeys a simple law called the *sine law*. This can be explained by the aid of Figs. 16 and 17, which are closely related to the two preceding figures.

Looking at the coil from the collector or front end (Fig. 13), the path described by the point p will be a circle, having its centre at O (Fig. 16); $p\ o$ being its zero or starting position, and 1, 2, 3, 4, 5, etc., successive points on its journey during one revolution of the coil.

5. GRAPHICAL REPRESENTATION OF AN ALTERNATING E.M.F.—Let us now see how the sine curve, or curve of e.m.f., is plotted. The procedure is a little difficult to follow at first, but the reader must persevere, since the matter is of fundamental importance. If it is not quite clear after the first two or three repeated readings, it should be returned to later.

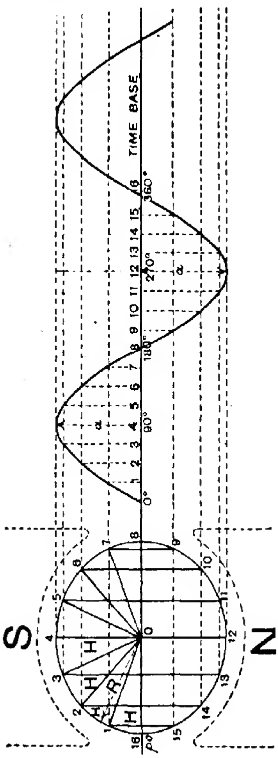

Fig. 17.

Fig. 16.—The Plotting of a Sine Curve.

Draw a horizontal line in the same straight line as the horizontal diameter of the circle in Fig. 16, and call it the *time base* (Fig. 17). Since the point p moves with uniform velocity round its circular path, distances measured along the time base may be taken to represent either "time from the beginning of measurement," or "distance moved by p round its circular path." p is connected to its "centre of travel" O, by the radius

R of the circle in which it moves; and this is clearly the greatest height to which it can rise, as in position O 4 (Fig. 16). We therefore take this height as the maximum height for our sine curve (Fig. 17), which represents the rise, fall, and reversal of e.m.f. The radius R will make with the horizontal diameter of the circle, an angle which will have zero value when p is in the position p o, and will increase as p travels round the circle, until, at position 4, the radius is 90° from its original position. To draw the e.m.f. curve, we must first take a length along the time base, and call it 360°; this being conveniently made equal to half the length of the circumference of the circle in which p moves.† This length is then divided into a number of equal parts corresponding with the number of points considered round the circular path in Fig. 16. Thus we get a straight line with divisions proportional to the distances moved by p along its circular path, or, what is the same thing, proportional to the angles made by the radius with its first position in its revolution round the centre O.* These divisions, as before pointed out, may also be taken to represent time, since p (Fig. 16) is supposed to be rotating at a uniform rate.

To draw the sine curve or wave (Fig. 17), we proceed as follows. Suppose p has reached the point 1, we take a distance along the time base equal to half † the circum-

* If p has moved from its zero position to position 2 (Fig. 16), the radius will have travelled round 45°. When p reaches the position 4, the radius will have travelled or have described an angle of 90°. When p has made one half-turn, *i.e.*, when it has reached the position 8, the radius may be said to have travelled 180° from its zero position. When p has made one complete revolution, we say that its radius has travelled round or described an angle of 360°.

† Distances along the time base are made *proportional* to circumferential distances, and may therefore be drawn to any desired scale. In the present case they are made equal to *half* the circumferential distances which they represent, this being the usual scale adopted.

ferential distance, 0 1, and at that point erect a perpendicular; where this cuts a horizontal line drawn through point 1 on the circle, we get one point on the curve. In the same way, for position 2, we take half the distance along the circumference 0 2, and mark this off on the time base, then erect a perpendicular, and where the latter cuts a horizontal line drawn through 2 on the circle we get the second point on our curve. This operation being repeated for different positions of p round its circular path (3, 4, 5, 6, etc.), a series of points is obtained, which, when connected, are found to lie on a wavy line called the sine curve.

The form of this curve depends on the relationship that the distance, H, of each position of p above (or below) the horizontal line bears to the radius, R. And the greater H is, that is, the greater the distance of p above or below the base line, the more effectively is it cutting the magnetic lines of the field, and the greater is the e.m.f. generated. H is a maximum at the positions 4 and 12, and these are consequently the maximum or "crest" or "peak" points on the curve. The connection between H and R is as follows:—

$$H = R \sin A,$$

where A is the angle which the radius, R, makes with the horizontal line, in the particular position taken.

The sine of the angle A (Fig. 18) (written *sin* A or *sine* A) is the number obtained by dividing the length of the perpendicular or height H by the length of the hypotenuse (side opposite the right angle) or third side R, in this case the radius of the circle,—*i.e.*,

$$\sin A = \frac{H}{R}$$

This ratio is dependent on the angle itself, not on the individual length of either of its sides.*

The foregoing means, shortly, that the e.m.f. generated at any position of the point p (Figs. 13 and 16) is not directly proportional to the angle which the radius R (joining p to o) makes with the horizontal, *but to the sine of that angle.* And as R is a constant, the e.m.f. is directly proportional to the values of H, *i.e.*

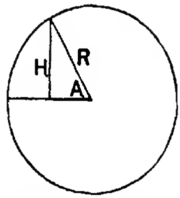

Fig 18.—Sine of an Angle.

$$\text{e.m.f.} \propto \text{sin angle} \propto H \text{ in Fig. 16.}†$$

Thus the various values of H, *i.e.* of the e.m.f., are indicated by the vertical dotted lines in Fig. 17.

6. **VARIATION IN THE SHAPE OF ACTUAL E.M.F. WAVES.**—The curve obtained in Fig. 17 shows the variation in the e.m.f. of a simple alternator, such as that illustrated in Fig. 13, during one revolution of its coil $a\ b\ c\ d$. The e.m.f. is at zero when the plane

* Let $A\ B\ C$ (Fig. 19) be any angle a, of which the sine value is required. Take any point D, in either side, say in $A\ B$, and drop therefrom a perpendicular $D\ E$, to the other side $B\ C$, cutting it at E. Then $B\ D\ E$ will be a right-angled triangle, of which $B\ D$ is the hypotenuse, and $D\ E$ the perpendicular. Now, in such, the ratio $\frac{perpendicular}{hypotenuse}$, *i.e.*, $\frac{D\ E}{D\ B}$, represents the sine value of the angle a. If the angle remain the same (in the present case it is 30°), no matter how long the sides $B\ D$ or $B\ E$ may be, or from which point or side the perpendicular is dropped, the ratio $\frac{perpendicular}{hypotenuse}$ will always be the same. In the present case, for instance, it is $\frac{1}{2}$, *i.e.*, sin 30° = ·5. Sine values may be obtained directly from the Table on page 271.

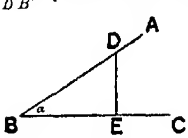

Fig. 19.—Sine of an Angle.

† The sign $\propto$ means *is proportional to.*

of the coil *a b c d* is at right angles with the lines of force of the field, and gradually rises, reaching a maximum when the plane of the coil is parallel with the direction of the field. The field in this case is assumed to be uniform; if it is not so, the simple sine law no longer holds good, and the e.m.f. curve will be more or less altered in form.

In practical alternators, owing to the non-uniformity of the fields, and the various shapes of coils used, the form of the e.m.f. curve may vary considerably from that of the true sine curve. The design of alternators, however, has been brought to such a pitch of perfection, that they may be made to give a true sine wave of e.m.f., or one which differs in shape therefrom, according to the ideas of the designer. The fact of thus being able to obtain variously-shaped waves of e.m.f. within certain limits, is of importance; and one question which naturally arises is:—What is the most efficient form of wave for a given circuit? This is a matter, however, beyond the scope of this book, but it may be stated that at the present day, general opinion is in favour of the simple sine-wave form (§ 24).

The actual drawing of a sine curve by the student in the manner described in § 5, will impress on him the approximate character of the e.m.f. variation in an alternating-current circuit. But it is seldom necessary, or possible, to draw such a curve in practice; since the predetermination of the interaction of the numerous coils and fields in an alternator in this manner, would be an exceedingly laborious task.

Photographs of alternating e.m.f. and current waves can be obtained by means of an *oscillograph*, which is a species of mirror galvanometer that is able to respond with exceeding quickness and exactness to the variations in the values of either e.m.fs. or currents. The

oscillations of the moving part of the apparatus are registered on a moving photographic film. The oscillograph, the best form of which is due to Mr Duddell, cannot be described here; but some photographs (*oscillograms*) of pressure and current curves are given in Figs. 41, 42, and 62.

7. FREQUENCY.—It will be evident, from Figs. 16 and 17, that during the first half-turn of the coil in Fig. 13, the induced e.m.f. increases from zero to a maximum, and afterwards decreases to zero; whilst during the second half-turn, the e.m.f. again rises and falls, but this time in the reverse direction. Hence, if the coil be connected up with an outer circuit, as seen in Fig. 13, in one revolution the induced e.m.f. (and the current due to it) will make two *half waves* or *half cycles*, *i.e.* one *complete wave* or *complete cycle.*

In Fig. 17, the portion of the curve from 0 to 180 is a half cycle, and the portion from 0 to 360 a whole cycle.

The time taken for one complete cycle is known as the *period* of the alternating e.m.f. and current. The *number of complete cycles per second,* which is termed the *frequency* (or sometimes *periodicity*) and is denoted by the symbol f, will depend upon the number of revolutions which the coil makes in that time. Thus, supposing it revolves 600 times in one minute, the frequency of the e.m.f. (and of the current set up) will be 10 cycles per second, or—briefly—10; while the period will be $\frac{1}{10}$th of a second.

Frequency is also denoted by the symbol $\sim$; but it is more convenient—especially in formulæ—to use f.

The frequency of alternating currents, as used for ordinary work in this country, varies from 25 to about 100 cycles per second. Elsewhere, there are systems working at 15 cycles; this low frequency being specially suitable for power transmission and railway work on a

large scale. For special purposes, such as wireless telegraphy, e.m.fs. of very much higher frequency than 100 are employed.

8. **FREQUENCY OF ALTERNATORS.**—It has just been shown that, in the case of a simple coil rotating in a 2-pole field, the frequency is equal to the number of revolutions per second. Practical alternators are, with few exceptions, constructed with multipolar field-magnets, as well as a number of coils: but the frequency may be got by simply multiplying together the revolutions per second and the number of pairs of poles.

Thus calling the frequency f, the number of revolutions per minute R, and the number of pairs of poles p—

$$f=\frac{Rp}{60}, \qquad (1)$$

R being divided by 60 to reduce it to revolutions per second.

Example.—An alternator has 12 pairs of poles (N and S), and runs at 300 revolutions per minute. In one revolution, each coil will pass through 24 fields, half of which are N and half S; and therefore there will be 12 complete reversals or waves of e.m.f. ($\sim$) in each revolution. Consequently, the resulting frequency will be:—

$$\frac{300}{60}\times 12=5\times 12=60.$$

Other examples of the use of this formula are given in § 91 and later.

9. **SELF AND MUTUAL INDUCTION. INDUCTANCE.**—In Fig. 10 it was shown that when a voltage is applied to a *non-inductive* circuit, the current reaches its maximum value immediately, and continues at that value while the voltage and the resistance are kept constant.

Now consider a series circuit (Fig. 20), comprising an

electro-magnet or choking coil C (§ 69) with a large number of turns in its windings, an ammeter A, switch S, battery B, and a voltmeter V shunted across the switch. The voltmeter should be of the moving-coil type, since such indicates both the magnitude and the direction of the voltage.

Let the values of the voltage and the current be represented as distances above the time base in Fig. 21, the voltage curve ($a\,b\,c\,e\,g\,h$) being dotted, and the current curve ($c\,d\,f\,k$) shown in full line. The normal voltage or p.d. at the switch terminals when the latter is open is represented by $o\,a$, this p.d. being the same as that of the battery. The voltmeter will indicate this value when the switch is off.

Suppose, to start with, that S is open, and that after the lapse of a second or so (o-c) the switch is closed; the reading on the voltmeter ($c\,b$) will suddenly drop to zero, and will remain at that value while the switch is closed. The current, ignoring the infinitesimal one passing through the high-resistance voltmeter, may be assumed to be at zero while the switch is open ($o\,c$), and to start at the moment (c) that the switch is closed. The current, however, does not reach its maximum value instantly, but increases gradually, as shown by the full line starting from c. It finally attains a steady value at point d, and remains at that value as long (d-f) as the circuit conditions are unaltered. The ammeter A in Fig. 20 will show the growth of the current and the steady value arrived at.

The explanation of the above is as follows: When a continuous current begins to flow along a conductor, it sets up a magnetic field around it, *i.e.*, there is an *increase* (from nothing in this case) in the number of lines of force surrounding the conductor, especially in that part of it which is wound on the iron core.

Now, according to the laws dealing with electromagnetic induction,* whenever the current in a circuit is varied, there is also a change in the number of lines of force passing through or interlinked with that circuit; and this change induces an e.m.f., the magnitude of which depends upon the rate at which the lines of force are changed. Further, the direction of this induced e.m.f. is always such as to tend to prevent any

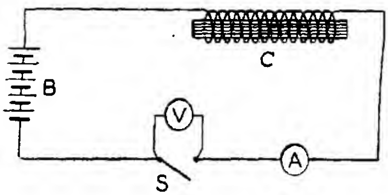

Fig. 20.—Inductance in a Continuous-Current Circuit.

alteration in the value of the current. This means that when the current increases from *c* to *d* in Fig. 21, an e.m.f. will be induced in the coil *C* (Fig. 20) in the opposite direction to that in which the current is flowing, and so prevents the current immediately reaching its maximum value. This effect will be shown by the ammeter. Between *d* and *f*, the current is constant, and so also must be the number of magnetic lines linked with the circuit: consequently, there will then be no e.m.f. induced in the coil.

Next, let us suppose that, at the instant *e* (Fig. 21) the switch S (Fig. 20) is opened. The current will then decrease very rapidly, and the magnetic field about *C* will collapse. This will be equivalent to a very rapid change in the number of magnetic lines in the circuit,

* *See* the Author's *Electric Lighting and Power Distribution*, Vol. I., Seventh Edition.

and a very high e.m.f. will consequently be induced in C, as indicated by $e\ g$ on the voltage curve. This e.m.f. will be in the same direction as that of the battery, and so will tend to maintain the current: and it will cause a small arc or spark to be formed at the separating contacts of the switch while the latter is being opened. But the arc is extinguished when the current becomes zero, *i.e.*, when the circuit is quite broken, as at k; and

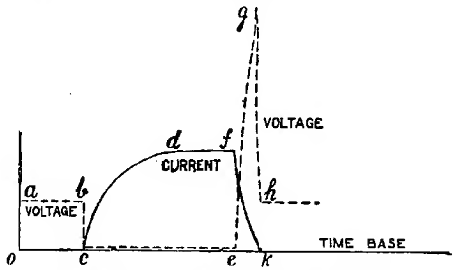

Fig. 21.—Variation of Voltage and Current in an Inductive Circuit.

the voltage then falls to its original or open-circuit value, as at h. The momentary high voltage ($e\ g$) will be indicated on the voltmeter (Fig. 20) by a sudden "kick" of its needle towards the high part of its scale.

Circuits which give rise to the above phenomena, namely, of always tending to prevent any increase or decrease in the value of the current, are said to possess *self-induction*, or to be *inductive*. As will be seen later, the self-induction of any given circuit is not necessarily a fixed quantity.

The value or magnitude of the self-induction observed when the current varies at the rate of one ampere per second, is known as the *co-efficient of self-induction*, or as the *inductance*: and it is expressed as so many *henries* (§ 28). The relation between these

terms—(self-induction and co-efficient of self-induction or inductance)—may be more clearly understood by considering the case of the increase of volume of metals under the application of heat. This phenomenon is known as *expansion*, just as *self-induction* is the name given to the phenomena observed in the experiment just described. In order to compare the increase of volume of different materials under similar conditions, the term *co-efficient of cubical expansion* is used, this being the increase of volume of 1 c.c. when the temperature is increased 1° C. Likewise, the terms *co-efficient of self-induction* or *inductance* are introduced to compare the self-induction of different circuits under similar conditions, viz., when the current is varied at the rate of one ampere per second. The term "inductance," on account of its brevity, is adopted throughout this book, and is more fully dealt with in § 28.

It is on account of the sudden increase in the voltage (noted overleaf) when opening an electromagnetic circuit, that it is dangerous suddenly to open the shunt circuit of a continuous-current machine. The extra voltage so induced, which may be several times the normal voltage, might be sufficient to break down the insulation on the field coils.

If we start with a given length of current-carrying circuit conductor, and if that conductor be stretched out straight, its whole length will be surrounded by circular concentric lines of force, whose number (per given length of conductor) will depend on the strength of the current. If several yards of this conductor be then coiled up into a solenoid, the lines of force in that portion will act together as if most of them passed right through the solenoid. If an iron core be introduced, the number of lines of force is greatly increased, the paths of the lines being roughly as shown in Fig. 22.

In this figure practically *all* the lines set up link through *all* the turns of the coil; consequently, any variation in the current gives a large variation in the number of lines through *each* turn, and a correspondingly large e.m.f. is induced in *each* turn. And as all these e.m.fs. are in series, the total e.m.f. induced is considerable, as has been seen.

In short, the self-induction of a given length of circuit wound on an iron core and carrying a given

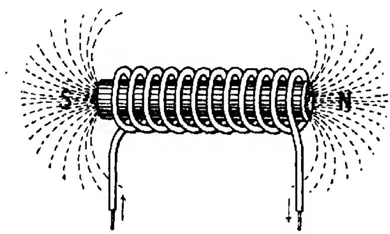

Fig. 22 —Distribution of the Lines of Force of an Electromagnet.

current, is tremendously greater than it would be were that part of the circuit stretched out straight.

Mutual induction is produced when the lines of force due to a current flowing in one circuit cut a neighbouring circuit. Thus, suppose two coils C and K are placed close together in line as shown in Fig. 23; and that C is connected-up to a battery, key, and galvanometer G', while K is joined-up to another galvanometer G. The two galvanometers must be out of reach of the direct magnetic influence of the coils.

When the key is depressed, lines of force due to the current in C cut K, and induce a momentary reverse current therein, as will be shown by G. On opening the battery circuit, the lines of force due to C collapse,

i.e., there is another change in the number of lines of force passing through K; consequently, an e.m.f. will be induced in the latter, but in the reverse direction to that induced on depressing the key; and this e.m.f. will be indicated by the deflection of G being in the opposite direction.

If the battery and the key were inserted in series with K and G, instead of as shown in Fig. 23, the setting-up and the stopping of the current through K would cause momentary currents in C; and these currents would be indicated by G'.

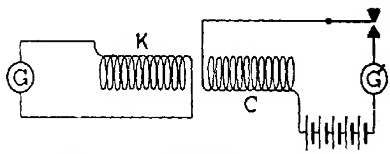

Fig. 23.—Circuits possessing Mutual Induction.

The two circuits C and K, therefore, possess mutual induction; that is, a current through either induces an e.m.f. in the other The value of the mutual induction between two coils is expressed as the *co-efficient of mutual induction* of the coils, and can be denoted in henries.

Although the action of transformers (Chap. V.) and of induction motors (Chap. VI.) depend upon mutual induction, it will not be necessary in this book to mention either the term or its co-efficient when dealing with these apparatus.

10. **INDUCTANCE IN A CIRCUIT.**—From what was said in the preceding section, it should be clear that it is impossible suddenly to start a current at its full value in an inductive circuit, and equally impossible to stop it suddenly. Because of the effects of inductance, the current takes time to grow and time to

die away. It is thus even more out of the question suddenly to *reverse* a current in such a circuit.

Although it is possible to arrange a simple circuit or to wind a coil so that it shall have little or no inductance, (as shown in Fig. 24, where each half of the circuit or coil neutralizes the other's magnetic effect), the conductor may have appreciable capacity;* and if so, this will also exercise a disturbing effect on the current. Moreover, a coil such as that shown in Fig. 24 would be useless for solenoids or electromagnets, as it would have no magnetic field.

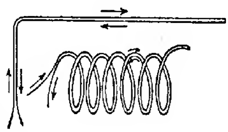

Fig. 24.—Non-Inductive Winding.

It therefore follows that every working circuit containing motors, arc lamps, and other electromagnetic apparatus exercises more or less disturbing effect; and also that, in the case of an alternating current, this disturbing effect is continuous, and is the greater the higher the frequency.

11. **INDUCTANCE IN AN ALTERNATING-CURRENT CIRCUIT.**—The effect of inductance in an alternating-current circuit is to cut down the current. The following experiment conclusively proves this.

The circuit L S B (Fig. 25) is fed at a virtual † alternating pressure of, say, 200 volts, from the mains M. B is a laminated iron core, built up of thin wires; and on this are coiled several turns of thick copper wire of negligible resistance, which may be short-circuited by the switch S. B obviously possesses con-

* *See* the Author's *First Book of Electricity and Magnetism*, Fourth Edition; or his *Electric Lighting and Power Distribution*, Vol. I, Seventh Edition.

† *See* § 23.

siderable inductance, whereas the rest of the circuit has very little. The light given by L depends upon the strength of the current passing through it, and is a convenient indicator of it. A carbon-filament lamp is more satisfactory than a metal-filament one for this experiment, because it is more susceptible to changes in the current strength.

Suppose that the lamp is fully lighted when S is closed so as to cut out B; then when B is put in circuit by opening the switch S, the lamp will burn dimly, or perhaps show no light at all, proving that the effect of

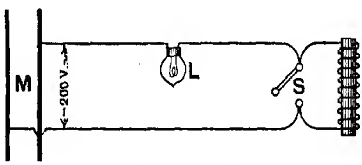

Fig. 25.—Experiment on Effect of Inductance.

inductance is to reduce the current permanently. This effect is the same as if a back e.m.f. had been introduced into the circuit, which, in fact, is the case, the back or counteracting e.m.f. being that due to the inductance of B. B is then acting as a choking coil (§ 69).

If a continuous current be used, the insertion or cutting out of B will make no appreciable difference, as its resistance is small; except, perhaps, a faint flicker of the lamp at the moment of closing or opening S; but this would be hardly noticeable.

Now suppose the circuit in Fig. 25 is once more fed with alternating current, and that the switch S is left open. If the iron core of B be gradually pulled out, the lamp L will at the same time gradually increase in brightness. When the core of B has been quite withdrawn however, L will still not burn quite as

brightly as when S is closed, showing that the solenoid without its core possesses some inductance. If the core be now gradually replaced, the lamp will be gradually "dimmed" once more.

The last two experiments show that a hollow solenoid has some inductance, and that the latter is increased by putting more and more iron inside it.

12. **ALTERNATING-CURRENT WIRES OR CABLES IN METAL CONDUITS.**—In alternating-current work care must be taken not to run single conductors (or

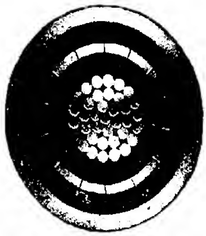

Fig. 26.—Two-Core Concentric Cable for Pressures up to 2200 volts. (*Helsby Cables.*)

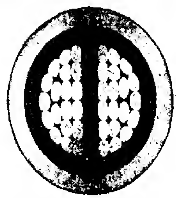

Fig. 27.—Two-Core Cable for Pressures up to 660 volts. (*Henley's.*)

conductors connected with one pole of the system only) through a metal pipe or tube for any distance. The effect of so doing, if the pipe were of iron or steel, would be to increase the inductance of the circuit and set up *eddy currents* in the pipe; and this inductance would naturally result in a considerable drop of pressure. In other words, the conductor (or conductors) and the pipe would act like a sort of elongated choking coil (§ 69). If the tube were of non-magnetic metal, there would be very little inductance, but eddy currents would still be induced in the conduit, tending to heat it and waste energy. (§ 74.)

With a "two-core" cable, that is to say, a cable

containing two conductors, disposed concentrically or otherwise (Figs. 26 and 27), and when these conductors carry a current to and from a certain point, there can be only a small inductive or eddy-current action, as the field of the current in one conductor practically neutralizes the field of the current in the other conductor. Such effects are also very small when a three-core cable (Fig. 28) carries a three-phase

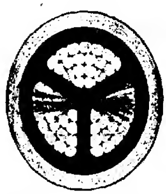

Fig. 28. — Three-Core Cable for Pressures up to 660 volts. (*Henley's.*)

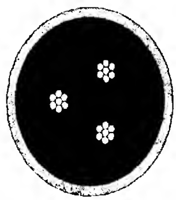

Fig. 28A. — Three-Core High-Tension Cable for Pressures up to 20,000 volts. (*Helsby Cables.*)

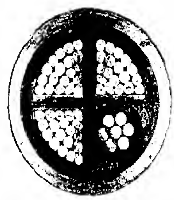

Fig. 28B. — Four-Core Cable for Four-Wire Three-Phase System and for Pressures up to 660 volts. (*Henley's.*)

current; for, as shown in § 47, the algebraic sum of the currents, and therefore of their magnetic fields, at every instant is zero. A three-core concentric cable (which resembles Fig. 26 except that it has a third conductor) is practically never used in alternating-current work, as the disposition of the conductors prevents the currents in them from entirely neutralizing each other's inductive effects.

13. **CAPACITY IN ALTERNATING-CURRENT CIRCUITS.** — One very important difference between the action of continuous and alternating currents is shown by the experiments illustrated in Figs. 29 and 30. Here two circuits are depicted, each containing a source of e.m.f., a glow lamp L, and two condensers

C, *C*; but in the first the e.m.f. is due to a dynamo *D*, and in the second to an alternator *A*.

In Fig. 29 it is clear that no current can flow through the lamp, even if one of the condensers be removed, for each of them interposes a break in the continuity of the circuit.

In Fig. 30, on the other hand, if the condensers are suitable in capacity, the lamp *L* will light up. At first sight this result may seem most inexplicable; but when we consider the action of the condenser,* and the fact that the alternator is keeping up a constant surging of electricity backwards and forwards between the plates *a* and *b*, it becomes evident that there must also be a corresponding alternating flow of electricity in the lamp circuit, between the plates *c* and *d*. The results would be precisely the same if one condenser only were employed in each experiment; but the use of two makes the effect in the latter case (Fig. 30) all the more remarkable.

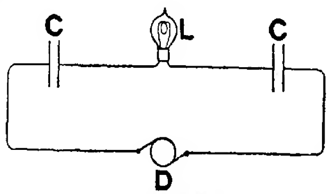

Fig. 29.—Continuous-Current Circuit with Condensers.

It will be noticed that in the experiments just described, the capacity is in series with the circuit, *i.e.*, there is no complete conducting path. This state of things effectually prevents the flow of a continuous current, but does not stop the "action" of an alternating one.

A fuller explanation of the second experiment (Fig.

* *See* the Author's *First Book of Electricity and Magnetism*, Fourth Edition.

30) is as follows. Before the alternator is working, the whole circuit may be assumed to be filled with electricity evenly distributed, and at zero potential or pressure. Now suppose the alternator to work. During the first half cycle, *i.e.*, while its e.m.f. is in one direction (§ 7), it pumps electricity through itself from *a* to *b*, causing a p.d. between *a* and *b* about equal to its own e.m.f.: *b* is consequently +ly. electrified and *a* –ly. electrified, as indicated by the signs + and –. Influence (electrostatic induction) now takes place across the condenser dielectrics, causing a rush of electricity through the lamp from right to left, so that *c* is + and *d* –. During the second half cycle, that is when the reversal of the alternator e.m.f. occurs, electricity is pumped from *b* to *a* through the alternator, so that *a* becomes + and *b* –. A rush consequently takes place at the same time from *c* to *d*, *c* becoming – and *d* +; and so on. Thus the alternating flow of electricity in the alternator circuit causes a corresponding alternating flow in the lamp circuit.

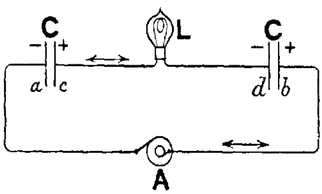

Fig. 30.—Alternating-Current Circuit with Condensers.

It has been stated that the same results would have been obtained with one condenser only in circuit. This will be understood from what follows. In Fig. 31, *A* is an alternator, with two wires joined to its terminals; one of the wires being severed, and a lamp, *L*, inserted. The ends of the wires approach very closely, as at *a* and *b*, but are not in contact, a sheet of glass or other

dielectric, *d*, being interposed to prevent sparking across. The alternator circuit is consequently not metallically complete. Now the ends of the wires *a* and *b*, and the dielectric *d*, virtually form a condenser of extremely small capacity; and the alternator pumps electricity backwards and forwards, between *a* and *b*. But in this case very little electricity passes at each reversal of the e.m.f., owing to the small capacity of the ends of the circuit; and an ordinary lamp will consequently show no indication of a current. The wires are supposed to be suspended in mid air, and not running side by side or near other bodies, as we wish to consider the circuit as only having appreciable capacity at its ends.

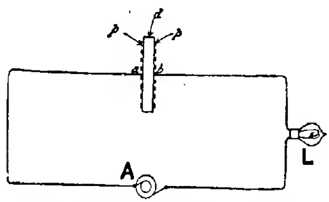

Fig. 31.—Alternating-Current Circuit with Condensers.

When the alternator is pumping in one direction, say from *a* to *b*, a quantity of electricity will pass sufficient to make the p.d. between *a* and *b* equal to the e.m.f. of the alternator; or, in other words, the condenser *a b* will be charged to the potential of the alternator. Now, the smaller the capacity of a condenser, the less is the displacement of electricity necessary to raise the p.d. between its coatings to a given amount. In the present case, because of the extremely small capacity of the ends of the circuit, only a very minute quantity of electricity will pass from *a* to *b*. When the alternator reverses its e.m.f., another small quantity of electricity will be pumped

from b to a. Thus every cycle of the e.m.f. will set up a to-and-fro current.

By putting metal plates on each side of the dielectric, d, as shown by the dotted lines p, p, the capacity of the adjacent ends of the circuit (*i.e.*, of the condenser) will be greatly increased, and a much greater quantity of electricity will pass to and fro through the lamp. But the current will still be insufficient to light it with a

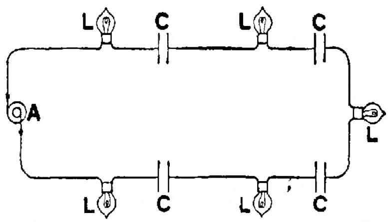

Fig. 32.—Alternating-Current Circuit with Condensers.

simple two-plate condenser such as this, unless of very unwieldy dimensions, or unless an enormously high e.m.f. be employed. It will be seen, however, that by using a multiple-plate condenser of sufficient capacity, an ordinary e.m.f. will cause enough electricity to pass to and fro to light a lamp, or, if need be, a number of lamps.

It has been explained how what is practically an alternating current can be kept up all round the circuit, even if one or two condensers be inserted therein (Figs. 30 and 31); and the reader should now be able to understand that the fanciful arrangement of things depicted in Fig. 32 is possible; any number of lamps, L, and condensers, C, being joined consecutively in the circuit of an alternator, A. The lamps will burn brilliantly if the condensers are of sufficient capacity, and the e.m.f. high enough.

When one or more condensers are inserted in an alternating-current circuit, there is a considerable drop of voltage in each. It has therefore been proposed to use them instead of transformers for reducing the pressure on lamp circuits in very small installations, to enable low-voltage low-candle-power metal-filament lamps to be used. It is claimed that the condenser provides a more economical means of doing this than a transformer. As far as is known, however, this system has made little headway.

As will be presently explained, every electric circuit possesses more or less capacity, owing to the proximity of the conductors to each other and to the Earth.

14. **INDUCTANCE, CAPACITY, AND RESISTANCE IN A CONTINUOUS-CURRENT CIRCUIT.**—In continuous-current work it is generally sufficient to liken a current to a steady flow of water through a pipe, the rate of flow representing current, the pressure on the water—e.m.f., and the resistance of the pipe—resistance in the electrical circuit. But here there is no good analogy for inductance, or for capacity; which two quantities are nearly always present in an alternating-current circuit. Consequently, some other help is necessary to enable us to picture in our minds the phenomena of an alternating current, and to compare them with those of a continuous or direct current.

In a course of lectures delivered at the Royal Institution in 1895, Professor George Forbes, F.R.S., employed various mechanical analogies to illustrate electrical phenomena. Somewhat similar but considerably-extended analogies will herein constitute the "other help" mentioned above.*

* The teacher or student should not be content with merely explaining or reading through the account of the following experiments. The contrivances depicted in Figs. 33, 34, etc., should be actually made and experimented with.

In Fig. 33, *R* is a short length of rigid rod, such as steel, mounted between steel centres *s, s,* on a stout iron frame *F*, so that it is quite free to turn thereon. *F* may be held in a bench-vice, or it may be secured to a bench by screws passing through its base, which should be at least one inch in width.

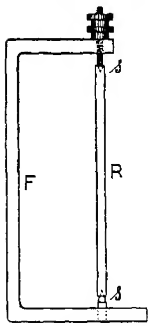

Fig. 33.—Mechanical Analogy for a Circuit without Resistance, Inductance, or Capacity.

Twist the top of the rod with the fingers and thumb of the right hand continuously round in one direction, and assume the twisting force applied to correspond with the e.m.f. in the electric circuit, and the rotation of *R* to represent the current.

R then corresponds with an electric circuit in which there is practically no resistance, inductance, or capacity; for it may be set rotating and kept rotating without appreciable effort, *i.e.*, the current may be started at once, and kept up with a very small expenditure of energy. If there were absolutely no friction at *s*, *s*, and no air-friction, *R*—when once set in motion—would keep in rotation for an indefinite time. Similarly, if we could get an electric circuit with absolutely no resistance of any kind (which would be impossible), a current—when once started—would keep flowing after the e.m.f. had ceased.

In the electrical case, the current is usually stopped by breaking the circuit, since this suddenly introduces a great resistance therein. In the mechanical case, the friction set up by slightly pinching *R* between the fingers is sufficient to bring the latter immediately to rest.

In Fig. 34, *R* is a similar rod to that in the previous figure. It has a stiff paper vane *V* fastened to it, and is placed in the frame *F* (Fig. 33) instead of the plain rod. The effect of *V* is to oppose continuous air resistance to the rotation of *R*, although it does not appreciably retard the setting-up or stopping of that rotation. This air resistance may be compared with electrical resistance, and the arrangement then corresponds with a circuit in which there is appreciable resistance, but practically no inductance or capacity. If the same twisting force be applied as in Fig. 33, the rotation of the rod will not be so rapid; in other words, with a given e.m.f., the current is less the greater the resistance.

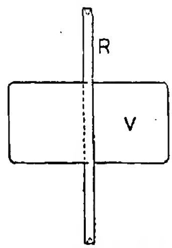

Fig. 34.—Mechanical Analogy for a Circuit with Resistance only.

In Fig. 35, *R* is another steel rod, with a disk of lead *L*, tightly mounted on it. There is also an indicating finger or pointer *p* attached to *R*. The whole is put in the frame *F* (Fig. 33). Now, the air offers little or no resistance to the turning of *L*, on account of its shape; but the latter, because of its inertia,* opposes considerable momentary resistance to the setting-up of motion in *R*; and it also tends to prevent the sudden stopping of *R*. This mechanical inertia is comparable with the inductance (sometimes called *electromagnetic inertia*) in the electric circuit, the effect of which is to

* *Inertia* is that property of a body in virtue of which it resists being set in motion, having its motion changed, or being stopped when in motion. The inertia of a body depends upon its weight (or, more strictly, its mass), and also upon its shape. Force is necessary to overcome inertia, for it requires considerable force to set a heavy body (a flywheel, for instance) in motion, and also considerable force to stop it. When a body is in motion, it is said to have *momentum*.

oppose momentarily the starting, changing, or stopping of a current (§§ 9, 10). Fig. 35 thus presents the mechanical analogy of a circuit with inductance only.

If, in a circuit with inductance, the e.m.f. be removed without breaking the circuit, the inductive e.m.f. and current will subside quietly. If the circuit be broken by means of a switch (which is equivalent to suddenly introducing an enormous resistance therein), the stored-up energy in the magnetic field (§ 41) will be sufficient to set up a momentary spark across the air gap at the switch, as already explained in § 9. This is equivalent to trying to bring R to rest while it is rotating under the influence of L. It will be found impossible to do this suddenly, however tightly the rod R be pinched between the finger and thumb. The great friction then set up is the equivalent of the high-resistance gap which has to be introduced to break the electrical circuit.

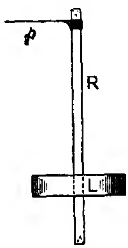

Fig. 35.—Mechanical Analogy for a Circuit with Inductance only.

The rod R, vane V, and lead disk L in Fig. 36, represent a circuit possessing both resistance and inductance. This combination, when placed in the frame F (Fig. 33), requires a greater effort to set R rotating than in Fig. 35, but a smaller one to stop the rotation, since the resistance of V is always such as *to tend to prevent rotation. Hence, in an inductive circuit containing resistance, the tendency of the current to **persist** or keep on is the less the greater the resistance.*

In the above examples, e.m.f. has been likened to a twisting or rotating force; current to the speed of

rotation; electrical resistance to air friction, the friction at the pivots, and that due to pinching the wire; and inductance to inertia. We must now get something to represent capacity.

In Fig. 37, there is the same frame *F* and steel rod *R* as in Fig. 33. In addition, there is a helical steel spring *S*, and a pointer *p*. One end of *S* is attached to the frame and the other to *R*; and *p* serves to indicate the movement of the latter. The frame *F* is supposed to be screwed to a bench, or clamped in a vice.

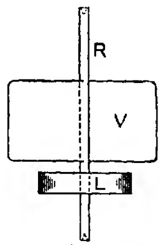

Fig. 36.—Mechanical Analogy for a Circuit with Inductance and Resistance.

It will now be shown that this arrangement is typical of a condenser. If a continuous or direct e.m.f. be applied to a condenser, there will be a momentary current due to the rush of electricity into one of its coatings (or set of coatings) and out of the other; the amount of which will depend upon its capacity: and the displaced electricity will represent the charge in the condenser. If the e.m.f. be removed, and the condenser left insulated, it will retain its charge: but when the condenser terminals are connected by a conductor, it will discharge itself, there being a sudden rush of electricity (momentary current) in the opposite direction.

Turning now to our mechanical analogy (Fig. 37), on applying a twisting force (e.m.f.) to the top of *R*, there will be a certain rotation of *p* (current) until the force with which *S* tends to untwist equals the twisting force. The amount of twist (charge) that can be put upon *R* depends on the flexibility of *S* (capacity), and on the twisting force (charging e.m.f.) applied. When

R has been twisted as much as possible, let its top be fixed (insulated) by means of a clamp; R and S will then represent an insulated charged condenser. Now release the clamp, and R and S will fly round, as indicated by p; this being equivalent to discharging the condenser, the momentary movement of p representing the momentary current of discharge.

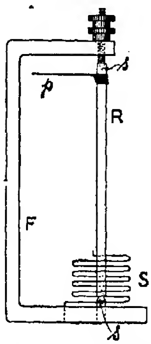

Fig. 37. — Mechanical Analogy for a Circuit with Capacity only.

The discharge of a condenser, after it has been charged by a steady pressure, is really oscillatory in nature; and the mechanical analogy fits the actual conditions, for, on release, p flies round a little past its normal position of rest, then returns, then goes back, thus making a certain number of diminishing oscillations before it comes to rest.

It will be noticed that the pointer p (Fig. 37) always tends to return to its original position, whereas in Fig. 35, the effect of inertia was to cause L to continue its rotation. Thus the two effects, capacity and inductance, are directly opposite in nature.

Fig. 38 illustrates a circuit with capacity (due to the spring S) and resistance (due to the vane V), but practically no inductance. Now begin to twist the top of the rod R. It will be found that a greater force (e.m.f.) will be required to rotate p at a certain speed (current) than was necessary in Fig. 37. This is due to the resistance of V. When the top of R is released, the pointer will return to its original position, but the time taken will be longer than in the previous case, this again being due to the resistance offered by the air to

the vane V. This is equivalent to charging and discharging a condenser through a resistance, and shows that the time taken to do either is greater the greater the resistance in the condenser circuit.

Next consider the case of a circuit with inductance (due to the lead disk L) and capacity (due to S), as

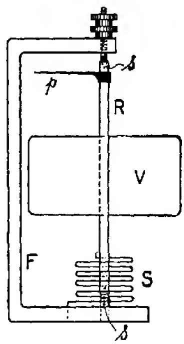

Fig. 38.—Mechanical Analogy for a Circuit with Resistance and Capacity.

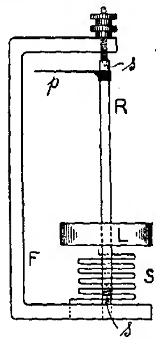

Fig. 39. — Mechanical Analogy for a Circuit with Inductance and Capacity.

shown in Fig. 39. As before, apply a twisting force to R. Both L and S will tend to resist the motion of p —owing to the inertia of the former, and the elasticity of the latter. If, after rotating p through a certain angle, the twisting force be withdrawn, it will be found that L tends to rotate p still further, whereas S is endeavouring to reverse the rotation, *i.e.*, the two effects are acting in opposition. The pointer p will consequently come to rest for a moment. At the instant that it does so, the force with which S is endeavouring to reverse the rotation will be at its maximum, whilst the momentum will be zero, hence the pointer p will

start moving backwards. When it has reached its original position, S will have untwisted, but the mass L will then be moving at a good speed, and because of its inertia, will cause p to swing still further, until it is brought to rest again by the action of S, L and p thereupon starting back again. This to-and-fro or oscillating motion would go on indefinitely if it were not for the friction at the pivots, of the air, and in the material of the spring; causing the swing of the pointer to decrease gradually, and bringing it eventually to rest in its original position.

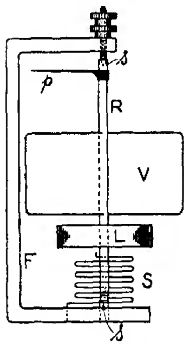

Fig. 40.—Mechanical Analogy for a Circuit with Resistance, Inductance, and Capacity.

The above experiment represents the discharge of a condenser (S) in a circuit possessing large inductance (L) and very little resistance (air friction). The condenser is alternately discharged and recharged in the reverse direction until the electrical energy has been dissipated in the form of heat in whatever resistance is present in the circuit, it being impossible to get a circuit entirely without resistance. It should be noted that the motion of p described above was caused by only one application of the external twisting force (initial charging of a condenser).

Fig. 40 shows a circuit possessing capacity (due to S), inductance (L), and resistance (V). On twisting the top of the rod R and then withdrawing the twisting force, it will be found that the effect is somewhat similar to that in Fig. 39. But the swinging of p will be appreciably "damped"; *i.e.*, instead of continuing

to oscillate from one side to another for a comparatively long time, the pointer will oscillate or swing to and fro very few times, the oscillations rapidly decreasing in amplitude, and the pointer soon coming to rest. This is equivalent to discharging a condenser through a circuit possessing inductance and a large resistance;

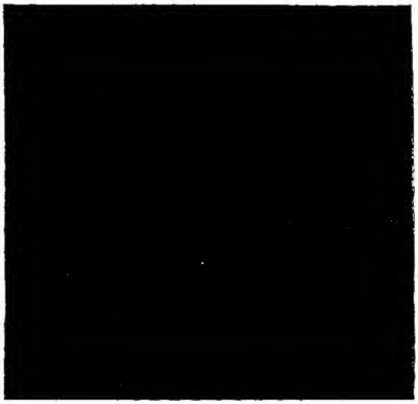

Fig. 41.—Oscillogram showing the Discharge of a Condenser.
(*Cambridge Scientific Inst. Co.*)

and clearly shows that the greater the resistance the less will be the number of electrical oscillations, since the electrical energy is quickly absorbed and converted into heat by the resistance.

15. **EFFECT OF INDUCTANCE AND RESISTANCE ON THE DISCHARGE OF A CONDENSER.**—The contrivances in Figs. 39 and 40 served to illustrate (mechanically) the facts that the oscillatory discharge of a condenser in a circuit with much inductance and

little resistance is comparatively prolonged; and that the addition of resistance reduces the oscillations and the time of discharge.

These statements are proved by the oscillogram curves in Figs. 41 and 42. These were obtained by means of an oscillograph (§ 6).

The first figure shows the variation of the condenser discharge current when it took place in a circuit with considerable inductance and very little resistance. Allowing for the fact that the illustration does not quite show the finish of the discharge, we may assume that it consisted of about ten oscillations. In Fig. 42 the number of oscillations is only about half, this being the result of adding considerable resistance to the previous circuit and again discharging the condenser.

Fig. 42.—Oscillogram showing the Discharge of a Condenser. (*Cambridge Scientific Inst. Co.*).

The signals in wireless telegraphy are made up of discharges like that in Fig. 41, each signal consisting of a large number of such discharges. Thus if the signalling key be kept down for one second for a "dash," in that time from 200 to 300 discharges will occur.

16. **INDUCTANCE AND RESISTANCE IN AN ALTERNATING-CURRENT CIRCUIT.**—In the preceding § 14 mechanical illustrations of the phenomena of the con-

tinuous-current circuit were given. We will now apply similar analogies to the alternating-current circuit, with the help of the same mechanical devices.

In Fig. 43, *R* is a piece of steel rod, mounted on centres *s*, *s*, in a frame *F*, as already described on p. 50. Twist *R* rapidly to and fro, giving a turn first in one direction and then in the other. This represents the application or "impression" of an alternating e.m.f. to or on the circuit. As *R* is supposed to have no inertia or flexibility, and as it is presumed that little resistance is opposed to its rotation, it may be taken to represent an imaginary circuit with no inductance or capacity, and low resistance. The direction of twist (e.m.f.) and rotation (current) may be changed immediately, and one might almost say that the rate of rotation (strength of current) was uniform, though rapidly alternating in direction. This case may, therefore, be taken as analogous to the alternating current represented by the graph or "curve" in Fig. 10.

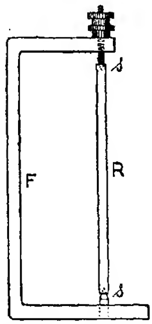

Fig. 43. — Mechanical Analogy for a Circuit without Resistance, Inductance, or Capacity.

In Fig. 44 the stiff paper vane *V* represents resistance in the electrical circuit; but it must be supposed not to possess any inertia. Then, if the same alternating twisting force (e.m.f.) be applied to the rod as before (Fig. 43), the rate of rotation (strength of current) will be less than in the first instance, in consequence of *V*; but there being no inertia, as we suppose, the rotation (current) will change directly the twisting force changes, and will always be at the same rate for a given force. Here we have the representation of an

alternating-current circuit, in which there is practically only resistance, and no appreciable inductance or capacity.

In Fig. 45 is shown the mechanical analogy of a circuit with inductance only (due to L). Apply an alternating twisting force (e.m.f.) to R. It will be found that the maximum force will have to be applied at the

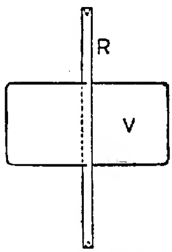

Fig. 44.—Mechanical Analogy for a Circuit with Resistance only.

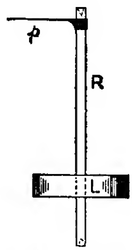

Fig. 45.—Mechanical Analogy for a Circuit with Inductance only.

start, and that less force will be necessary as L gets into motion. To stop L and reverse its direction of rotation (current), the twisting force must be applied in the opposite direction. Thus the pointer p can be kept oscillating (alternating current) by applying a twisting force to oppose its outward swing on each side, and continuing that force until p is well in motion on the return swing.

The relation between the alternating twisting force (alternating e.m.f.) and the motion of p (alternating current) can be shown by means of the graph in Fig. 46. It is assumed that the disk L makes less than a complete revolution in either direction, and that a certain position of p corresponds with the zero or

reversing point of the force (e.m.f.). This position may conveniently be that when p is pointing directly to the frame F (Fig. 43) in which R L and p (Fig. 45) are mounted.

Let the horizontal line in Fig. 46 represent the time base (as in Fig. 17), and let the distances above and below this line represent either the alternating twisting

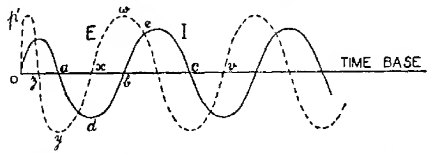

Fig. 46.—Curves showing Relation between Alternating Twisting Force (E.M.F.) and Speed (Current) with Inertia (Inductance) only in Circuit.

force (alternating e.m.f.) applied at R as in curve E, or the *angular velocity* * of p (alternating current) as in curve I.

To begin with, a rather sharp twist has to be given to R to start L, as shown by the peak p' at the beginning of E. The force necessary then decreases to zero at z, and is reversed (z to y) in order to bring L to rest. This instant corresponds to time a in the figure. Since the twisting force is still maintained (y to x), L begins moving in the opposite direction, as indicated by the continuation of curve I below the time base past the reversing point a. The twisting (or electromotive) force E, having reached its negative maximum

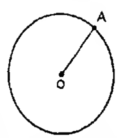

Fig. 47.—Angular Velocity.

* Angular Velocity.—If a line O A (Fig. 47) rotates about a centre O, the point A will describe a circle. The rate at which A moves along the circumference of that circle is called the *angular velocity* of O A, and depends on the angle traversed by O A in one second.

at y, is gradually decreased until, when the negative movement of the pointer is maximum at d, the force is zero at x. While the force was reduced from y to x, the speed of L or p (current) has been continually increasing (from a to d), so that its momentum will tend to carry it on; thus the twisting force at R will again have to be reversed (after x) so as to retard and reverse the motion of L, until at the end of time $o\ b$, E will be maximum at w, and I zero at b. Consequently, the pointer begins to swing backwards again, as from b to e.

These reversals and speed variations (alternating current) continue to take place as long as the alternating twisting force (alternating e.m.f.) is maintained.

The waves of e.m.f. and current in Fig. 46 differ in shape at the start from what they are later, it being remembered that the point O represents the *beginning* of the setting-up of the alternating current in the inductive circuit.

It is important to notice that the zero and the maximum values of the motion or current curve I occur a quarter of a cycle after those of the twisting-force or e.m.f. curve E. For example, the force is at its maximum at y, and the current at its maximum at d; the force is zero at x and the current is zero at b: the intervals y-d and x-b being a quarter-cycle apart. A whole cycle would be from a to c or from x to v. Hence, *if an alternating-current circuit contained inductance only, the current would "lag" a quarter of a cycle behind the e.m.f. applied to the circuit. In other words, the two would be "in quadrature."* (*See* Appendix A.)

The above statement may be made clearer by means of Fig. 48, which is a plan of Fig. 45. R is the rod that is being subjected to an alternating twist, p is the pointer attached thereto, and L is the disk of lead.

R F represents the zero position, while *R B* and *R A* are the extreme positions between which the pointer swings.

Let us consider the variations in the twisting force (e.m.f.) applied to *R*, and in the speed of *p* (current), during the swing from *R B* to *R A*; and let the particular time considered be the half cycle during which the e.m.f. passes through *y x w* and the current through *a d b* in Fig. 46.

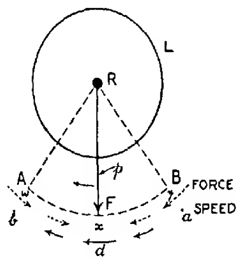

Fig. 48.—Variation of Twisting Force and Speed during one Swing of *p* in Fig. 45.

The pointer is stationary for a moment in the position *R B*, and the twisting force is at its maximum acting inwards. This condition corresponds to points *a* and *y* respectively in Fig. 46. The pointer then starts moving towards the centre position *R F*, its speed (current) increasing as shown at *a d* (Fig. 46), while the force decreases and is zero at *x* when the pointer arrives at *R F*, which corresponds with *d* in Fig. 46. The twist (e.m.f.) is then reversed (*x w*), and attains its maximum when *p* has reached the position *R A*, corresponding with points *b* and *w*.

The variations in magnitude and in direction of the twisting force (e.m.f.) during this half oscillation (half cycle) are shown in Fig. 48 by the length and the direction of the dotted arrows. The corresponding values of the speed (current) are represented by the lengths of the full-line arrows, which all point in the same direction, since the current does not reverse during

the time considered. The dots *a* and *b* in Fig. 48 are meant to signify that there is no movement or current at these points.

It should now be evident that the maximum value of the twisting force (maximum e.m.f.) occurs at *B*, while the maximum speed (maximum current) in the same direction occurs at *F*, a quarter of a period later. In other words, the current lags 90° behind the applied voltage.

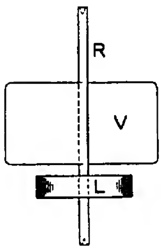

Fig. 49.—Mechanical Analogy for a Circuit with Inductance and Resistance

Fig. 49 illustrates a circuit with inductance (*L*) and resistance (*V*). On applying an alternating twisting force in this case, we get a result somewhat similar to that considered in the previous section, except that the action of the vane *V* (resistance) renders it more difficult to accelerate the mass *L*, and assists to bring that mass to rest. In fact, *V* retards or reduces the motion in either direction. The result is that the motion or current curve *I* (Fig. 46) will still lag behind the force curve *E*, but the amount of the lag will be smaller than in the previous case; that is to say, the force and current maxima or zeros will be something less than a quarter-cycle apart, as there is resistance as well as inductance present.

17. **INDUCTANCE, CAPACITY, AND RESISTANCE IN AN ALTERNATING-CURRENT CIRCUIT.**—In Fig. 50, the spring *S* is introduced to represent capacity in the circuit. On applying an alternating twist to *R*, the elasticity of *S* continually tends to cause the pointer *p* to return to its original or zero position, *i.e.*, it assists to bring *p* back to that position, and opposes any attempt to cause *p* to rotate outwards in either

direction. This is equivalent to saying that when an alternating e.m.f. is applied to a circuit containing capacity only, the effect of the latter is always to assist in reversing the current.

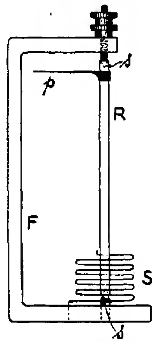

Fig. 50. — Mechanical Analogy for a Circuit with Capacity only.

This effect is shown very clearly by the diagram in Fig. 51. At the instant of starting, a small twisting force will cause the pointer p (Fig. 50) to move quickly, as shown by the peak p'. The opposing force of the spring, however, increases with the deflection of p, until the speed of the latter is reduced to zero; the opposing force of the spring being then equal to the twisting force applied, as at the points y and a in Fig. 51. This is exactly analogous to what happens when a capacity (condenser) is charged.

The twisting force is then decreased to nothing, *i.e.*, from y to x on curve E, and then increased in

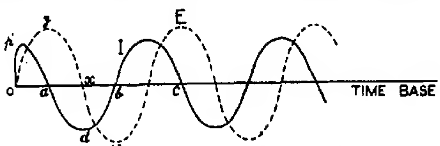

Fig. 51.—Curves showing Relation between Alternating Twisting Force (E.M.F.) and Speed (Current), with Spring (Capacity) only in Circuit.

the opposite direction, *i.e.*, to w. The pointer (Fig. 50), consequently travels back to its farthest position in the opposite direction, with a speed (current) as indicated by $a\ d\ b$ on curve I. An oscillating movement of the

pointer can thus be obtained by applying an alternating twist to R.

It will be seen in Fig. 51 that the zero and maximum values of the motion or current curve I occur a quarter of a cycle *before* those of the twisting-force or e.m.f. curve E. For example, the force is at its maximum at w, and the current at its maximum at d: and the force is at zero at x, and the current at zero at a: the intervals d-w and a-x being a quarter-cycle apart. Thus, *in a circuit containing capacity only, an alternating current "leads" by a quarter-cycle in front of the e.m.f.*

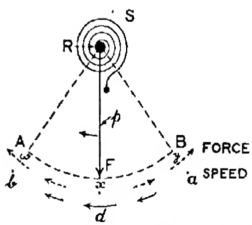

Fig. 52.—Variation of Twisting Force and Speed during one Swing of p in Fig. 50.

Capacity, therefore, has exactly the opposite effect to that of inductance, which, as we have seen (p. 62) causes the current to *lag* behind the e.m.f.

As in Fig. 46, so in Fig. 51, it should be noticed that the waves of e.m.f. and current differ in shape at the start from what they are later, it being remembered that the point O represents the *beginning* of the setting-up of the alternating current.

The fact that the current in a circuit with capacity only is 90° in advance of the e.m.f. may be made clearer with the help of Fig. 52. This diagram resembles Fig. 48, except that a spring S replaces the disk L; that is to say, it is a plan of Fig. 50. The variations in the twisting force (e.m.f.) applied to the rod R and in the speed of the pointer p (current) are represented by dotted and full-line arrows respectively.

Fig. 52 represents the conditions during the time

the e.m.f. is passing from y to w and the current from a to b in Fig. 51. The pointer p has reached the position F where the twisting force (e.m.f.) is reversed, and where it is consequently at its zero value. But at this same point, its speed (current) d is at its maximum. A quarter of a cycle later, when the pointer reaches the position RA, the force (w) is maximum and the speed (b) zero.

In other words, the current leads in front of the applied e.m.f. by 90°.

The addition of a resistance (V) to a circuit containing capacity is shown in Fig. 53. The result is that the vane V will retard the swinging of p, and will decrease the amount by which the current curve I leads in front of the pressure curve E in Fig. 51.

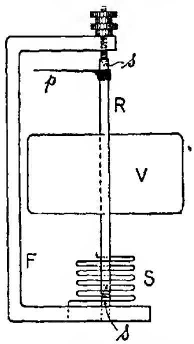

Fig. 53.—Mechanical Analogy for a Circuit with Resistance and Capacity.

The point to remember in connection with the above and the previous section is that *the addition of resistance to a circuit containing either inductance or capacity, decreases the lag of the current in the first case and decreases the lead of the current in the second case.*

18. **INDUCTANCE, CAPACITY, AND RESISTANCE IN AN ALTERNATING-CURRENT CIRCUIT** (*cont.*). **ELECTRICAL RESONANCE.**—Fig. 54 illustrates a circuit with both capacity and inductance. It has already been explained (p. 56) how the pointer p can be made to oscillate something like a pendulum by applying an initial twisting force. If a small twist be afterwards applied at the beginning of each swing, in the direction

of that swing, it will be found that the *amplitude*, or amount, or extent of the oscillations will increase.

A well-known example of this effect, which may be termed *mechanical resonance*, is found in the method of setting a swing-boat in motion. Small pulls are applied to the rope at the correct moments, with the result that in a short time the boat is swinging high, *i.e.*, with great amplitude; and the swings or oscillations can be maintained with very much less exertion than at the start.

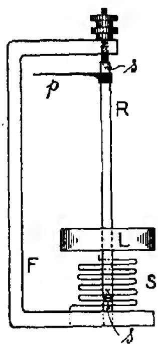

Fig. 54. — Mechanical Analogy for a Circuit with Inductance and Capacity.

The above shows that in the electrical circuit under consideration, *i.e.*, having both inductance and capacity, large currents (represented by the high speed of the mass *L* in Fig. 54) and high voltages (represented by the great strain produced in the spring at the end of each swing) may be set up by the application of a comparatively small external e.m.f., under certain conditions. These conditions are (*a*) that the frequency of the applied or impressed alternating e.m.f. must be the same as that of the electrical oscillations occurring in the circuit; and (*b*) that the two must be always in the same direction.

This phenomenon—the setting-up of high voltages within the conductor—is known as *electrical resonance*; and it was first observed on the early long transmission cables of the Deptford Station of the London Electric Supply Corporation. Great care is nowadays exercised in the design of long covered or underground cables, to ensure that the frequency of the electrical oscillations

produced therein shall be very high compared with that of the supply. If this care were not taken, the high resonance voltages that would be set up would very likely break down the insulation.

In an overhead transmission line, the capacity is very small, so that the possibility of electrical resonance is exceedingly remote.

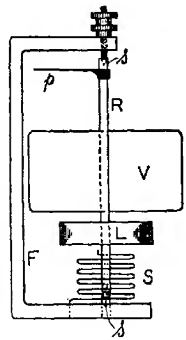

Fig. 55. — Mechanical Analogy for a Circuit with Resistance, Inductance, and Capacity.

Electrical resonance is also of importance in connection with wireless telegraphy, but in this case it is wanted. The inductance and capacity in the receiving circuit are adjusted so that the oscillations in that circuit possess the same frequency as those of the transmitting circuit. The receiver is then said to be *in resonance with* or to be *tuned to* the transmitter.

Fig. 55 is analogous to a circuit possessing inductance (L), resistance (V), and capacity (S). On applying an alternating twisting force at R, it will be found that the effects of L and S tend to neutralize each other. In such a case, the current either lags behind or leads in front of the e.m.f. It lags if the inductance preponderates, and it leads if the capacity preponderates.

It may be pointed out that one particular in which the mechanical analogies illustrated in several of the foregoing figures do not exactly fit the true condition of things, is that the resistance (air friction), inductance (inertia), and capacity (flexibility), are contained in

separate parts of the circuit (rod). In the real electric circuit these properties are, as a rule, more or less intermingled along the whole of its length. Though this fact does not materially affect the application of the analogies, it should be borne in mind.

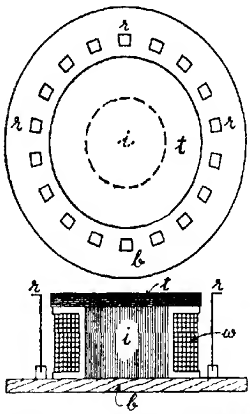

Fig. 56.—Principle of a "Resonance" Frequency Indicator. (*Everett, Edgcumbe & Co.*)

19. **"RESONANCE" FREQUENCY INDICATOR.**—A *frequency indicator* or *frequency meter* is an instrument for indicating the frequency in a circuit (§ 7). The type described below gives a very good example of the application of mechanical resonance, and is one of various makes acting in this way.

The principle of this instrument is shown in Fig. 56. There is a central laminated iron core *i*, with circular flat laminated iron top *t*. Around this core is a high-resistance coil or winding *w*, the ends of which are connected by two terminals on the outside of the case to the supply mains. Arranged in a circle around the coil, and in close proximity to the flat top, are a number of thin steel reeds *r r r*, with bent-over flat tips; these reeds being securely fixed at their lower ends in the brass base-plate *b*.

Now if a continuous current were sent through the coil, and if *t* were, say, a N. pole, all the reed tips would be S. poles and would be drawn in towards *t*. But with

an alternating current in the coil, *t* and *r r*, etc., are being rapidly and continuously magnetised and demagnetised; and the tendency is for the latter to be set in vibration. Each individual reed, however, is "tuned" to a different rate or frequency of vibration, and they are ranged in order with gradually increasing frequencies. Consequently, when an alternating current traverses the instrument, that particular reed whose frequency of vibration corresponds with the frequency of the current, will be set into vibration. The bent-over ends or tips of the reeds are painted white, and show up well against the dark background of the interior of the instrument, as seen in Fig. 57.

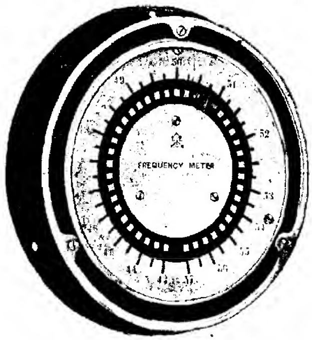

Fig. 57.—"Resonance" Frequency Indicator. (*Everett, Edgcumbe & Co.*)

The particular reed which responds to the circuit frequency will do something more than vibrate regularly, *i.e.*, it will *resonate;* the amplitude of its vibration increasing until the little white flag or tip is oscillating vigorously over an appreciable distance.

This action of the instrument is well shown in Fig. 57, where the reed corresponding to a frequency of $50\frac{3}{4}$ is shown resonating. Those on either side of it are vibrating to a less extent, while all the others are apparently quiescent. The particular instrument here shown has a range of from 43 to 57 cycles.

The impedance (§ 27) of the coil of the instrument is generally sufficiently high to permit of its direct connection across the circuit; but in some cases an external resistance is connected in series with the instrument.

20. **LAG AND LEAD.**—It was explained in § 16 that the presence of inductance in an alternating-current circuit causes the current wave to *lag* behind the e.m.f. wave.

This effect is depicted in Fig. 58, where the dotted curve, *E*, represents the e.m.f. or pressure wave; and

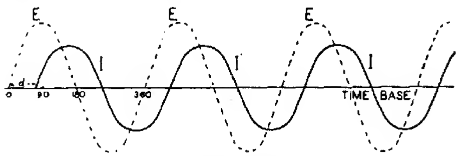

Fig. 58.—Curve of Lagging Current.

the other curve, *I*, the current wave. Beginning at the left-hand end of the horizontal line or time base, it will be seen that the current starts from zero after the e.m.f. starts from zero, and reverses after the e.m.f. reverses, and so on. In other words, the current *lags in phase* behind the e.m.f., although its frequency is exactly the same.

The amount of *lag* is measured in degrees as set out along the time base. Thus the lag is indicated by the distance, *d*, between a zero point on the pressure curve and the adjacent zero point on the current curve, and is in this case about 70°. The lag due to inductance may be anything up to 90° (a quarter-period), but cannot exceed this.

The differences between Figs. 46 and 58 should be noted. In the first place, as previously mentioned, Fig. 46 represents the actual starting of a current in an

inductive circuit; the pressure and current waves there being of different form at the beginning, due to the first effect of the inductance. In Fig. 58 both waves are supposed to have settled down to their regular forms. In the second place, it will be noticed that in Fig. 46 the current lags behind the voltage by 90°, since the circuit is assumed to be purely inductive; whereas in Fig. 58, the angle of lag is only about 70°. To give the latter value, the circuit must possess both inductance and resistance.

In § 17 it was shown that capacity in a circuit

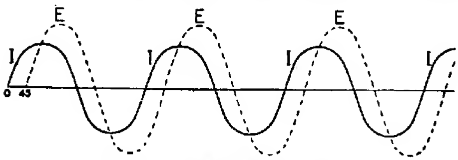

Fig. 59.—Curve of Leading Current.

causes the current to lead in front of the e.m.f. or pressure. This is depicted in Fig. 59, which should be compared with Fig. 51. The latter figure represents the starting of a current in a capacity circuit, the waves being different in form at the beginning. Fig. 59 shows the regular form of the waves when the current has once been started. In this figure the angle of lead is 45°; in Fig. 51 it is 90°—which is a theoretical maximum that is never reached in practice, since all conductors possess some resistance, however small it may be.

There has been some objection to the terms "lead of current" or "lead in phase," principally on the ground that they tend to convey the idea that the effect precedes the cause—*i.e.*, that the current is set up before the e.m.f. causing it, which of course would be impossible. The phenomenon should be clear from the consideration

of Figs. 51 and 52, where it was shown that, when the circuit has capacity only, the time of applying the maximum twisting force (maximum e.m.f.) in the direction of the swing occurs a quarter of a period later than the maximum speed (maximum current) in the same direction. In other words, although at the beginning, as seen in Fig. 51, the applied e.m.f. and current start together; yet after starting, the current leads in front of the applied e.m.f.

As a general rule, alternating currents lag more or less in phase, as the inductance usually greatly preponderates over the capacity: but, on very long cables, or by purposely introducing capacity into a circuit, the lag may be neutralized or even exceeded by the lead, and the current will then be either in phase with the pressure or leading in phase.

Lag and lead are further dealt with in §§ 26, 33, 36, etc.

21. **AMPLITUDE AND PHASE.**—The *amplitude* of an alternating e.m.f. or current is the maximum value or height of each wave. Thus, in Fig. 17, the distances *a a* represent the amplitude of the waves of e.m.f.

Both e.m.f. and current undergo periodic changes of strength, or in other words, they pass through different cycles or states. When the current rises, falls, and reverses exactly at the same time as the e.m.f., the current is said to be *in phase* or *in step* with the e.m.f. But, as already explained, the current is seldom in step with the pressure, its wave being more often *out of phase* with the e.m.f. wave, owing to the effects of inductance or capacity, or of both.

The frequency of the current is, however, always the same as that of the e.m.f.

22. **"SKIN EFFECT" OR CONDUCTOR REACTANCE.**—When a continuous current begins to traverse a con-

ductor, it commences to flow first at the surface, and then at last penetrates to the interior: when it stops, it leaves off first at the surface and lastly in the interior.

This effect is due to the inductance of the conductor, and may be explained as follows. Imagine the conductor to consist of a number of separate small insulated wires packed closely together side by side (Fig. 59A). Each separate wire would produce a field of its own, which, in accordance with Lenz's law, would set up a reverse e.m.f. in itself, and also in the neighbouring wires. Near the centre, each wire is completely

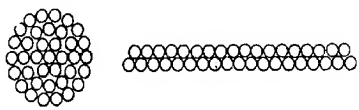

Fig. 59A.—"Skin Effect" or Conductor Reactance.

surrounded by other wires, whereas on or near the outer surface, the wires are only partly surrounded. Consequently, the total reverse e.m.f. produced in a conductor near the centre would be stronger than that near the surface; so that a continuous current, when starting, would find less momentary opposition near the surface than in the interior of the conductor. In other words, the conductor would possess greater inductance at its centre than in the outer layers.

On account of this higher inductance at the centre of a conductor, an alternating current, when it commences to flow in it, starts first at the outer surface and then penetrates more or less to the interior. The current, however, will have reversed before it has had time to distribute itself evenly over the section of the conductor. Consequently, the greater part of the current

flows near the surface, whilst little or no current passes through the centre.

This phenomenon has been given the name of *skin effect*, and depends upon the shape of the section of the conductor. Thus if we take two conductors of equal

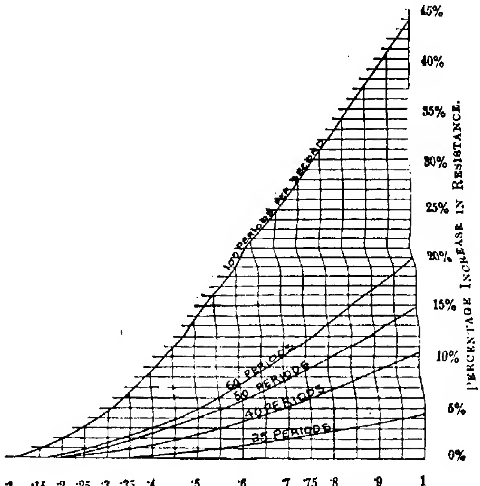

Fig. 60.—Increase of Resistance due to Conductor Reactance.
(*British Insulated and Helsby Cables, Ltd.*)

length, resistance, and cross-section; one being of circular section and the other in the shape of a ribbon (Fig. 59A); it will be found that the so-called "skin effect" of the first is greater than that of the second, for the reason that the latter conductor is more spread out, *i.e.*, the imaginary separate wires composing it are not so much under each other's inductive influence.

The term "skin effect" would lead one to think that with two or more conductors of equal cross-section, the phenomenon in question would be more marked in the one with the greater surface, whereas the reverse is the case. It is here suggested that *conductor reactance* would be a better and more expressive name for this effect.

The actual extent to which this "skin effect" or conductor reactance increases the resistance of a conductor is very clearly seen in Fig. 60. From this it will be evident that the larger the sectional area of the cable, and the higher the frequency, the greater will be the increase in the resistance.

23. **VIRTUAL VOLTS AND AMPERES.**—The e.m.f. of a practical alternator is continually rising, falling, and reversing, in much the same manner as described in § 5, and the current in the circuit must rise, fall, and reverse in sympathy (though not necessarily in step) with the e.m.f., as seen in Figs. 58 and 59.

It is clear that we cannot take the maximum points of the pressure or current wave as the nominal value, for the pressure or current are only at these *maxima for comparatively short periods. What is rightly called* an alternating e.m.f. of, say, 100 volts, must at some instants be considerably above 100 volts, and at other instants be zero. Similarly, an alternating current of, say, 10 amperes, is continually and very rapidly varying from zero to something more than 10 amperes in either direction.

We must take a value, called the *virtual value*, which is equivalent to that of a direct e.m.f. or current which would produce the same effects. And those effects are taken which are not affected by rapid changes in direction and strength; in the case of e.m.f. or pressure—the reading on an electrostatic voltmeter; and in the case of current—the heating effect.

Thus, a *virtual e.m.f.* of 100 volts is one that would produce the same deflection on an electrostatic voltmeter as a direct or continuous e.m.f. of 100 volts: and a *virtual current* of 5 amperes is that current which would produce the same heating effect as a continuous current of 5 amperes—in some kind of electric heater for example. But both pressure and current will be continually varying above and below these values.

The virtual value of an alternating e.m.f. or current having a sine-wave form (Fig. 17) is ·707, or somewhat less than three-fourths of its maximum value. For example, an e.m.f. which alternates between maximum values of 100 volts in one direction, and 100 volts in the other, will have a virtual value of about 70·7 volts. Similarly, a current which alternates between maxima of 10 amperes in one direction, and 10 amperes in the other, will have a virtual value of about 7·07 amperes. The reciprocal* of ·707 is 1·41, so that if any virtual value of pressure or current be multiplied by this number, the product will give the approximate maximum value. Thus, a virtual alternating pressure of 220 volts alternates between maxima of ($220 \times 1{\cdot}41 =$) 310 volts in either direction; and a virtual current of 50 amperes between ($50 \times 1{\cdot}41 =$), say, 70 amperes in one direction, and 70 amperes in the other.

A given virtual alternating pressure throws more strain on the insulation of a circuit than a continuous pressure of the same value (§ 37): and in this connection it should be remembered, as we have just pointed out, that any given virtual pressure fluctuates between

* The reciprocal of any number, n, is obtained by dividing it into unity—*i.e.*, reciprocal of $n = \frac{1}{n}$. Thus reciprocal of $\cdot 707 = \frac{1}{\cdot 707}$ $= 1{\cdot}41442 \ldots$ or say, 1·41. The product of any number multiplied by its reciprocal is unity: thus $\cdot 707 \times 1{\cdot}41442 \ldots = 1$.

maximum values nearly half as high again as its virtual value. If the wave of pressure differs from the sine-curve form, a matter which depends on the design of the alternator, as mentioned in § 6, the maxima may be as much as twice the virtual value. In such a case, the form of the current wave will also be affected.

It has been stated that an alternating current may be measured by its heating effect. Now as the latter, with a given resistance, is proportional to the square of the current, what we measure on the ammeter is the *mean or average of the square of the current*; and the instrument is calibrated to give the square root of this mean value, *i.e.*, the virtual value. This explains why the virtual value is often referred to as the *root-mean-square* or *r.m.s.* value.

It must be noted that the virtual or r.m.s. value of an alternating current or voltage is *not* the same as the arithmetical mean or average value, which is only ·637 of the maximum value, for a sine curve. The virtual or r.m.s. value (·707 of the maximum) is 1·11 times the arithmetical mean value.

24. **FORM FACTOR.** — In the preceding section it was stated that the r.m.s. value of an alternating voltage or current is not the same as its average value. The ratio of the former to the latter is known as the *form factor* of the voltage or current wave, *i.e.*,

$$\text{Form Factor} = \frac{\text{r.m.s. (or indicated) value of voltage or current.}}{\text{arithmetical mean or average value of voltage or current.}}$$

The *indicated value* is—of course—that shown by the voltmeter or ammeter.

In any given case, the form factor depends upon the shape of the wave. Thus, for a pure sine wave, the form factor is 1·11. For a peaked wave, such as that in

Fig. 61, it is higher than the above value; whilst for a flat-topped wave it is lower.

Suppose an alternating current to have the following successive values at successive equal intervals during one cycle, beginning from zero:—0, 2, 10, 3, 0, −2, −10, −3, 0. A curve corresponding to these values is shown

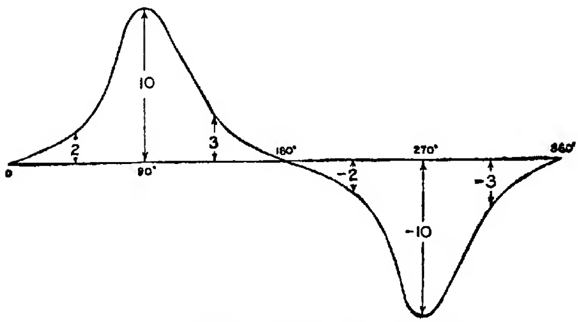

Fig. 61.—Wave of Current plotted from Given Data.
(In practice, a larger number of ordinates would be taken.)

in Fig. 61. The average value of this current during the first interval is $\frac{0+2}{2}=1$. Similarly, during the succeeding intervals, the average values are:—

$$\frac{2+10}{2}=6,\quad \frac{10+3}{2}=6{\cdot}5,\quad \frac{3+0}{2}=1{\cdot}5,\quad -1,\quad -6,\quad -6{\cdot}5,$$
$-1{\cdot}5$.

The arithmetical mean value for the first half cycle

$$=\frac{1+6+6{\cdot}5+1{\cdot}5}{4}=3{\cdot}75$$

and that for the second half cycle $= -3{\cdot}75$.

Further, the r.m.s. value for the first half cycle

$$= \sqrt{\frac{(1)^2+(6)^2+(6\cdot5)^2+(1\cdot5)^2}{4}} = 4\cdot51$$

and for the second half cycle $= -4\cdot51$.

Finally, the form factor in this case

$$= \frac{4\cdot51}{3\cdot75} = 1\cdot2.$$

The importance of the form factor lies in the fact that it is—to some extent—an indicator of the goodness (or badness) of the design of a machine. For example, in an alternator, the r.m.s. voltage induced depends not only upon the total number of lines of force per pole, but also upon their distribution. If the flux is concentrated, the e.m.f. wave will be peaked; but if the same amount of flux is evenly distributed over the pole face, the e.m.f. wave is flattened, and the r.m.s. voltage will be smaller than before.

The simplest method of determining the form factor of an alternating e.m.f. or current is to obtain the exact shape of the wave by means of an oscillograph (§ 6). Examples of such curves are shown in Fig. 62, which is an oscillogram reproducing the e.m.f. and current curves (marked "V" and "C" respectively) in a certain circuit. By measuring-up, say the voltage wave, in the manner shown in Fig. 61, its form factor can be determined in the way described above.

25. **VECTOR DIAGRAMS.**—It was shown in Figs. 16 and 17 that when a point p rotates at a uniform rate round a circle, its distance above or below a horizontal line bisecting the circle will follow a sine curve when plotted on a time base.

If we represent to scale the *maximum* value of an alternating e.m.f. by a straight line OP (Fig. 63), and suppose OP to be rotating about O, like one arm of a

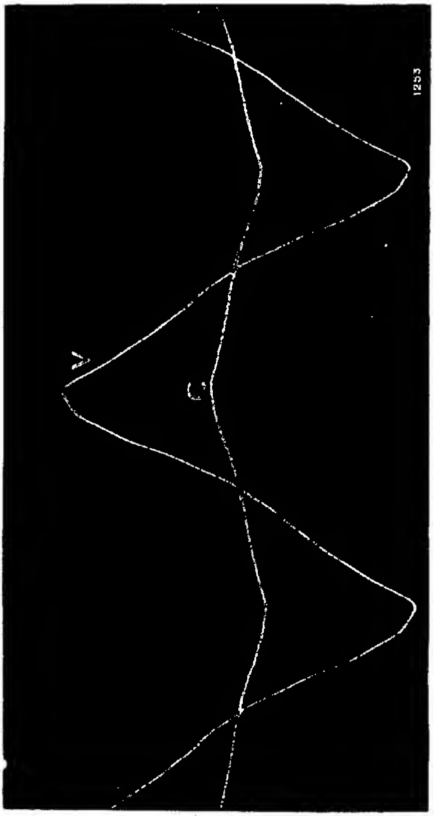

Fig 62.—Oscillogram showing E.M.F. and Current Waves (*Cambridge Scientific Inst. Co.*)

windmill, then the point P will describe a circle, and a complete rotation will correspond to a complete cycle (§ 7).

Next, if in any of the positions of *OP*, a line *PQ* be drawn parallel to the horizontal line *OA*, *OQ* will give the height of *P* above *OA*: and if these heights were plotted on a time base, we should get a sine curve similar to that in Fig. 17. Thus when an alternating e.m.f. is of a true sine-wave character, the length of *OQ* will represent to scale the value of the e.m.f. at any particular instant. At the start of the cycle, *OP* will coincide with *OA*, and *OQ* will be zero. Assuming that *OP* begins to rotate at a constant speed in a counter-clockwise direction,* it is evident that *OQ* will increase until *OP* coincides with *OB*. Then *OQ* will be equal to *OP*, *i.e.*, the e.m.f. will be at its maximum. As *OP* rotates still further, *OQ* will diminish; and when *OP* has revolved through 180° from its original position *OA*, the e.m.f. will have dropped to zero. After passing this point, *OQ* gradually increases again, but this time in the opposite direction, and the e.m.f. will have a maximum negative value at 270°, *i.e.*, at *OC*. Beyond this, *OQ* diminishes until *OP* once more coincides with *OA*, when the e.m.f. will be zero again. Further rotation of *OP* will simply give a repetition of the above values of *OQ*. Hence one rotation of *OP* is equivalent to the passing of the e.m.f. through one complete cycle.

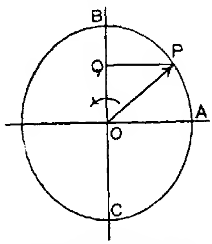

Fig. 63.—A Rotating Vector.

It should now be understood that alternating currents as well as voltages can be represented by

* This direction of rotation is now practically always adopted in these diagrams. Fig. 16, by the way, is based on clockwise rotation, but it will serve for comparison.

rotating lines whose lengths are proportional to their maximum values; and that the projection of any one of these lines on the vertical axis, at any instant, represents the *instantaneous value* of that quantity, that is to say, the value at that particular instant. Thus OQ in Fig. 63 is the instantaneous value of the e.m.f. vector OP at the moment it is in the position there shown.

The line OP in Fig. 63 is called a *rotating vector*, and diagrams in which such vectors are employed are called *vector diagrams.* As will be seen presently, vector diagrams are indispensable in many alternating-current calculations, and every endeavour should be made to grasp them thoroughly. A representative selection of such diagrams faces page 1; and their various uses will be understood in due course.

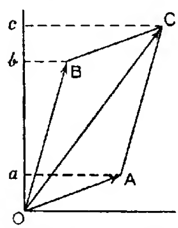

Fig. 64.—To Find the Resultant of Two Vectors.

In practice, alternators are seldom or never connected in series; but suppose, for the sake of furnishing an example of vectorial working, two such machines are so connected, and suppose further that they have the same frequency but that their e.m.fs. are out of phase.* Their resultant e.m.f. can be determined vectorially, as shown in Fig 64, where OA represents the maximum e.m.f. of one alternator, and OB that of the other. This is a case, by the way, where the e.m.fs. of the machines are unequal: if they were equal, OA would be equal in length to OB.

Now suppose OA is drawn first, it may be drawn

* It should be obvious that it would be useless to connect alternators either in series or in parallel, if their frequencies differed; for no regular wave of e.m.f. could then be obtained from them.

out at any angle with the base line. OB is next drawn so that the angle AOB is equal to the lag (§ 20) in degrees of OA behind OB. The resultant maximum e.m.f. of the two machines is found by completing the parallelogram $OACB$, *i.e.*, drawing BC parallel to OA and AC parallel to OB, and then drawing the diagonal OC. The length of OC as compared with OA and OB, will be equal to the resultant maximum e.m.f. This is proved by considering the instantaneous values Oa, Ob, and Oc of the respective e.m.fs.; the sum of Oa and Ob being obviously equal to Oc.

The above example illustrates the *finding of the resultant of two vectors*, a process which will be repeated in numerous other kinds of example that follow.

26. **EXAMPLES OF USE OF VECTOR DIAGRAMS.**—It was explained in § 16 that in a circuit containing resistance only, an alternating current is in phase with the e.m.f. In a vector diagram these quantities would be represented by two coincident lines as shown in Fig. 65. Here OI stands for the current, and OE for the voltage; Oi and Oe representing their values at the instant when their vectors OI and OE are in the positions shown.

If a circuit could have inductance only, the current would lag 90° behind the impressed e.m.f., as mentioned on p. 62; and this condition may be represented (Fig. 66) by a current vector OI 90° behind the e.m.f. vector OE.

If OE in Fig. 66 be produced backwards so that $OE_i = OE$, OE_i will represent the counter e.m.f. induced in the circuit, this—in an imaginary circuit with inductance only—being equal and directly opposed to the impressed e.m.f.

The student will probably wonder why a current flows in such a circuit when there are two equal and

opposite e.m.fs. present. Now it should be remembered that the counter e.m.f. E_i induced in the circuit is due entirely to the variation in the value of the current; so that if there were no current, no e.m.f. would be induced. But as an e.m.f. is applied to the circuit, there must be a current; and as that circuit is inductive, there must be an e.m.f. induced in it. The resistance and the capacity

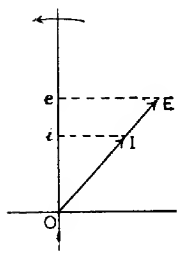

Fig. 65.—Vector Diagram for a Circuit with Resistance only.

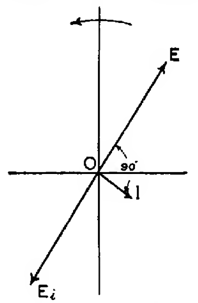

Fig. 66.—Vector Diagram for a Circuit with Inductance only.

of the circuit are here assumed to be negligible, consequently the impressed e.m.f. E is all absorbed in neutralising the induced e.m.f. E_i, *i.e.*, the two e.m.fs. must be equal and opposite, as shown in Fig. 66.

This point may be understood more clearly from the mechanical analogy in Fig. 45.

It is evident from one of the laws of motion—"*every action has an equal and opposite reaction*"—that the force exerted on the fingers (counter e.m.f.) by the rod R is exactly equal and opposite at all instants to the twisting force to which R is subjected (impressed e.m.f.), since we are assuming that there is no friction present. This is equivalent to saying that in an

alternating-current circuit possessing inductance only, the back e.m.f. induced in the circuit would be equal and opposite to the voltage applied.

The above, by the way, is quite an imaginary state of things, since it would be impossible to have a practical

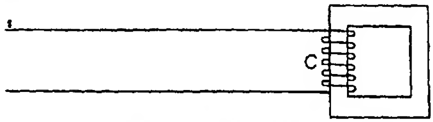

Fig. 67.—Circuit with Inductance and hardly any Resistance.

circuit with inductance but without any resistance or capacity. Fig. 67 represents the nearest approach to a circuit with inductance only, the leads to and from and the winding on the choking coil C, being of extremely low resistance.

If a circuit could have capacity only, the current (represented by OI in Fig. 68) would lead 90° in front of the impressed e.m.f. OE, as explained on p. 66. The discharge e.m.f. produced in the circuit by the capacity, otherwise termed the *capacity e.m.f.*, can be represented by producing OE backwards so that $OE_c = OE$.

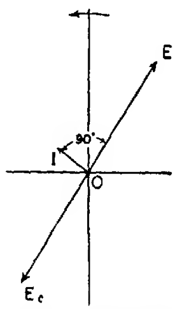

Fig. 68.—Vector Diagram for a Circuit with Capacity only.

As regards this capacity e.m.f., let us refer back to Fig. 50. Here it should be evident that the force exerted on the fingers by R is exactly equal and opposite to that applied, since friction is assumed to be absent.

In this case again, we have an imaginary condition

of things, viz., a circuit with capacity but no resistance or inductance. A circuit closely approaching this is depicted in Fig. 69, where K is a condenser, the leads to and from which possess a very low resistance, and practically no inductance.

The conditions represented in Figs. 66 and 68, are, as already stated, quite imaginary, for it is impossible to have a practical circuit without resistance. In other words, the opposing e.m.f. of inductance or of capacity can never become quite equal to the original or impressed

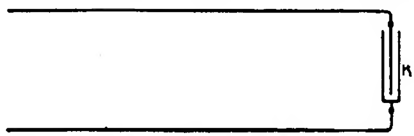

Fig. 69.—Circuit with Capacity and hardly any Resistance or Inductance.

e.m.f. in the circuit; since there must always be a surplus of the latter to provide for the voltage drop in the resistance. The chief object of these examples (Figs. 66 and 68) is to show how vector diagrams can deal with such cases.

Many further applications of vector diagrams will be given in succeeding sections.

27. **REACTANCE AND IMPEDANCE.**—The resistance offered by a conductor to a steady flow of electricity is expressed in ohms; and this value is the same whether the conductor be coiled up or stretched out, and is unaffected by the presence of neighbouring conductors. With a constantly-changing current, such as an alternating one, the *apparent resistance* offered to its flow is greater if the circuit conductor be coiled up than if it be straight; it is affected by the presence of neighbouring conductors, and also depends upon the frequency,

Even if the conductor be straight, there is something more than ordinary resistance in the circuit, especially if the conductor be very long, and if the frequency be great (§ 22). In short, Ohm's simple law cannot be applied to alternating-current circuits.

The additional or "extra resistance" in an alternating-current circuit is due to self-induction or to capacity, or to both; and it is called *reactance.**

The combined effect of the resistance and the reactance in a circuit, *i.e.*, the total opposition to the flow of an alternating current, is called the *impedance* of that circuit.

The terms "reactance" and "impedance" must not be confused. It should be easy to remember that *reactance* refers only to the *reactive effect* in the circuit; in other words, the "extra opposition" set up when the flow of electricity is not steady. *Impedance*, on the other hand, implies the "total effect" *impeding* the flow of an alternating current.

As will be seen presently, values of resistance, reactance, and impedance are all expressed in ohms.

28. **INDUCTANCE AND ITS UNIT.**—It is most important to understand that, if saturation be neglected, the inductance of a circuit, or of any part of a circuit, such as a coil, is a *constant quantity*. Thus the inductance of a coil depends simply on the square of the number of turns on the coil and on the permeance † of the magnetic path through it (page 94). It follows that a coil on an iron core has far more inductance than one without. The inductance of any uncoiled portions of a circuit, such as the straight wires or cables, is

* The meaning of the terms self-induction and inductance is explained in § 9.

† The *permeance* of a magnetic path is a measure of its capability for conducting lines of force. (*See* Author's *Electric Lighting and Power Distribution*, Vol. I., Seventh Edition).

practically negligible, except in the case of long transmission lines.

The inductance of any coil does not vary when different currents are sent through, *it is the effect of the inductance which varies*, the effect being the *reactive e.m.f. induced.* It is not usual to employ the latter term when dealing with continuous currents, but it is not incorrect to do so. Consider the sparking which would take place at the contacts of a switch controlling any electromagnetic apparatus, when the circuit is opened. The apparatus, to start with, has a certain inductance depending entirely on its construction, *i.e.*, on the quality and shape of its iron core and on the size, shape, and number of turns in its coil or winding. When the circuit, with a given current, is opened, the inductance will give rise to a certain e.m.f. of reactance, and the latter will be the cause of the sparking. With a greater or less current, the same inductance will give rise to greater or less reactive e.m.f.

Reactive e.m.f. is set up in capacity circuits as well as in inductive circuits. In the latter case it is frequently referred to as inductive e.m.f., as will be noticed herein.

The inductance of any given apparatus is a constant quantity so long as the permeance of its magnetic circuit remains unaltered; just as its resistance is a constant quantity if the temperature of the conductor be maintained at a steady value. Now the heat generated in a given coil depends primarily on its resistance, for if there were no resistance there would be no heat. The heat also depends upon the current that is passed through the coil. Thus the heating of a coil and the reactive volts induced in it are effects due primarily and respectively to the resistance and inductance of the coil; the latter depending not only

on the coil but also on its core. But no heat or reactive e.m.f. will be generated unless a current is passed round the coil, and the amounts of either will depend upon the current. The generation of heat goes on constantly as long as any current (either continuous or alternating) is flowing. The generation of reactive e.m.f., however, is only constant in an alternating-current circuit. With a continuous current, the e.m.f. of reactance is only produced at the moment that the current is started, stopped, increased, or diminished.

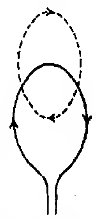

Fig. 70.—The Inductance of a Loop or Single Turn of Wire.

The *c.g.s. unit* * *of inductance* is the inductance of a loop of wire (such as that in Fig. 70) through which one magnetic line of force is set up when one c.g.s. unit of continuous current flows round the wire. Such a loop (in air) would have to be exceedingly small.

The practical unit of inductance is the *henry*, and is equal to 1000 million (10^9) c.g.s. units of inductance.† Thus a loop of wire possesses an inductance of 1 henry when a c.g.s. unit of continuous current produces a flux of 10^9 magnetic lines inside that loop. To get this relatively large amount of inductance with one turn, the loop would have to be made exceedingly large, even if it were wound round an iron core.

It follows from the above that the flux produced when a current flows through an inductance is pro-

* One unit of a system based on the centimetre, gramme, and second, as fundamentals; the system being known as the *C.G.S. System*.

† 10^9 means 1 with 9 noughts after it, *i.e.*, 1,000,000,000. Similarly 10^8 = 100,000,000; 10^6 = 1,000,000; and so on. The small figure is called the *power index*, and this system of *index notation* has the obvious advantage of brevity.

portional both to the current and to the inductance thus:—

If L = inductance of a loop in henries,

I = continuous current round the loop in amperes,

and F = flux produced inside that loop,

$$\mathsf{F} = L \times 10^9 \times \frac{I}{10} = LI \times 10^8 \text{ magnetic lines.} \qquad \textbf{(2)}$$

I is divided by 10 because one ampere $= \frac{1}{10}$th of a c.g.s. unit of current.

If the above flux F be set up or be decreased to zero in 1 second, then in either case

the average e.m.f. induced $= LI \times 10^8$ c.g.s. units,

$= LI$ volts, **(3)**

as one volt $= 10^8$ c.g.s. units of e.m.f.

The *average* value of the e.m.f. is taken because the e.m.f. varies between maximum and zero.

If I be one ampere, and L be one henry, the average e.m.f. would be one volt; hence we have the following definition of a henry:—

A circuit possesses an inductance of one henry if a variation (increase or decrease) in a continuous current by the amount of one ampere in one second induces an average e.m.f. of one volt in that circuit.

In other words, an inductance of one henry will cause an average e.m.f. of one volt to be induced when one ampere of continuous current is started or stopped in one second; or when any continuous current already flowing is increased or decreased to the extent of one ampere in one second. The greater the inductance, the greater the e.m.f. set up when one ampere of current is started or stopped in the second, or when a current already flowing is increased or decreased by one ampere in that time.

Next let us consider a coil, such as that shown in

Fig. 71, on which there are N turns; and let F be the flux or total number of magnetic lines produced inside the coil when a continuous current of I amperes is flowing round it. Now *an average e.m.f. of one c.g.s. unit is set up by the cutting of one turn by* 1 *line in* 1 *second*; so that, if, in the above coil, the flux F passes through all the turns, and is set up or reduced to zero in one second, the average e.m.f. induced in *each turn* is F c.g.s units. Hence:—

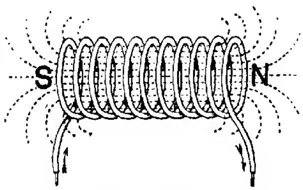

Fig. 71.—Magnetic Field of a Solenoid.

Total of the average e.m.fs. induced in the whole coil $= \mathsf{F}N$ c.g.s. units $= \frac{\mathsf{F}N}{10^8}$ volts **(4)**

as one volt $=10^8$ c.g.s. units of e.m.f.

If L stand for the inductance of this coil in henries, and if, as stated above, the continuous current I amperes and the flux F due to it are set up or reduced to zero in one second, then from Formula 3,

the average e.m.f. induced in the coil $=LI$ volts **(5)**

As (4) and (5) are both equal to the average e.m.f. induced in the coil, they are equal to each other, *i.e.*,

$$LI = \frac{\mathsf{F}N}{10^8},$$

$$\therefore L = \frac{\mathsf{F}N}{I \times 10^8} \text{ henries} \qquad \textbf{(6)}$$

The above formula enables us to calculate the inductance of a solenoid or magnet by sending a known current round it and measuring the total flux set up.

EXAMPLE.—*If a current of* 10 *amperes flowing round a coil of* 600 *turns (on an iron core) produces a total flux of* 100,000 *lines inside, calculate the inductance of the coil.*

By Formula 6:—

$$L = \frac{N\mathsf{F}}{I \times 10^8} = \frac{600 \times 100,000}{10 \times 10^8} = {\cdot}06 \text{ henry.}$$

As $\mathsf{F} = N \times$ flux produced by one turn, if we substitute this value of F in Formula 6, we get—

$$L \text{ (in henries)} = \frac{N^2 \times \text{flux produced by one turn}}{I \times 10^8}. \quad \textbf{(7)}$$

This means that the inductance of a coil is proportional to the flux produced by one ampere of continuous current flowing round one turn, and to the square of the number of turns with which that flux is linked. The flux produced by one turn is a measure of the permeance.

28A. **REACTANCE DUE TO INDUCTANCE.**—The matter in § 28 relating to inductance and reactive e.m.f. in a continuous-current circuit will help us to consider the effect of inductance in an alternating-current circuit.

If f be the frequency of a circuit, and L its inductance in henries, the reactance will be $2\pi f L$ *ohms.** Let us now see how this important expression is arrived at.

During one complete cycle of an alternating-current, the magnetic flux in the inductive part of the circuit (say a choking coil with N series turns) increases from zero to its maximum value F in one direction; it then decreases to zero, then reaches its maximum value in the opposite direction, and afterwards decreases once more to zero. That is, during one cycle, the total change in the lines of force passing through the coil will be

* π (Greek *pi*) stands for the ratio of the circumference of any circle to its diameter; *i.e.*, $\pi = 3{\cdot}1416$.

4 F. Hence, if f be the frequency, the total change in the number of lines of force, during one second, will be 4 F f.

$\therefore$ Average e.m.f. induced in one turn $=\frac{4Ff}{10^8}$ volts,

and total of the average e.m.fs. induced $=\frac{4FNf}{10^8}$ volts.

It was stated on p. 79, that for a sine wave—

Virtual e.m.f. induced $=1{\cdot}11\times$ average e.m.f. induced.

Therefore, in the above case—

$$\text{Virtual e.m.f. induced} = \frac{4{\cdot}44FNf}{10^8} \text{ volts.} \qquad \textbf{(8)}$$

Since F is the maximum value of the flux, it must be the flux produced by the maximum current $I_{max.}$, and it follows from Formula 6 that the inductance of the choking coil or other inductive part of the circuit

$$L=\frac{NF}{I_{max.}\times 10^8}.$$

On p. 78 it was shown that a virtual current (of sine-wave form) could be translated into its maximum value by multiplying it by 1·41, which is $\sqrt{2}$. That is to say—

$$I \text{ (virtual amperes)} \times \sqrt{2} = I_{max.}$$

Thus the inductance L in a circuit carrying I (virtual) amperes is

$$L=\frac{NF}{I\sqrt{2}\times 10^8}. \qquad \textbf{(9)}$$

For a purpose to follow, we may rearrange above as under—

$$NF = LI\sqrt{2}\times 10^8.$$

Let us now consider the factor 4·44 in Formula 8. As was there seen, it was made up of $4\times 1{\cdot}11$,

and this equals:—$4\times\frac{\cdot 707}{\cdot 636}$ or $4\times\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\pi}}$ or $\frac{2\pi}{\sqrt{2}}$.

Substituting these values for NF and **4·44** in Formula 8 we get—

Virtual reactive e.m.f. (otherwise the e.m.f. necessary to overcome the reactance due to inductance) $= \frac{2\pi}{\sqrt{2}} \times \frac{LI\sqrt{2} \times 10^8 f}{10^8}$.

$= 2\pi fLI$ volts. **(10)**

This expression is otherwise termed the *counter e.m.f. of reactance* or the *reactive drop* (of volts) due to inductance (Fig. 72), or the *inductive e.m.f.*; and if we eliminate the current factor (I), what is left ($2\pi f L$) is the *reactance* due to inductance (Fig. 73).

29. **CONNECTION BETWEEN INDUCTANCE, REACTANCE, IMPEDANCE, IMPRESSED VOLTS, AND VIRTUAL CURRENT.**—We saw in § 27 that the reactance in an alternating-current circuit depends upon the inductance and capacity therein.

In a circuit with negligible capacity, if L be the inductance, and f the frequency, the reactance will be $2\pi fL$, as shown at the end of the previous section.

Reactance, like resistance, is independent of the current; but the current must be taken into account when we wish to find the volts necessary to overcome either of these quantities. Thus, if I be any virtual current, RI denotes the volts necessary to force it through a resistance R. Similarly :—

$2\pi fLI$ will be the volts necessary to force the current I through a reactance $2\pi fL$. **(11)**

For example, if $I = 60$ amperes, $f = 80$, and $L = {\cdot}005$ henry, the counter e.m.f. of reactance (otherwise the reactive drop) will be $2 \times 3{\cdot}14 \times 80 \times {\cdot}005 \times 60 = 151$ volts.

It must be remembered, however, that we cannot have a circuit with inductance only; it *must* have some resistance. It seems to follow then that the total volts necessary to be impressed on the circuit will be equal

to the sum of the volts required to send the given current through the resistance, and the volts equal to the counter e.m.f. of reactance (*i.e.*, $RI + 2\pi f LI$). This, however, is *not* the case, owing to the fact that the e.m.f. required to overcome the inductance which causes the reactance is 90° in front of that required to overcome resistance.

If OI (Fig. 72) represents the direction of the current vector at a particular instant, then the e.m.f. to be impressed on the resistance to give a current I is RI* and is in phase with I, and can therefore be represented by OA in Fig. 72. OI simply stands for the current, but not necessarily to scale, and it is drawn slightly longer than the length necessary for RI, *i.e.*, OA.

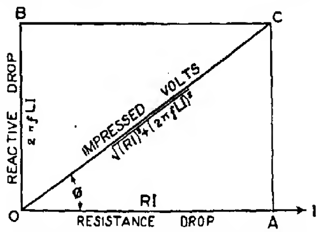

Fig. 72.—Vector Diagram for a Circuit with Inductance and Resistance in Series.

To send a current I through an inductance L only, would (by 10) require an impressed e.m.f. of $2\pi f LI$ volts; but the current would lag behind that e.m.f. by 90°, so that the voltage to overcome inductance can be represented by OB, 90° in advance of OI. Complete the parallel-

* The reader will probably ask himself here why the vector OA should not be $1 \cdot 41\ RI$, since I is the virtual current (§ 23), and since, in Fig. 63, the length of the rotating e.m.f. vector is made equal to the maximum value. The explanation is that if this course were adopted, then OB would be $2\pi fL \times 1 \cdot 41\ I$, and the max. impressed voltage would be $1 \cdot 41 \sqrt{(RI)^2 + (2\pi f LI)^2}$. In short, each side would be 1·41 times as long as in Fig. 72. Making the lines represent virtual or r.m.s. values instead of maximum values simply means that the figure is smaller in the ratio of 1 to 1·41; the relationship between the lengths of the lines remaining unaltered, and the calculations being greatly simplified

ogram $OACB$ and draw the diagonal OC. The length of OC, on the same scale as OA and OB, represents the resultant impressed e.m.f. necessary to overcome both the resistance and the reactance due to inductance.

Now OAC is a right-angled triangle, of which OC is the hypotenuse, *i.e.*, the side opposite the right angle. In such, the square of the hypotenuse is equal to the sum of the squares of the other two sides (by *Euclid*, I. 47).

Thus:—

$$OC^2 = OA^2 + AC^2 = OA^2 + OB^2, \text{ since } OB = AC.$$

Hence:—

$$OC = \sqrt{OA^2 + OB^2}.$$

That is:—

$$\begin{aligned} \text{Virtual impressed volts } (E) &= \sqrt{(RI)^2 + (2\pi fLI)^2} \\ &= \sqrt{I^2 \times \{R^2 + (2\pi fL)^2\}} \\ &= I\sqrt{R^2 + (2\pi fL)^2} \qquad \textbf{(12)} \end{aligned}$$

Now, as in Ohm's simple law—

$$E = IR,$$

and

$$I = \frac{E}{R},$$

we may write:—

$$(\text{Virtual})\ I = \frac{(\text{Virtual impressed})\ E}{\sqrt{R^2 + (2\pi fL)^2}} \qquad \textbf{(13)}$$

This may be termed the Ohm's law for alternating currents, $\sqrt{R^2 + (2\pi fL)^2}$ being, in fact, the *impedance* of the circuit (page 89).

The above may be written in words thus:—

$$\text{Virtual current} = \frac{\text{Virtual e.m.f.}}{\text{Impedance}}. \qquad \textbf{(13A)}$$

If, in a continuous-current circuit, we multiply

together the current and the resistance, the product will give the e.m.f. in the circuit; or :—

$$I \times R = E.$$

A similar result follows in an alternating-current circuit, for multiplying the virtual current by the impedance will give us the virtual e.m.f., *i.e.* :—

$$\text{(Virtual) } I \times \sqrt{R^2 + (2\pi f L)^2} = E \text{ (virtual)}$$

this being merely the foregoing equation (13) transposed.

It will be seen in Fig. 72 that in each of the three quantities — impressed volts, e.m.f. to overcome reactance (reactive drop), and that to overcome the resistance (resistance drop)—the quantity I (virtual current) occurs. Obviously I, being a common factor, may be eliminated in each case; and the quantities will then respectively represent impedance, reactance, and resistance, as shown in Fig. 73.

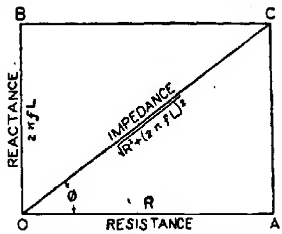

Fig. 73.—Impedance due to Inductive Reactance and Resistance in Series.

Thus :—

$$\text{Impedance}^2 = \text{resistance}^2 + \text{reactance}^2.$$

i.e.,—

$$\text{Impedance} = \sqrt{\text{resistance}^2 + \text{reactance}^2}. \quad \textbf{(14)}$$

In Fig. 72, the angle AOC indicates the amount of the lag of the current behind the impressed voltage; and the same angle occurs in Fig. 73, where it will be seen that its magnitude depends upon the ratio of the reactance to the resistance, and is therefore independent of the current. Thus the greater the reactance as com-

pared with the resistance, the greater will be the angle of lag; and *vice versa.*

It should be noted here that in most cases where e.m.f., voltage, or current are spoken of, the virtual value is meant; and that it is unnecessary to drag in the word *virtual* every time. Also, *impressed e.m.f.* or *voltage* merely means the volts giving rise to the current in a circuit.

If ϕ* = angle of lag of current behind voltage, then, from Figs. 72 and 73

$$\cos\phi = \frac{OA}{OC} = \frac{RI}{I\sqrt{R^2+(2\pi fL)^2}} = \frac{R}{\sqrt{R^2+(2\pi fL)^2}} = \frac{\text{resistance}}{\text{impedance}}. \quad \textbf{(15)}$$

It is important to note that the above relates to a series circuit. Thus if, in a series circuit, the resistance be divided by the impedance, the quotient will give the cosine of the angle of lag, *i.e.*, of the phase difference. The cosine value itself, represents the *power factor* of the circuit (§ 44).

The case of parallel circuits is dealt with in § 36.

30. **WORKING WITH VECTOR DIAGRAMS.** — It should now be clear that a vector diagram enables one or more alternating-current quantities to be found from others which are known; and that working with such diagrams is a sort of electrical mensuration.

It should be understood that—in practice—vector diagrams are drawn three or four times as large as they are shown herein. Also, that any given diagram may be drawn to any convenient scale. The larger the scale the easier it is to arrive at an accurate result.

* ϕ = Greek (*phi*), used for lag or lead values. Cos ϕ signifies the cosine of any angle ϕ. The cosine of angle a (Fig. 19), for instance, is the ratio of the adjacent side BE to the hypotenuse BD, or $\frac{BE}{BD}$. The cosine values of all whole-number angles may be obtained direct from the Table on p. 271.

Squared paper with inch and 0·1 in. squares is good for doing these "vector workings" upon; and each ·1 in. may represent a unit, or 2, or 5, or 10, or more units, according to the magnitude of the quantities and the size of the sheet.

Consider a case, like Fig. 72, where two vectors are drawn to start with, and then the parallelogram.

The first vectors having been drawn, the parallelogram is described, and then the diagonal vector is drawn. The length of the latter is then measured off with a rule, and the value of the quantity it represents is calculated on the same scale as that to which the first vectors were drawn.

Suppose, for example, that in Fig. 72 the values of the resistance drop and the reactive drop are 336 and 252 volts respectively; and that the size of the squared paper is 8 ins. by 6 ins. In such a case, a very suitable scale is that in which 1 inch represents 50 volts. Then OA is made $\frac{336}{50} = 6{\cdot}72$ ins. long, and OB is made $\frac{252}{50} = 5{\cdot}04$ ins. long. The parallelogram $OACB$ being completed, the length of the diagonal—by measurement—is found to be 8·4 ins.; so that the impressed voltage is $8{\cdot}4 \times 50 = 420$ volts.

Had a scale of, say, 1 inch to 100 volts been chosen in the above example, the diagram would have been only half the size; and the lengths of the lines could not have been measured as accurately as with the larger figure.

Centimetre and millimetre scales are useful, the small divisions rendering them very suitable for accurate measurements.

31. **TO FIND THE NECESSARY INDUCTANCE FOR A CHOKING COIL.**—Suppose we have a non-inductive resistance, such as a group of forty ·35-ampere glow

lamps in parallel, for which the supply pressure is too high. By means of an adaptation of the vector diagram in Fig. 72, we can ascertain the reactance and inductance which a choking coil must have in order to reduce the supply pressure to that required by the lamps, when it is connected in one of the leads.

In Fig. 74, the line OA is first drawn proportional in length (to a convenient scale) to the voltage required across the lamps. At A, AC is drawn perpendicular to OA. Next, with centre O, and radius equal to OC (representing to scale the supply pressure), an arc is drawn cutting the perpendicular from A at C. The parallelogram $OACB$ is then completed. On comparing this figure with Fig. 72, it is evident that OB stands for the p.d. at the choking-coil terminals, in other words, the reactive drop therein; it being assumed that the resistance of the choker is negligible. We thus get the following relation:—

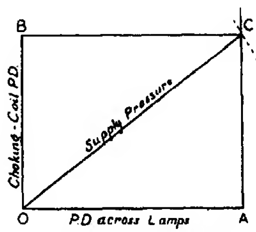

Fig. 74.—Vector Diagram for Choking-Coil Circuit.

Supply pressure^2 = (lamps p.d.)^2 + (choker p.d.)^2.

If the supply pressure is, say, 240 volts at 60 ∿, and the lamps are 200-volt ones, and the choker p.d. required is called x,

Then:—

$$240^2 = 200^2 + x^2$$
$$i.e., \quad x^2 = 240^2 - 200^2$$
$$x = \sqrt{240^2 - 200^2}$$
$$= 133 \text{ volts.}$$

The circuit may be diagrammatically represented as in Fig. 75, L being the group of lamps and C the choking coil.

The reader may be puzzled to find that the choking coil p.d. is greater than, instead of being equal to, the difference between the circuit pressure and that required by the lamps. This, as explained in § 29, is owing to the fact that the e.m.f. induced in the choker lags 90° behind the volts absorbed in the resistance (Fig. 66).

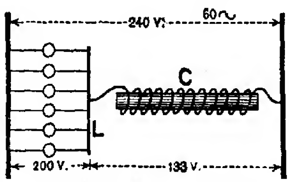

Fig. 75.—Choking-Coil Circuit.

The case taken is a simple one only, where the load (*i.e.*, the lamps) has no inductance, and where the choker is assumed to have little or no resistance. Generally, however, the load, as with arc lamps, has considerable inductance; and the resistance of the choker cannot always be neglected, so that the calculation is less simple. However, to proceed with the example, having found that the necessary choking p.d. is 133 volts, the next step is to find what inductance the coil must have.

The current required by the lamps is 14 amperes. The reactive drop in the choker may then be calculated (from Formula 10) as follows:—

$$133 = 2\pi f L I$$
$$= 2\pi 60 L 14$$

Thus its inductance $L = \dfrac{133}{2\pi \times 60 \times 14}$

$$= \cdot 025 \text{ henry.}$$

Having found its inductance, the details of construction of the choker, *i.e.*, the shape and quality of its iron

core, and the size and number of turns of wire on it, are matters of electro-magnetic design, with which we are not concerned here.

From Fig. 74, if desired, the angle of lag can be calculated thus:—

$$\frac{OA}{OC} = \cos \phi = \frac{200}{240} = {\cdot}833.$$

On a reference to a table of cosines it will be found that ·833 is the equivalent of 33·5°, the latter therefore being the angle of lag. (See page 271.)

As will be understood later (§ 41), cos ϕ is the *power factor* of the circuit.

32. **TO FIND THE IMPEDANCE OF A CIRCUIT WITH CAPACITY.**—As already stated (p. 89), the reactance in a circuit may be due to inductance, or to capacity, or to both.

Suppose a circuit to have capacity only, as in Fig 68. If K be the value of the capacity in farads,* then for a frequency f, the reactance due to that capacity will be:—

$$\text{Reactance} = \frac{1}{2\pi f K} \dagger \qquad \textbf{(16)}$$

This quantity is sometimes termed the *capacity-reactance*, to distinguish it from $2\pi fL$, which is sometimes termed *inductive-reactance.*

With a given capacity-reactance, the greater the current the greater will be the counter e.m.f. or the reactive drop due to the capacity, this being otherwise called the *capacity e.m.f.*

Thus the voltage E necessary to send a current I

* The *farad* is the unit of capacity, and 1 microfarad = ·000001 farad. *See* the Author's *Electric Lighting and Power Distribution*, Vol. I.

† For derivation of Formulæ 16 and 17 *see* Appendix B

amperes at f frequency through a capacity K* will be:—

$$E=\frac{I}{2\pi fK} \qquad (17)$$

The above expression thus stands for the *reactive e.m.f.* or *reactive drop* due to the capacity reactance with a given current I; and it also represents the virtual e.m.f. necessary to send the current I through the reactance.

For example, if $I=60$ amperes, $f=80$, and $K=1000$ microfarads, the reactive drop will be $\frac{60}{2\times3{\cdot}14\times80\times{\cdot}001}$ $=119$ volts; the capacity being expressed in terms of the farad—not in microfarads.

In practice it is impossible to get a circuit with capacity only; it must have *some* resistance. In such a case, the necessary voltage, for the current is obtained by adding *vectorially* the respective pressures required to overcome the capacity-reactance drop and the resistance drop.

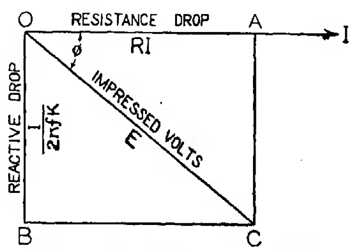

Fig. 76.—Vector Diagram for a Circuit with Capacity and Resistance in Series.

If the resistance of the circuit be R ohms, the resistance drop will be RI, and its value (OA) is scaled

* If the capacity be connected like Fig. 69, the current only apparently flows *through* it. The action of the alternating current in such a circuit was explained in connection with Figs. 30 to 32.

off on the current vector OI in Fig. 76. Since the current in a pure capacity circuit is 90° in advance of the reactive drop, the vector OB for the latter will be 90° behind OI.

Fig. 76 should be compared with Fig. 72. In the latter, OB is the reactive drop due to inductance, and is 90° in advance of the current. In Fig. 76, OB is the reactive drop due to capacity, and it lags 90° behind the current.

The parallelogram in Fig. 76 being completed, the diagonal OC represents the volts necessary for setting-up the current in the circuit.

$$\text{As } OC^2 = OA^2 + OB^2 \qquad \text{(page 98)},$$

$$= (RI)^2 + \left(\frac{I}{2\pi fK}\right)^2$$

$$\therefore \text{ required voltage } (E) = \sqrt{(RI)^2 + \left(\frac{I}{2\pi fK}\right)^2}$$

$$= I\sqrt{R^2 + \left(\frac{1}{2\pi fK}\right)^2}. \qquad \textbf{(18)}$$

What we set out to do was to find the impedance of the circuit, and this is done by eliminating the current factor I from the above.

Hence:—

$$\text{Impedance} = \sqrt{R^2 + \left(\frac{1}{2\pi fK}\right)^2} \qquad \textbf{(19)}$$

$$= \sqrt{(\text{resistance})^2 + (\text{capacity-reactance})^2}.$$

Here again, as in the two previous examples (Figs. 72 and 73), the power-factor (§ 44) and the angle of lag (or lead) can be calculated from the vector diagram. In Fig. 76, for example:—

$$\cos\phi = \frac{OA}{OC} = \frac{RI}{I\sqrt{R^2 + \left(\frac{1}{2\pi fK}\right)^2}} = \frac{\text{resistance}}{\text{impedance}}. \qquad \textbf{(20)}$$

In Formula 20, the impedance is due to resistance and capacity *in series*, whereas in Formula 15 it is due to resistance and inductance *in series*. In either case, however, the ratio of the resistance to the impedance gives us the important quantity cos ϕ, which is the power factor of the circuit (§ 44).

With parallel circuits the case is different, as will be be seen in § 36.

As will be found in the next section, impedance is expressed in ohms.

33. **TO FIND THE IMPEDANCE OF A CIRCUIT CONTAINING INDUCTANCE, CAPACITY, AND RESISTANCE IN SERIES; AND THE E.M.F. NECESSARY FOR SETTING-UP A GIVEN CURRENT THEREIN.**—Fig. 77 represents a circuit containing an inductance of L

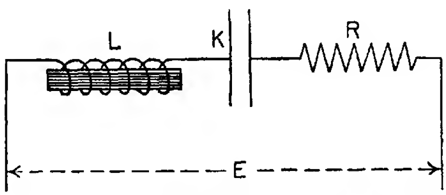

Fig. 77.—Circuit with Inductance, Capacity, and Resistance in Series.

henries, a capacity of K farads, and a resistance of R ohms, all these being in series. It is required to find the e.m.f. E necessary for setting-up a current I at a frequency f in this circuit.

The e.m.f. required to overcome the counter e.m.f. of reactance, otherwise the reactive drop due to inductance, is $2\pi fLI$ (10); that for the counter e.m.f. of capacity, otherwise the reactive drop due to capacity is $\frac{I}{2\pi fK}$ (17), and that for the resistance is RI. These

three values must be added vectorially, as shown in Fig. 78.

OA is the voltage for the resistance, and is in phase with the current vector OI. The pressure to overcome

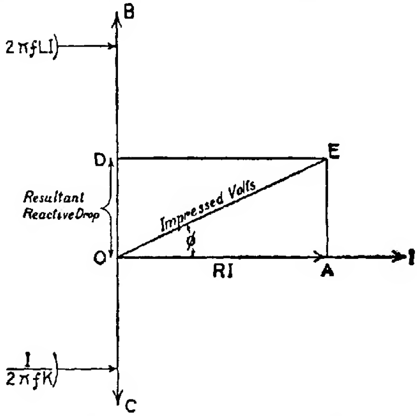

Fig. 78.—Vector Diagram for a Circuit with Inductance, Capacity, and Resistance in Series.

the inductive e.m.f. is given by OB, 90° in advance of OI, whereas that required to overcome the capacity e.m.f. is denoted by OC, 90° behind OI. Consequently OB and OC are in a straight line, and therefore tend to neutralize each other; and the resultant of the two is OD.

That is to say:—

OD (reactive drop) $= OB - OC$,

$= 2\pi f L I - \dfrac{I}{2\pi f K}$,

= inductive e.m.f. − capacity e.m.f.,

$$= I\left(2\pi f L - \frac{1}{2\pi f K}\right), \qquad \textbf{(21)}$$

= current (inductive-reactance − capacity-reactance).

It has been assumed in this example that the inductive e.m.f. is greater than the capacity e.m.f., this being usually the case.

The parallelogram $OAED$ (Fig. 78) is then drawn, and the impressed e.m.f. will be represented by the diagonal OE.

$$\text{Now } OE^2 = OA^2 + OD^2$$
$$= (RI)^2 + \left\{ I\left(2\pi fL - \frac{1}{2\pi fK}\right)\right\}^2,$$
$$= I^2\left\{R^2 + \left(2\pi fL - \frac{1}{2\pi fK}\right)^2\right\}.$$

Hence the necessary e.m.f.

$$OE = I\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fK}\right)^2}. \qquad \textbf{(22)}$$

In § 29 (on p. 99) it was shown that e.m.f. = impedance × current. Consequently, if we eliminate the current factor from both sides of the above equation we get:—

$$\text{Impedance} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fK}\right)^2} \qquad \textbf{(22A)}$$
$$= \sqrt{(\text{resistance})^2 + (\text{reactance})^2}.$$

The latter expression corresponds with Formulæ 14 and 19, and is thus true when the reactance is due to inductance, or to capacity, or to both; so long as the circuit is a *series one.* The case of parallel circuits is dealt with in § 36.

EXAMPLE.—If, in Fig. 77, $R = 2$ ohms, $L = \cdot 005$ henry, $K = 1000$ mfd. $= \cdot 001$ farad, and $f = 80$.

$$\text{Impedance} = \sqrt{2^2 + \left(2\pi \times 80 \times \cdot 005 - \frac{1}{2\pi \times 80 \times \cdot 001}\right)^2}$$
$$\doteq \sqrt{4 + (2\cdot 51 - 1\cdot 99)^2} = 2\cdot 07 \text{ ohms}.$$

Suppose $I = 60$ amperes,

Then necessary e.m.f. $= 60 \times 2{\cdot}07$,
$= 124{\cdot}2$ volts,
$= 124$ volts practically.

From an examination of Fig. 78, it will be seen that if OB is greater than OC, the diagonal OE lies above OI, *i.e.*, the current lags behind the impressed e.m.f. If, on the contrary, OB is less than OC, OE will be below OI, showing that in this case the current will lead in front of the e.m.f. Further, with given values of inductance and capacity, the amount of lag or lead will depend upon the resistance. Thus if, in Fig. 78, the resistance be increased, the angle EOA will become smaller, and the phase-difference or angle of lag between the current and the e.m.f. will be decreased.

The particulars in Fig. 78 enable the power factor (§ 44), and the angle of lag (or lead) to be calculated.

Thus:—

$$\frac{OA}{OE} = \cos\phi = \frac{RI}{I\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fK}\right)^2}} = \frac{\text{resistance}}{\text{impedance}}. \qquad \textbf{(22B)}$$

Thus $\cos\phi = \dfrac{2}{2{\cdot}07} = {\cdot}966 =$ power factor of the circuit.

In the Table on p. 271 it will be seen that $\phi = 15°$; and since the inductive-reactance is greater than the capacity-reactance, the current lags 15° behind the voltage.

34. **THE CONDENSER ACTION OF SOME ALTERNATING-CURRENT CIRCUITS.**—It was stated at the end of § 13 that every ordinary electric circuit possesses more or less capacity; and it is important to note here that this capacity is usually *in parallel with the circuit*, not in series with it as in Figs. 30 to 32 and 77.

In Fig. 79, C is an electric cable laid direct in the

ground, or in a conduit; the cable and ground (or conduit) then acting together like a condenser. The conductor forms one coating of the condenser, the insulation of the cable the dielectric, and the outer metal sheathing (if any), material of the conduit (if metal), or the Earth, form the other coating. This state of things may be diagrammatically represented as in Fig. 80, where we may imagine the conductor of the cable as joined at intervals to the coatings of condensers, the other coatings of which are connected with Earth. From this it is clear that the capacity is in parallel with the cable.

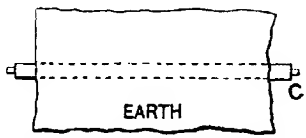

Fig. 79.—Condenser Action of a Cable.

If the cable has no metal sheathing, and if it be placed in a roomy conduit, the air surrounding it and also the conduit (if non-metallic) form part of the dielectric of our imaginary condenser system; and the capacity is then much less, as the "coatings" of the condenser are further apart.

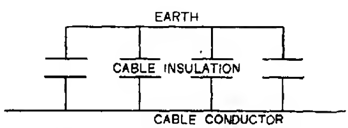

Fig. 80.—Condenser Action of a Cable.

Suppose there are two cables (which may or may not form part of the same circuit) running side by side in a pipe or conduit, or in the ground, as represented in Fig. 81. We may then look upon the two cable conductors as the respective coatings of the condenser, and the two insulating coverings, etc., in between, as the

dielectric. The conception of the condenser action here is not so easy as in the case of a single cable; but it is made clearer in Fig. 82, which represents a section of what lies between the conductor of one cable and that of the other.

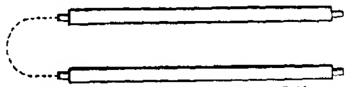

Fig. 81.—Condenser Action of Adjacent Cables.

The greater the length of the cables, and the closer together or to Earth they are, the greater their capacity. As a rule, armoured cables have greater capacity than unarmoured ones; and the capacity of unarmoured cables is increased when they are placed in iron conduits. The capacity of underground armoured mains varies from ·3 to ·6 microfarads per mile. It depends somewhat upon their size and construction, and is reduced by employing paper instead of indiarubber as the dielectric.

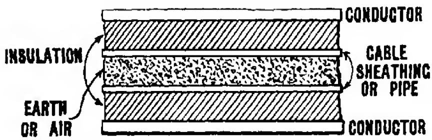

Fig. 82.—Condenser Action of Adjacent Cables.

In some paper-insulated cables, for telephone work, the paper is usually wrapped comparatively loosely round the conductor, a certain amount of air being thereby imprisoned in the folds. The object of this construction is to reduce the condenser effect.

35. **EFFECT OF CAPACITY IN THE CIRCUIT.**—As has already been shown by mechanical analogies and vector diagrams, the effect of capacity in an alternating-current circuit is exactly opposite to that of

inductance, for it assists or tends to assist the current to rise to its maximum value sooner than it would otherwise do, whereas inductance retards or tends to retard the current (§ 9). The capacity phenomenon is the same whether the capacity is in series or in parallel with the circuit, but the numerical results differ. (*See* §§ 33 and 36.)

In Fig. 83, *A* is an alternator, the mains from which run for a long distance side by side, and feed a number of transformers, etc. For convenience, we place the transformers, *T T*, at the far end of the circuit, and think of the condenser effect of the first portion. The alternator is constantly pumping electricity backwards and forwards between the mains *M M M* and *M′ M′ M′*, and these may be looked upon as the opposite coatings of a condenser. Let us suppose the alternator first pumps from *M* to *M′*; electricity will be heaped up, so to speak, on *M′*, and a deficit left on *M*, *M′* being + and *M* −. Now, neglecting for the moment the far end of the circuit, suppose the alternator were suddenly stopped: there would then be a momentary return flow of electricity from *M′* to *M* through the alternator; in other words, the condenser would discharge itself. If the alternator goes on working, however, it is obvious that the electricity heaped up on *M′* helps or increases the flow when the alternator begins to pump from *M′* to *M*. *M* then becomes + and *M′* −, and when the alternator again reverses its e.m.f., the + charge on *M* flows across to *M′*, and helps the ordinary current.

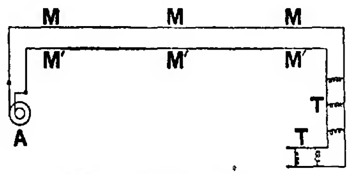

Fig. 83.—Capacity in a Circuit.

This auxiliary current, if we may so call it, is generally termed the *condenser current*, and is clearly greater the greater the capacity of the mains. In the above explanation we have to suppose that the alternator is pumping to and fro very slowly, whereas in reality the reversals of e.m.f. really take place many times a second (§ 7).

When the "go" and "return" mains do not run side by side, the condenser action may be pictured as follows: Suppose the alternator to pump from left to right (Fig. 84), a surplus is heaped-up on the right-hand

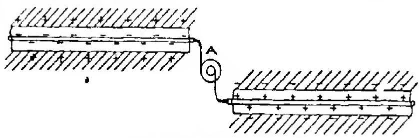

Fig. 84.—Capacity of Cables.

cable, and a deficit created in the left-hand one; influence or electrostatic induction takes place, and − and + charges respectively are influenced (or induced) on the outsides of the cables, as shown by the signs. If the alternator e.m.f. suddenly stopped, there would be a momentary current from right to left through the alternator. It is clear, therefore, that when the alternator reverses its e.m.f., there will be a greater transference of electricity (from right to left) than there was when the alternator first started and pumped from left to right. The left-hand cable will then become +ly charged, and the right-hand one −ly charged, and the discharge will help the alternator when it again reverses its e.m.f.

36. **TO FIND THE RESULTANT CURRENT (AND WHETHER IT LAGS OR LEADS) IN A CIRCUIT CONTAINING RESISTANCE, CAPACITY, AND INDUCTANCE**

IN PARALLEL.—Consider an alternator circuit containing resistance R, capacity K, and inductance L in parallel, as in Fig. 85: the voltage of the alternator being E and its frequency f.

The currents through the three portions of the circuit are as follows—that (I_R) through R is $\frac{E}{R}$, and is in phase with the voltage; that (I_K) in the condenser

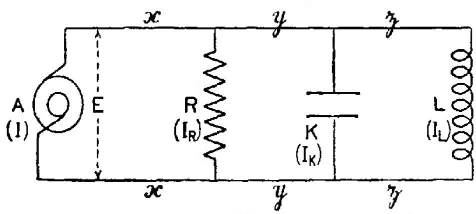

Fig. 85.—Resistance, Capacity, and Inductance in Parallel.

is $E \times 2\pi f K$ (from Formula 17, p. 105), and is 90° in advance of the impressed e.m f.; whilst that (I_L) through L is $\frac{E}{2\pi f L}$ (from Formula 10, p. 96), and lags 90° behind the applied voltage.

These currents can be represented by a vector diagram, as in Fig. 86. OA is the current through the resistance and is in phase with the impressed voltage OE; OB represents the capacity current leading 90° front of the impressed e.m.f.; and OC the current in the inductance, lagging 90° behind the impressed e.m.f.

In this figure it is assumed that I_L is greater than I_K; and OD is made equal to $I_L - I_K$, which is their resultant. Completing the parallelogram $OAFD$, we get the diagonal OF representing the current I which

has actually to be supplied by the alternator, and which lags behind the impressed e.m.f. by the angle FOA.

From Fig. 86 we have:—

Resultant current $I = OF$,

$$= \sqrt{OA^2 + AF^2} = \sqrt{OA^2 + OD^2},$$
$$= \sqrt{OA^2 + (OC - OB)^2},$$
$$= \sqrt{I_R^2 + (I_L - I_K)^2}. \qquad (23)$$

Also, if ϕ be the phase angle between the current and the voltage—

$$\cos\phi = \cos FOA,$$
$$= \frac{OA}{OF} = \frac{I_R}{I}. \qquad (24)$$

$\frac{I_R}{I}$ thus gives the power factor of the circuit (§ 44).

It is evident from Fig. 86 that the current will lag

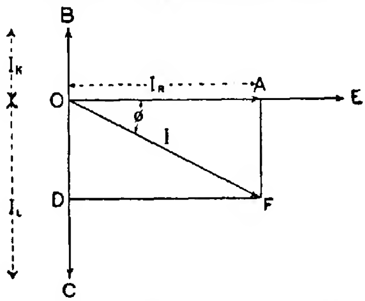

Fig. 86.—Vector Diagram of Currents in Fig. 85.

behind or lead in front of the voltage according as OC is greater or less than OB; *i.e.*, if $(I_L - I_K)$ is positive, the current lags; whereas if $(I_L - I_K)$ is negative, the current leads.

The differences between the vector diagrams for

series (Fig. 78) and for parallel (Fig. 86) circuits should be carefully noted. In the former diagram, we start with the vector that is common to every part of the circuit, namely, the current vector OI; the voltage vectors are then drawn in their respective directions relative to this vector, as already described. In the case of parallel circuits, however, it is the voltage that is the same for each circuit, hence in Fig. 86, we start with the voltage vector OE, and the vectors representing the currents in the different circuits are drawn at their proper angles with OE.

It is very important to notice that for series circuits (pages 100, 106, and 110),

$$\cos\phi = \frac{\text{resistance}}{\text{impedance}}, \tag{25}$$

whereas, for parallel circuits, as in the present case,

$$\cos\phi = \frac{\text{current through resistance circuit}}{\text{total current}}. \tag{25A}$$

And cos ϕ is the power factor (§ 44).

To proceed with the example, Figs. 85 and 86, if R, K, and L are arranged in the order shown in Fig. 85, the current in the two parts zz of the circuit is represented by OC in Fig. 86, while that in yy is OD, *i.e.*, the resultant of OC and OB. The current in xx is the same as that in the alternator, namely that represented by OF.

It should be clear from Fig. 86 that should I_R be small, and I_L large, the current I_L at the end of the circuit remote from the alternator, *i.e.*, zz in Fig. 85, would probably be greater than that (I) through the alternator itself and through the first part xx of the circuit. This effect is of importance when we are considering the best place to install an apparatus to neutralize the effect of inductance in a distributing system.

An examination of Fig. 86 will also make it evident that if the capacity and the inductive currents are the same, *i.e.*, if $I_K = I_L$, the resultant current OF will be equal to I_R, and will be in phase with the voltage. In other words, the capacity and inductance will neutralize each other's effect.

For the purpose of reducing the effect of excessive inductance in working circuits, condensers, or machines (such as over-excited synchronous motors) which act like condensers, are sometimes put in parallel with the inductive load. This matter is further explained in § 44.

EXAMPLE.—*A resistance of* 100 *ohms, an inductance of* 0·5 *henry, and a capacity of* 50 *microfarads are connected in parallel across* 600-*volt mains working at a frequency of* 40. *Calculate the current in each circuit, and the total current taken from the mains. Also find the phase difference between the total current and the applied voltage, and whether the current leads or lags.*

Assume that the above circuits are arranged exactly as those in Fig. 85.

Then, current through resistance $= I$,

$$= \frac{E}{R} = \frac{600}{100} = 6 \text{ amps.}$$

Current through inductive circuit $= I_L$,

$$= \frac{E}{2\pi f L} = \frac{600}{2\pi \times 40 \times \cdot 5} = 4{\cdot}76 \text{ amps.}$$

Current through capacity circuit $= I_K$,

$$= 2\pi f K E = 2\pi \times 40 \times \cdot 00005 \times 600 = 7{\cdot}54 \text{ amps.}$$

From Formula 23, we have :—

$$\text{Resultant current} = \sqrt{I_R{}^2 + (I_L - I_K)^2},$$

$$= \sqrt{(6)^2 + (4{\cdot}76 - 7{\cdot}54)^2},$$

$$= 6{\cdot}6 \text{ amps.}$$

Again, from Formula 24,

$$\cos \phi = \frac{I_R}{I},$$

$$= \frac{6}{6 \cdot 6} = \cdot 91.$$

$$\therefore \quad \phi = 24 \cdot 5^\circ$$ (See Table of Cosines, p. 271.)

As $I_L - I_K = 4 \cdot 76 - 7 \cdot 54 = -2 \cdot 78$; *i.e.*, as $(I_L - I_K)$ is negative, the current leads in front of the voltage by 24·5°.

The power factor of the circuit is ·91 (§ 44).

37. **EFFECTS OF AN ALTERNATING CURRENT ON THE INSULATION OF A CIRCUIT.**—In a conductor carrying an alternating current at a given virtual voltage, there is a greater tendency for the electricity to leak through or break down the insulation than in the case of a continuous current at the same voltage. The reasons for this are that in the first case the current is moving rapidly backwards and forwards, and that the maximum value of the impressed e.m.f. is 1·41 times its virtual value (p. 78); while in the latter case it is flowing steadily in one direction, and the e.m.f. is also steady. Thus, to send a continuous current of 30 amperes through 10 ohms would require a voltage of 300; and the voltage would remain steady at this value. With alternating current, the virtual voltage would be 300, so that the actual voltage would vary between 0 and a maximum of 423 volts on either side of zero.

The insulation of the alternating-current circuit would thus—roughly speaking and (considering the effect of voltage only)—have to be $1\frac{1}{2}$ times as good as that of the continuous-current circuit; for the maximum voltage in it would be nearly $1\frac{1}{2}$ times as great.

There is also another difference as regards the effect on the dielectric or insulation when alternating voltage

is used. In Fig. 87 is given a section of a cable conductor, its insulation, and the surrounding sheathing, pipe, or Earth. Let the conductor be carrying a steady continuous current, and suppose that particular portion of it under consideration to be at a higher potential than the Earth;* it will then have a steady +charge. Influence (electrostatic induction) will take place, and a − charge will be induced on the inner surface of the metallic sheathing of the cable, or other surroundings, the system acting like a condenser. A stress will consequently be set up in the insulation, due to the fact

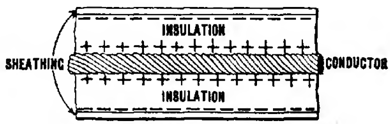

Fig. 87.—Electrification of Conductor Dielectric.

that these charges will mutually attract each other. With an alternating voltage, the charges will be constantly alternating in sign, so that the stress in the insulation will be reversing at the same rate. The insulating material, in consequence, becomes *fatigued*, and will be less effective than if the pressure were steady, as with continuous current. If *break-down tests* were applied to similar samples of cable, with alternating and continuous voltages respectively, it would be found that the insulation would fail sooner in the first case. A break-down dielectric test is one in which a high and gradually increasing pressure is applied until the dielectric is actually pierced by a spark.

The *dielectric-fatigue* effect just mentioned is very

* Though there is a gradual fall of pressure or potential along a conductor carrying a continuous current, there is almost always bound to be a p.d. between nearly every part of the conductor and the Earth, the conductor being either +ly or −ly charged with respect to the Earth.

similar to the phenomenon which occurs with metals when they are subjected to alternate tension and compression. It is well known that under such conditions, a steel rod (for example) will break at a far smaller load than it would if the load were kept steady either as tension or compression.

An alternating stress in an insulating material is accompanied by a small loss of power. The dielectric in fact offers a certain opposition to the setting-up of alternating charges. This is known as *dielectric hysteresis*, and is somewhat analogous to magnetic hysteresis.

From the foregoing it will be seen that the insulation of an alternating-current circuit is subjected to more severe stresses than that of a continuous-current circuit working at the same voltage: firstly, because the maximum voltage is higher; and secondly, because of dielectric fatigue.

CHAPTER II.—QUESTIONS

In answering these Questions, give Sketches wherever possible.

NOTE.—Questions marked * range slightly beyond the subject-matter of this Book. Those marked † can only be partly answered therefrom.

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1. What effects are produced by passing a current through a coil of wire in which a solid iron core is placed, and would the effect be the same with direct as with alternating current? (*Wiremen's, Grade I.*, 1914.)

2. What are the causes of loss of power in a three-phase cable? Would you use a three-core steel-armoured cable in

preference to three single-core steel-armoured cables, the cores of each cable having the same section? In which system would you expect to get the greater loss? In what respect does the core of an extra high tension cable differ from that of a low tension cable? (*Ord.*, *A.C.*, 1910.)

†3. What is the object of armouring cables? Why must all the conductors of an alternating-current circuit lie within the same armouring? (*Grade II.*, *A.C.*, 1913.)

4. Is there any objection to using steel conduit or piping on an installation supplied with alternating current, and does it matter how many conductors are run in one pipe? (*Wiremen's, Grade I.*, 1914.)

5. How does the presence of self-induction in any circuit affect an alternating current therein (*a*) when the current is of low frequency; (*b*) when the current is of very high frequency; (*c*) when the circuit is one in which a condenser has been interposed? (*Ord.*, *A.C.*, 1908.)

6. Describe what is meant by "skin effect." (*A.M.I.E.E. Exam.*, 1914.)

†7. Show that if for any reason a current does not distribute itself with equal density through the cross-section of a conductor of given material, the energy lost by heat in that conductor will be greater than would be the case if the current density were uniform all over the cross-section. Also show why with high-frequencies the resistance offered by a cylindrical conductor is greater than that offered by the same conductor to an equal current of lower frequency. (*Ord.*, *A.C.*, 1909.)

8. If the form of the wave-curve of an alternating electromotive force be given, show how to find, by a graphic construction, the virtual (root-mean-square) value of the electromotive force. (*Ord.*, *A.C.*, 1909.)

9. What is the meaning of the term "form-factor," as applied to an alternating quantity? Suppose an alternating current to have the following successive values at successive equal intervals during one period, beginning from zero:—3, 4, 4·5, 5·5, 8, 10, 6, 0, −3, −4, −4·5, −5·5, −8, −10, −6, 0.

Find the root-mean-square (or "virtual") value, and find also the form-factor. In what conditions is the form-factor equal to 1·11? (*Ord., A.C.*, 1910.)

Ans. R.M.S. value = 5·67 : form-factor = 1·105.

10. Explain how electrical quantities, which are varying periodically, and their relations can be represented by vector diagrams. (*Grade II., A.C.*, 1913.)

†11. What is meant by the *wave form* of an alternating current? Show how such forms are represented graphically (*a*) by rectangular co-ordinates, (*b*) by polar co-ordinates. Draw, both as rectangular and polar curves, the form of the electromotive-force wave generated by a circular coil rotating about a diameter that is at right-angles to a uniform magnetic field. (*Ord., A.C.*, 1908.)

†12. What is the "co-efficient of self-induction" of a coil? An iron magnetic circuit has a mean length of path of 50 centimetres and a cross section of 20 square centimetres. It is wound with 200 turns of wire. Find the total flux in the magnetic circuit when the coil carries a current of 1 ampere. Neglecting the resistance, find the voltage which must be applied to the terminals to make the current rise at the rate of 2 amperes per second. Take the permeability of the iron as constant at 2000 c.g.s. units. (*Grade II., A.C.*, 1914.)

AUXILIARY NOTE.—Ampere-turns =

$$\cdot 8 \times \frac{\text{mean length of magnetic circuit (cms.)} \times \text{total flux through coil.}}{\text{permeability} \times \text{cross-section of magnetic circuit (sq. cms.).}}$$

Hence total flux can be determined.

The inductance is then calculated by Formula 6.

According to Formula 5, if the current is varied at the rate of I amperes per second,

Average e.m.f. induced $= LI$ volts.

Hence this will be the voltage to be applied to the coil to cause the current to rise at the rate of I amperes per second.

Ans. 2×10^{6} lines ; ·8 volts.

13. In an inductive circuit with inductance L and resistance R, determine the relation between current and potential difference when the circuit is carrying an alternating current

of f cycles per second. Find also the phase difference between current and potential difference for such a circuit. (*Grade II., A.C.*, 1913.)

14. An electromotive force of 100 volts, virtual value, is impressed on a circuit consisting of a resistance of 5 ohms in series with an inductance of 0·01 henry. State current and power factor. The frequency is 50 cycles per second. (*Ord., A.C.*, 1911.)

Ans. 17 amps.; p.f. = ·85.

15. Calculate the current in an inductive circuit supplied at 200 volts 50 periods, when the resistance is 10 ohms and the inductance 0·02 henry. (*Wiremen's Final*, 1914.)

Ans. 17 amps.

16. A sine wave alternating current of 15 amperes at 50 frequency flows through a coil, which has a resistance of 3 ohms and an inductance of 0·313 henry. Draw vectors to represent this current, the voltage drops in the resistance and inductance, and the impressed terminal voltage. (*Grade II., A.C.*, 1914.)

Ans. 45 volts, 1475 volts, 1476 impressed volts.

17. Explain the effect of applying to the terminals of a condenser, the resistance of which is many megohms, an alternating electromotive force. What electromotive force will be required to drive 10 virtual amperes through a circuit containing a condenser of which the resistance is 1200 megohms, and its capacity 22 microfarads, the frequency of supply being 80 periods per second. (*Ord., A.C.*, 1908.)

Ans. 905 volts.

18. An alternating potential difference of 2000 volts at 50 frequency is applied to test a cable of 14 microfarads capacity through a resistance of 110 ohms. Find the current, which will be taken from the supply, and the phase difference between the current and the impressed potential difference. (*Grade II., A.C.*, 1914.)

Ans. 7·9 amps. and 64°.

†19. Explain the terms: reactance, power factor, hysteresis, impedance. (*A.M.I.E.E. Exam.*, 1914.)

20. Give the physical (non-mathematical) reasons why a condenser produces a lead, and why a self-induction produces a lag, in an alternating current, At what frequency will a capacity of 1 microfarad, and a self-induction of 1 henry exactly annul one another's effects? (*Ord.*, *A.C.*, 1910.)

Ans. 159.

21. A coil, having an inductance of 0·08 henry and a resistance of 2 ohms, is connected in series with a capacity of 30 microfarads. A potential difference of 50 volts is applied to this circuit and the frequency is varied. It is found that for one value of the frequency the current through the circuit reaches a maximum. Calculate this frequency, and find the potential difference between the terminals of the condenser. (*Grade II.*, *A.C.*, 1913.)

Ans. 103 and 1290 volts.

22. A resistance of 100 ohms, an inductance of 2 henries, and a capacity of 1·25 microfarads, are all put in series between the terminals of an alternating supply of a frequency of 80 periods per second, at an electromotive force of 1000 volts (virtual, or r.m.s. volts). Find the current and the power-factor, also the voltage across the terminals of the capacity and that across the terminals of the inductance. (*Ord.*, *A.C.*, 1909.)

Ans. 1·68 amps., ·168 p.f., 2680 volts, 1690 volts across inductance.

*23. Explain what is meant by "Electrical Resonance." If a capacity of 10 microfarads is placed in series with a choking coil and the whole circuit supplied with current at a frequency of 50 cycles per second, what value of inductance of the choking coil will give resonance? (*Ord.*, *A.C.*, 1911.)

Ans. 1·015 henries.

AUXILIARY NOTE.—The physical meaning of resonance has been explained in § 18. The actual condition for electrical resonance to occur is that the inductive reactance should equal the capacity reactance, *i.e.*, that $2\pi fL = \frac{1}{2\pi fK}$. Transposing, we have $f^2 = \frac{1}{4\pi^2 LK}$. $\therefore f = \frac{1}{2\pi\sqrt{LK}}$.

The value $\frac{1}{2\pi\sqrt{LK}}$ is known as the *natural frequency* of the circuit, and resonance takes place if the frequency of the applied voltage coincides

with the natural frequency. In the above question, f and K are known and L has to be determined.

24. Give an instance of the circumstances in which resonance may arise at some point of a three-phase supply system, and explain the cause of such resonance. (*A.M.I.E.E. Exam.*, 1914.)

25. A certain choking coil of negligible resistance takes a current of 8 amperes if supplied at 100 volts, at 50 periods per second. A certain non-inductive resistance, under the same conditions, carries 10 amperes. If the two are transferred to a supply system working at 150 volts, at 40 periods per second, what total current will they take (*a*) if joined in series; (*b*) if joined in parallel? (*Ord., A.C.*, 1908.)

Ans. (*a*) 10·6 amps.; (*b*) 21·2 amps.

26. Between two terminals, with a potential difference of 2000 virtual volts at 50 frequency, are connected in parallel a resistance of 400 ohms, an inductance of 1 henry, and a capacity of 10·3 microfarads. Give the current in each of the circuits and the total current taken from the terminals and its power factor. Illustrate your answer by a vector diagram. (*Grade II., A.C.*, 1912.)

Ans. $I_R = 5$ amps., $I_L = 6·37$ amps., $I_K = 6·47$ amps., $I = 5·001$ (say, 5) amps., p.f. = ·9998 (say, 1).

27. Between two terminals with a potential difference of 1000 virtual volts at 50 frequency are connected in parallel a resistance of 200 ohms, an inductance of 0·5 henry, and a capacity of 20·6 microfarads. Give the current in each of these circuits, and the total current taken from the terminals and its power factor. Illustrate your answer by a vector diagram. (*Ord., A.C.*, 1911.)

Ans. $I_R = 5$ amps., $I_L = 6·37$ amps., $I_K = 6·47$ amps., $I = 5$ amps., p.f. = 1.

*28. A circuit, having a resistance of 200 ohms and a coefficient of self-induction of 1 henry in series, is connected across the terminals of a 10,000-volt alternating supply. A capacity of 6 microfarads is also connected across the same

terminals. State the total current from the supply at a frequency of 50 periods per second. (*Ord.*, *A.C.*, 1910.)

Ans. 14·9 amps.

AUXILIARY NOTE.—The simplest method to work out this exercise is first to calculate the current and the phase displacement for each circuit. A vector diagram is then drawn, as in Fig. 87A, to represent these

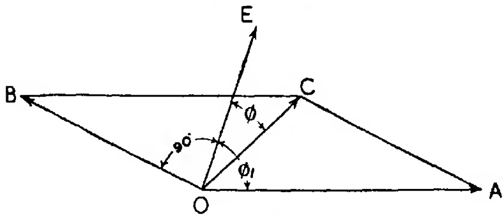

Fig. 87A.

quantities to scale. Thus, *OA* is the current in the inductive circuit, lagging ϕ_1 degrees behind the impressed volts *OE*, ϕ_1 being determined by Formula 15; and *OB* is the capacity current leading 90° in front of *OE*. The resultant current is given by the diagonal *OC* obtained by completing the parallelogram *OACB*.

The phase difference ϕ between the total current and the impressed voltage is obtained by measuring the angle *COE* in Fig. 87A.

[It should be noted that Fig. 87A has *not* been drawn to scale.]

*29. An alternating electromotive force of 3000 volts (virtual or r.m.s. volts), of a frequency of 50 periods per second, is maintained between two supply terminals. The electromotive force follows a simple sine-curve. The terminals are connected by two circuits, one containing a resistance of 200 ohms in series with an inductance of 0·5 henry, the other a capacity of 5 microfarads but no resistance. Determine the total current taken from supply terminals and the power factor of the united circuit. (*Ord.*, *A.C.*, 1909.)

Ans. 9·62 amps. and ·96 p.f.

CHAPTER III.

POWER, POLYPHASE CURRENTS, ETC.

38.—POWER IN ALTERNATING-CURRENT CIRCUITS.—The power in a continuous-current circuit is obtained simply by multiplying together the pressure and the current. If the circuit has no back e.m.f. in it, the power therein may also be obtained by multiplying together the square of the current and the resistance. The product in either case represents watts.*

It might be thought that by taking the product of the volts and amperes in an alternating-current circuit, we should also obtain the actual power developed. Such would be true in a sense; but the product would only represent *real power* when the current was in phase with the e.m.f. In any case, the product would give the *volt-amperes* or *apparent power* or *apparent watts.*

The phase difference or angle of lag or lead (§§ 20 and 36) has to be taken into account in an alternating-current circuit; and the greater this is, the less is the real power developed with a given pressure and current. In fact, if the phase difference be very great, *i.e.*, if there be a large amount of either inductance or capacity in a circuit of comparatively low resistance, the current will be largely *reactive*, or in other words, more or less *wattless.* The *real power* will then be far less than the *apparent power.*

* The *watt* is a unit of power or rate of doing work. *See* the Author's *Electric Circuit Theory and Calculations.*

The idea of a *wattless current*, *i.e.*, a current with little or no power in it, is difficult to grasp. If there be any current at all, it is not easy to understand why it cannot do some useful work. But when it is remembered that a flow of electricity—as of water—must have pressure behind it to enable it to do work, and when we are dealing with alternating pressure and flow, and can conceive that they may be more or less out of step with each other, comprehension becomes fairly simple.

The following analogy affords a rough but useful explanation. Let *PP* (Fig. 88) represent a pipe filled with water (or a conductor forming a closed circuit), and W, W_1, W_2, and W_3, water-wheels to which an alternating movement may be given by means of the handles *h*, *h*, *h*, *h*. These water-wheels are to be regarded as alternators, and when we are considering any one of them, the other three are supposed to be absent. Let *WW* be a fifth water-wheel, to which a reciprocating motion is imparted by the to-and-fro movement of the water in the pipe. The motion of water in the lower part of the pipe-circuit may be considered as analogous to "active current," or current with power in it; and the consequent movement given to *WW* as analogous to the real power.

We will first consider a case where there is no phase difference—that is, when the watermotive-force of *W* acts directly in line with the circuit, as indicated by the dotted line. These conditions represent the e.m.f. of *W* acting with best effect upon the electricity (or water) in the circuit; and the current (or motion given to the water by *W*) acts with best effect in operating *WW*. Then the volt-amperes (*i.e.*, e.m.f. × current) will also represent the real watts (actual power given to *WW*).

To illustrate the effect of a small phase difference, we will next consider the water-wheel as placed slightly

skew with the circuit, as at W_1: the angle of lag (or lead) being represented by the angle a between the two lines. Suppose the frequency (rate of reversal) and e.m.f. of W_1 to be the same as in the first case (W); the current or actual movement of electricity (water) immediately about W_1 will also be the same. But as part of the pressure will be uselessly employed in driving the water against the sides of the pipe, the repelling effect of which may be looked upon as analogous to the counter e.m.f. of reactance; the pressure and the current about WW (the product of which is the real power) will be less than those generated at W_1. The backwash from the sides of the pipe is thus analogous to reactive or wattless current.

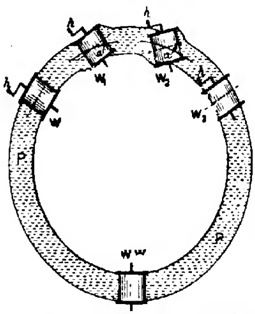

Fig. 88.—Hydraulic Analogy of a Wattless-Current Circuit.

If there be a still greater phase difference, as at W_2, though the e.m.f., current, and therefore also the volt-amperes given out be the same as in the first case (W), the actual or real power at WW will be still further lessened.

If the phase difference be 90°, as at W_3, where the water-wheel is placed at right angles with its most effective position, we may suppose that the current will be wholly reactive or wattless, *i.e.*, that there will be no real power developed, and consequently no movement of WW. In practice, however, the phase difference can never become so great as this.

As already stated, the above analogy is a very

rough one; but it will serve its purpose if it helps the reader to understand that a given number of volts and amperes in an alternating-current circuit may represent, under different conditions, different amounts of *real power*.

39. **POWER IN ALTERNATING-CURRENT CIRCUITS** (*cont.*).—For further analogies showing the effect of the phase difference between the current and the voltage upon the actual power, let us reconsider some of the mechanical devices used in the earlier part of the book.

In Fig. 44, representing a circuit with resistance only, the direction in which V is moving (current) corresponds at every instant with the direction of the alternating twist (e.m.f.) applied at R; it being assumed that the device (circuit) has absolutely no momentum (inductance) or elasticity (capacity). The current is then in phase with the e.m.f. When the twisting force is stopped, the vane immediately stops also, or would do so if it really had no momentum. In other words, after the e.m.f. has been removed from a circuit with resistance only, there is no tendency for the current to "persist" or keep on. If it did do so, it would give back some of the energy absorbed.

It is necessary in some of the following arguments to assume that the e.m.f. (twisting force) is stopped without opening the circuit. Opening the circuit immediately increases its resistance enormously, and is equivalent to suddenly pinching or clamping the rod R between the fingers in order to stop its movement.

In Fig. 45, when the twisting force is in one direction only (continuous or direct e.m.f.), if that force be stopped (without opening the circuit), the mass L tends to continue its rotation in the direction in which it happens to be moving, just as a top spins. But as it is

impossible to devise any moving system (or electrical circuit, including the source of e.m.f.) entirely devoid of friction (resistance), the continued movement of L (inductive current) is eventually subdued; the energy stored in L (inductive e.m.f. and current) being gradually transformed into heat. If the e.m.f. be cut off by opening the circuit (suddenly clamping R), it will be found impossible to stop it immediately. This is the equivalent of the inductive spark at the break in the circuit.

When R is subjected to an alternating twist (Fig. 45), the mass L will be alternately absorbing and giving out energy; and assuming that there was no friction loss (no resistance in the circuit), the energy absorbed would be equal to that given out, so that the net energy absorbed would be zero. It is evident from Figs. 46 and 48 that the speed of L (current) lags a quarter of an oscillation behind the twisting force (applied e.m.f.). Hence if there could be a phase difference of a quarter cycle (90°) between the current and the voltage, the nett energy absorbed would be zero; and the current would then be entirely reactive or wattless.

In Fig. 50, the spring S returns to its original position when the pressure on R is released, and in so doing, gives back the energy absorbed by it while it was being twisted. Since we are assuming that there is no friction, the energy given back by the spring must be exactly equal to that absorbed by it. And it was shown in Figs. 51 and 52 that the speed of S (current) was a quarter of an oscillation in front of the twisting force (applied e.m.f.). Hence in a circuit containing capacity only, the net energy consumed would be zero.

In a circuit possessing capacity and inductance but no resistance, as in Fig. 54, it was explained on p. 56, how after giving R a twist and then letting it go, the

pointer p will oscillate backwards and forwards. If there were no friction at all, this movement would continue indefinitely. In this case, we have the mass L being retarded by the torsion of S, the latter stress afterwards accelerating L in the opposite direction. In other words, when being retarded by S, L is delivering energy to it; while at a later instant, S is giving back that energy to L in accelerating it. All this takes place without any energy being absorbed from any outside source. In practice it would be impossible to have an electric circuit with inductance and capacity only; there must always be some resistance, so that there is always some power being absorbed.

The above analogies show that though the twisting force (applied e.m.f.) and the speed of the moving system (current) may be large, the power absorbed will be small if the phase difference that exists between the two quantities is considerable.

40. **POWER AND ENERGY IN ALTERNATING-CURRENT CIRCUITS.** — The effect of a difference in phase between the current and the voltage upon the power absorbed in a circuit is very plainly indicated in the following graphs or curves.

Fig. 89 shows the current (I) and the voltage (E) in phase with each other. At any instant, such as XX, the power is the product of the values of E and I at that instant. The variation of the power is shown by curve P, each point on which is got by multiplying together the corresponding values of pressure and current. The power curve in this case is always positive, since the current and the voltage are always either both positive or both negative; the product of two + or two − values being always +. The total energy consumed in the circuit in a certain time is represented by the shaded portions. In other words, the area of those portions

would represent to scale so many kelvins* of work done in the time denoted by the length along the time base.

If the current and voltage were 90° out of phase, the net power and energy absorbed by the circuit would be zero, because the current would be entirely reactive or wattless, as has been previously explained. This state of things is diagrammed in Fig. 90, where it will be

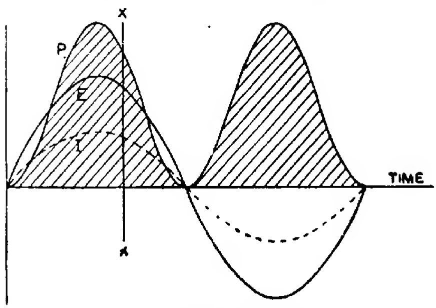

Fig. 89.—Curves showing A.C. Power in a Circuit with Resistance only.

noticed that the power curve P and the shaded energy-areas are alternately above and below the time base.

Let us first see why the power curve takes this form. At the moment b on the time base, the pressure is represented by the length ba and is +, while the current is represented by the length bc and is −. The product of these two lengths (*i.e.*, the power) is bd, and is negative; it is therefore marked off below the time base. At the instant e, both pressure and current values are +, and therefore the power is also +. At the instant f, the

* The *kelvin* or *Board-of-Trade unit* is the unit of electrical energy or work. *See* the Author's *Electric Circuit Theory and Calculations*.

pressure and current values are both –, so that their product, the power, is again +.

Let us now try to understand why the alternation of the power between + and – has a very different effect from the alternation of pressure or current. In Fig. 89 it was seen that though the pressure and the current were both alternating "in sign" (as they always do in an a.c. circuit), the power was wholly positive.

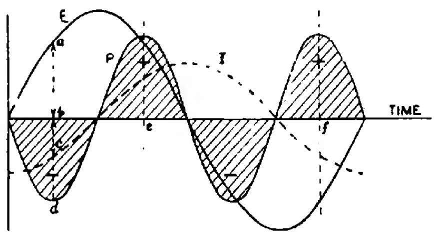

Fig. 90.—Curves showing A.C. Power in an Imaginary Circuit with Inductance only.

This means that the power was at every instant being absorbed by the circuit. When the power becomes negative, it signifies that it is being *returned* by the circuit under the reactive effect of inductance or capacity. In Fig. 90, representing the imaginary case of a wholly reactive circuit, it is evident, as already explained, that all the power and energy given to the circuit is returned.

Figs. 89 and 90 represent the two extremes as regards power in an a.c. circuit; and though the conditions in Fig. 89 sometimes occur, it never happens that a circuit has inductance only, as in Fig. 90.

For an imaginary circuit possessing capacity only, the shape of the curves would be exactly as those in

Fig. 90, except that curve I would be leading 90° in front of instead of lagging 90° behind curve E.

A circuit always has resistance, and more or less inductance and capacity. Fig. 91 shows the power conditions in a circuit with resistance and some inductance, *i.e.*, one in which the voltage and current are only a little out of phase. Here the curve P represents the variation of power from instant to instant; and it

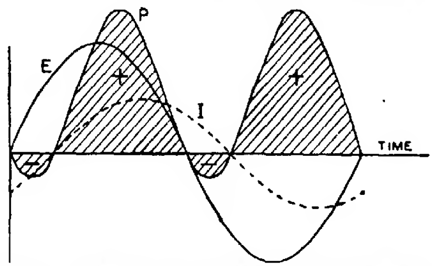

Fig. 91.—Curves showing A.C. Power in a Circuit with Resistance and Inductance.

will be seen that the sum of the energy-areas marked + is greater than that of those marked −. The net energy absorbed is represented by the difference between the + and − areas. The nearer the curve I comes into phase with curve E, the larger will be this net area, and *vice versa*.

In a circuit with resistance and capacity the current would lead instead of lag; but the result, so far as the power was concerned, would be more or less the same as in Fig. 91.

In a circuit with resistance, inductance, and capacity, there would be similar E, I, and P curves giving alternations of + and − power and energy; the value of which would vary with the phase difference: and the

latter would depend on the resultant effects of the resistance, inductance, and capacity.

Thus with a given voltage and current, the real power developed, and the energy absorbed, are the greater the less the phase difference.

41. **POWER PHENOMENA IN REACTIVE CIRCUITS.**—It must be borne in mind that a reactive or wattless current is only wattless or ineffective so far as its power of doing *useful work* is concerned. If there is any current at all, it must do something. Let us see what is done in a case where the circuit contains inductance only, and let us refer again to Fig. 90. The + and – power and energy alternations represent the power and energy given to and returned by the alternating magnetic field of the circuit. As the current increases from zero to its maximum in either direction, energy is absorbed in setting-up the magnetic lines about the circuit. As the current decreases from its + or – maximum, the magnetic field collapses, and in so doing, gives back its energy to the circuit in the form of an induced e.m.f. and current.

Suppose an alternator to be supplying a purely inductive load, such as that in Fig. 67. During the quarter cycle when the current is increasing from zero to maximum, power is being supplied by the alternator to build up the magnetic field of the circuit. During the next quarter, the current is decreasing, and therefore power is being returned by the magnetic field as it collapses or weakens, and this returned power tends to drive the alternator as a motor. If, as in this case, the circuit be assumed to have no resistance, the energy taken from the alternator during the first quarter of a cycle would be equal to that given back during the second quarter of the same cycle, and the resultant energy absorbed by the circuit would be zero. In

other words, the current would be wholly reactive or wattless.

The same effect would take place if the circuit consisted of capacity only, as in Fig. 69; the alternate charging and discharging of the condenser alternately taking energy from and returning it to the alternator.

In practice no circuit can be entirely devoid of resistance, so that the current could not be wholly reactive in the two cases just considered. Thus more or less electrical power would be converted into heat in all parts of the circuits.

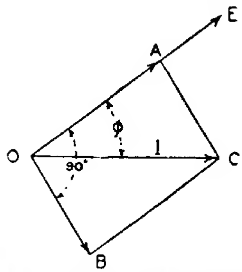

Fig. 92.—Vector Diagram of Active and Reactive Components of a Current.

42. **ACTIVE AND REACTIVE COMPONENTS OF A CURRENT.**—When a current I lags behind the e.m.f. E by an angle ϕ, as in Fig. 92, that current can be considered as being made up of two components or parts; viz., one component in phase with the voltage, and the other component lagging 90° behind the voltage. The values of these components are obtained by drawing CA from C perpendicular to OE, and completing the parallelogram with OB and BC parallel with AC and OA respectively. Then OA represents the current component in phase with the e.m.f., and OB the component in quadrature therewith, *i.e.*, 90° behind it.

Since the component OA and the voltage are in phase, the power absorbed in the circuit is given by the product of the two, as already explained in connection with Fig. 89. On the other hand, it was shown in

Fig. 90 that when the voltage and the current are 90° out of phase, the current is entirely reactive or wattless, and the net power absorbed is zero. Hence, in the case illustrated in Fig. 92, the component OA is known as the *active*, or *power*, or *useful component* of the current; and OB as the *reactive* or *wattless* or *useless component* thereof; OC being the current itself.

From Fig. 92, it follows by trigonometry (pp. 31 and 100) that:—

$$\text{the active component of the current} = OA = I \cos \phi \qquad \textbf{(26)}$$

and that:—

$$\text{the reactive component of the current} = OB = AC = I \sin \phi. \qquad \textbf{(27)}$$

In words, Formula 26 shows that the useful component of a current is got by multiplying the actual current by the cosine of the angle of phase difference. And Formula 27 shows that the useless component of a current is obtained by multiplying together the actual current and the sine of the angle of phase difference.

The reactive component of an alternating current is objectionable in central-station work for three reasons.

Firstly.—The maximum current that can be carried by the armature of an alternator is determined by the permissible limit of temperature-rise in that armature. The greater the phase difference, the greater must be the reactive component of that maximum current, and the less must be its active or useful component. In other words, the greater the reactive component the less is the maximum working load that may be put upon that alternator; since the total or apparent power (in volt-amperes) developed with a given e.m.f. is limited by the maximum current the armature conductors can carry.

Secondly.—If the current lags behind the e.m.f., as is nearly always the case, *i.e.*, if it has a reactive com-

ponent, it tends to demagnetize the field. To compensate for this effect, a stronger field is necessary; and the power to be expended in excitation is consequently increased.

Thirdly.—Owing to the greater current necessary for a given useful load, the cross-section of the cables has to be increased in order to keep the voltage-drop and their temperature-rise within permissible limits.

The moral of all this is that the phase difference should be kept as small as possible.

43. **CONNECTION BETWEEN POWER, PRESSURE, CURRENT, AND PHASE DIFFERENCE.**—It was shown in the previous section that the useful power absorbed in a circuit is the product of the voltage and the active component of the current. And by Formula 26, (p. 139) the active component of any current $I = I \times \cos\phi$, ϕ being the phase difference. Thus, if E be the e.m.f., I the current, ϕ the angle of lag or lead, and P_W the real or useful power (in watts) then,—

$$P_W = \text{voltage} \times \text{active component of the current},$$
$$= EI\cos\phi. \qquad \textbf{(28)}$$

On reference to the Table on p. 271 it will be seen that the cosine of 0 degrees is unity, or 1. Thus, in the formula above, if the current and pressure are in phase —*i.e.*, if there is no phase difference (angle of lag or lead)—the real power will be obtained simply by multiplying together the volts and amperes in the circuit.

As the phase difference increases, the cosine values decrease below unity, thus $\cos 10° = \cdot985$, $\cos 30° = \cdot866$, $\cos 80° = \cdot174$, $\cos 90° = \cdot000$; and the real watts become proportionately less than the apparent watts. Thus, supposing the phase difference $\phi = 60°$, the real watts will be only half the apparent watts, for $\cos 60° = \cdot5$. It will thus be seen once more how important it is to keep the phase difference as low as possible.

In practice, ϕ cannot be worked out directly with any degree of accuracy, for it varies with every variation in the conditions of the circuit; though it can, of course, be calculated for any given case. The real watts may be ascertained by means of a wattmeter (§ 54).

44. **POWER FACTOR.**—From what has gone before, it will be clear that the apparent watts put into a circuit feeding arc lamps, motors, or other inductive apparatus, as calculated from the indications of a voltmeter and ammeter at the generating-station end, may be greatly in excess of the actual power conveyed to the lamps, etc. Thus the indications of these instruments may give the idea that a far larger number of consuming devices are in circuit than is actually the case, owing to the reactance in the circuit.

The following record of actual observations furnishes an instructive example. The current supplied by an alternator to an inductive load was 44 amperes, and the pressure 2050 volts. The exciting current of the alternator for a corresponding non-inductive load would have been between 50 and 55 amperes; but this had to be increased to about 75 amperes in order to maintain the 2050 volts pressure at the alternator terminals. The reason for this was explained under *Secondly* on page 139.

By tests made with a wattmeter, which measures real power, it was found that the latter was only 56,000 watts.

Thus, in this case—

$$\begin{aligned} \text{Apparent watts} &= 2{,}050 \times 44, \\ &= 90{,}200, \\ \text{and} \qquad \text{Real watts} &= 56{,}000. \end{aligned}$$

Consequently the ratio between the real and apparent watts was $\dfrac{56{,}000}{90{,}200} = \cdot 62.$

This ratio is known as the *power factor* (abbreviated *P.F.*, or *p.f.*) of the circuit.

Thus—

$$\text{Power factor} = \frac{\text{real watts}}{\text{apparent watts}}, \qquad \textbf{(29)}$$

$$= \frac{\text{volts} \times \text{amperes} \times \cos\phi}{\text{volts} \times \text{amperes},} \qquad \textbf{(29A)}$$

$$= \cos\phi. \qquad \textbf{(29B)}$$

Hence the power factor is given by the cosine of the angle of lead or lag, *i.e.*, the cosine of the angle of phase difference; and it is the fractional quantity or "factor" by which the apparent power has to be multiplied in order to determine the real power.

The determination of the power factor with an inductance or capacity in a series circuit, and with both in series or in parallel circuits, was described on pp. 100, 106, 110, 116, and 117 respectively.

The necessity of keeping the phase difference small and the power factor of a circuit as near unity as possible should now be obvious, and this is one of the chief problems in alternating-current distribution and application. It is in this connection that the question crops up of introducing capacity to reduce the lag, as alluded to in § 36.

Condensers are now manufactured specially for power factor correction purposes. They are coming into fairly general use. Synchronous motors, however (especially in the form of rotary converters), when strongly excited, behave like condensers in so far that they take a leading current; and they have been largely employed for improving the power factors of distributing systems.

During the last few years, devices of different forms, known as *phase advancers*, have been introduced. These, when connected to the rotors of induction motors (which

are the worst offenders in lowering the power factor of a supply system) improve the power factors of their circuits. They can even be arranged to cause their motors to take a leading current, and so tend to compensate for the lagging currents taken by other induction motors working off the same supply but not fitted with phase advancers. The theory of the action of these apparatus is too complicated for treatment in this elementary book.*

Whether the phase difference be due to excessive inductance or capacity in the circuit, the effect of either may be partly or wholly neutralized by the introduction of the opposite effect; and the power factor will consequently be improved. This will be evident from Figs. 78 and 86.

As a rule, a power-factor value relates to a circuit in which the current is lagging. When it refers to a circuit in which the current is leading, the distinction may be made by adding the word *lead* or *leading*, thus:—p.f. = ·71 leading.

44A. **EXAMPLE ON THE CALCULATION OF POWER.** —*A resistance of* 200 *ohms, an inductance of* 2 *henries, and a capacity of* 3 *microfarads are all connected in series across mains working at* 500 *volts and* 50 *cycles. Find the power consumed in the circuit.*

1st Step.—Find the impedance.

This we can do by means of Formula 22A, p. 109, thus:—

$$\text{Impedance} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fK}\right)^2}$$

In the present case, $R = 200$ ohms, $L = 2$ henries, $K = \cdot000003$ farads, and $f = 50$.

$$\therefore \text{Impedance} = \sqrt{(200)^2 + \left(2\pi \times 50 \times 2 - \frac{1}{2\pi \times 50 \times \cdot000003}\right)^2}$$

$$= 477 \text{ ohms.}$$

* Readers are referred to *Power Factor Correction*, R. Clayton, M.I.E.E. (*Pitman*).

2nd Step.—Find the current.

$$\text{Current} = \frac{\text{voltage}}{\text{impedance}} \quad (\textit{see}\ \text{Formula 13A, p. 98}),$$
$$= \frac{500}{477} = 1{\cdot}05 \text{ amps.}$$

3rd Step.—Find cos ϕ and the real power.

When resistance, inductance, and capacity are in series, cos ϕ is given by Formula 15, p. 100.

$$\textit{i.e.,}\quad \cos\phi = \frac{\text{resistance}}{\text{impedance}} = \frac{200}{477} = {\cdot}419.$$

$$\therefore \text{power-factor} = \cos\phi = {\cdot}419.$$

$\therefore$ by Formula 28, p. 140,

$$\text{True power consumed} = IE \cos\phi$$
$$= 1{\cdot}05 \times 500 \times {\cdot}419$$
$$= 220 \text{ watts.}$$

45. POWER FACTOR AND EFFICIENCY OF A MOTOR.—From what has been said in the foregoing sections, it will be evident that the matter of power factor is an important one to the user of alternating-current motive-power, as the cost of such power depends very much upon the power factor.

The power factor of an alternating-current motor at any given frequency and load depends largely upon the design of the machine. When the motor is under test, its p.f. may be found by dividing the apparent watts delivered to the motor into the real watts absorbed. The less the reactance of the machine, the greater will be its power factor, as the driving current will then be more nearly in phase with the applied voltage.

It follows from this that to get a given power out of a motor, the less its power factor the greater must be the current sent through it as compared with that which would have been necessary had the motor pos-

sessed unity power factor, which—by the way—is not possible with ordinary types of motor. This excess current, though useless for power, still exercises its full heating effect upon the conductors, and so further increases the losses in the motor circuit.

Hence the input to such a motor is generally expressed in *volt-amperes* (abbreviated *VA*) or in *kilo-volt-amperes* (*kVA*) instead of in watts or kilowatts.

For example, if an a.c. motor is taking a current of 20 amperes, and the pressure is 400 volts, it is better to call the product $(400 \times 20) = 8000$ volt-amperes or 8 kVA instead of 8000 apparent watts, or 8 apparent kilowatts. If it were a continuous-current motor we should know that the 8000 VA, or 8 kVA, would be really 8000 watts or 8 kW.

Thus—

$$\text{Real input in kilowatts} = \text{kVA} \times \text{p.f.} \qquad \textbf{(30)}$$

or

$$\text{kVA} = \frac{\text{real input in kW}}{\text{p.f.}} \qquad \textbf{(30A)}$$

The *commercial efficiency* of an a.c. motor is the ratio of the mechanical power given out to the true power absorbed, both quantities being expressed in the same units (generally watts or kilowatts).

Thus :—

$$\text{Percentage efficiency of a motor} = \frac{\text{output}}{\text{real input}} \times 100, \qquad \textbf{(31)}$$

$$= \frac{\text{real input} - \text{losses in motor}}{\text{real input}} \times 100. \qquad \textbf{(31A)}$$

Since the losses in a motor depend to a certain extent upon the power factor, the latter consequently affects the efficiency. As, in addition, a low power factor possesses the three disadvantages mentioned in § 42, it is necessary to secure as large a power factor

as possible. To help in this, the machine should be of thoroughly good design, it should be speeded as high as permissible, and the lower the frequency of the supply the better. The lower the frequency, the less is the reactance of the motor circuit (p. 96).

The power factor of an ordinary polyphase motor depends upon its type and size, and at full load may be anything between ·7 for slow-speed machines and ·93 for high-speed ones (see Table on p. 377). The power factor of any given motor generally decreases or becomes worse as the load decreases.

EXAMPLE.—**To find**, The Input necessary for an Alternating-Current Motor: **Given**, Its Output, Efficiency, and Power Factor.

A three phase motor has an output of 15 b.h.p. What is its k.v.a. or apparent input, if its efficiency is 86 per cent., and the power factor ·87?

1st Step.—Convert b.h.p. to kW.

$$15 \text{ b.h.p.} = 15 \times 746 \text{ watts} = 11{,}190 \text{ watts} = 11{\cdot}19 \text{ kW}.$$

2nd Step.—From Formula 31, p. 145,

$$\text{Real input} = \frac{\text{output}}{\text{efficiency}},$$

i.e.,

$$\text{Real input} = \frac{11{\cdot}19}{{\cdot}86} = 13 \text{ kW}.$$

3rd Step.—By Formula 30A, p. 145,

$$\text{apparent or kVA input} = \frac{\text{real input}}{\text{power-factor}},$$

i.e.,

$$\text{kilo-voltamperes} = \frac{13 \text{ kW}}{{\cdot}87} = 15 \text{ kVA}.$$

All three steps may be briefly worked together by the following formula—

$$\text{Apparent input (kVA)} = \frac{\text{h.p.} \times 746}{1{,}000 \times \text{eff.} \times \text{p.f.}} \qquad \textbf{(32)}$$

The 1000 is inserted to give the result directly in kilo-voltamperes. If it be omitted, the result will be in volt-amperes. The division by 1000 could be effected at once by changing the 746 into ·746.

Substituting the known values in the above formula we get:—

$$\frac{15 \times 746}{1{,}000 \times \cdot 86 \times \cdot 87} = \frac{11{,}190}{748 \cdot 2} = 15 \text{ kVA}$$

which agrees with the result in the 3rd Step of the first solution.

It is important to note that, in working the above problem, it is necessary to take the p.f. into consideration only when getting the apparent input of the motor; the real input being found directly from the output and efficiency simply.

It is necessary to know the apparent or kVA input in order to find the actual current to be carried. Knowing the latter, the size of cable for connecting the motor to the source of supply can easily be found, for it will depend simply on the current to be carried, and the permissible loss of power or of pressure in the cable.*

46. **POLYPHASE CURRENTS.**—The kind of alternating current dealt with in the preceding portion of this book is that known as the *one-phase*, *single-phase*, or *monophase* current. There are other kinds of alternating current called *polyphase currents.* These are the *two-phase* (or *diphase*), and the *three-phase* (or *triphase*) currents; which may respectively be likened to two or three simple alternating currents set up in different circuits. These two-phase or three-phase currents are of equal frequency and strength, but they differ in phase; that is to say, they lag one behind the other.

As will be seen later, polyphase currents are better

* *See* the Author's *Electric Circuit Theory and Calculations.*

adapted than the single-phase current for the transmission of power over great distances, and for most motive-power work, except railways, etc.

It may be mentioned that 6- and 12-phase currents are sometimes employed; but as their application is very limited, they will not be considered in this book.

47. **TWO- AND THREE-PHASE ALTERNATORS AND THEIR CIRCUITS.**—A simple single-phase alternator

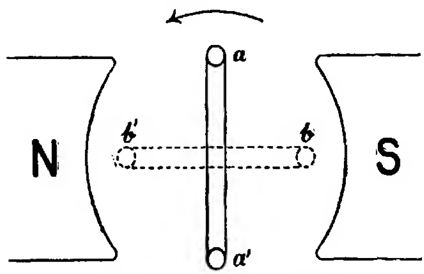

Fig. 93.—Diagram of Simple Two-Phase Alternator.

was illustrated in Fig. 13. A *two-phase alternator* is wound with two distinct coils or series of coils, and the e.m.fs. generated in these are generally applied to two distinct circuits.

Fig. 93 shows the principle of a simple two-pole alternator, with two coils aa' and bb' fixed at right angles to each other, and revolving in a counter-clockwise direction. When the coils are in the position indicated, the e.m.f. being induced in aa' is zero, whilst that in bb' is maximum. As the coils rotate, these e.m.fs., and the currents due to them, vary in magnitude and direction in the manner shown in Fig. 94, where it will be seen that curve b lags 90° or a quarter-cycle behind curve a

The two circuits of a two-phase alternator are generally kept quite distinct from each other, as in Fig. 95, where a and b represent the two coils or series

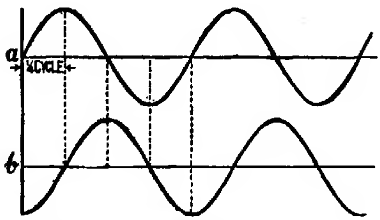

Fig. 94.—Two-Phase E.M.Fs. or Currents.

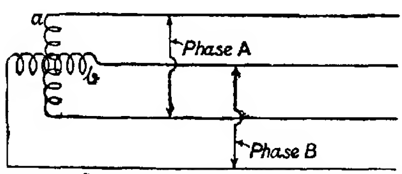

Fig. 95.—Connection of Two-Phase Alternator to Four-Wire Mains.

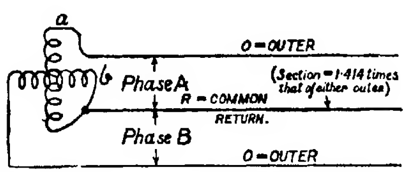

Fig. 96.—Connection of Two-Phase Alternator to Three-Wire Mains.

sets of coils: and four distribution conductors are then necessary. Sometimes, however, the phases are connected together as in Fig. 96, thus dispensing with the

fourth conductor.* The common return wire or main † has to be of sufficient cross-section to carry the resultant current of the two phases.

If OA and OB in Fig. 96A be the vectors representing the currents in the two phases, the one lagging 90° behind the other, then the resultant current is given by the diagonal (OC) of the parallelogram $OBCA$. Now :—

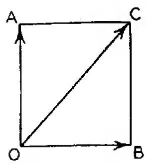

Fig. 96A.—Vector Diagram for Three-Wire Two-Phase Mains.

$$OC^2 = OB^2 + BC^2 = OB^2 + OA^2 = 2\ OA^2$$

since $OA = OB$ when the two phases are equally loaded.

Hence :— $OC = \sqrt{2}\ OA$,

i.e., the resultant current in the common main is $\sqrt{2}$ (or 1·414) times the phase current, so that the common main in Fig. 96 usually has 1·414 times the sectional area of each of the two "outer" mains. **(32A)**

In Fig. 96, the voltage between the outers is 1·414 (or $\sqrt{2}$) times the phase voltage. **(32B)**

The two-phase system is generally only installed when an existing single-phase system is being converted into a polyphase system; the original single-phase alternators being paired-off between the two phases. Thus in Figs. 95 and 96, *a* and *b* might each represent one or more single-phase alternators. But the two machines (or sets of machines in parallel) would have to run 90° apart in phase.

Two-phase distribution is never resorted to in entirely new work, for the chief reason that it requires more copper than a three-phase one.

* In Figs. 95 and 96 the alternator windings appear to be interconnected where they cross, but such is not really the case.

† The different systems of distribution are generally referred to as *two-wire*, *three-wire*, *four-wire*, etc.; but the actual conductors, of course, are really *cables*—not *wires*, except in some isolated cases of overhead distribution.

In *three-phase alternators* there are three coils (or series of coils) displaced equally from one another.

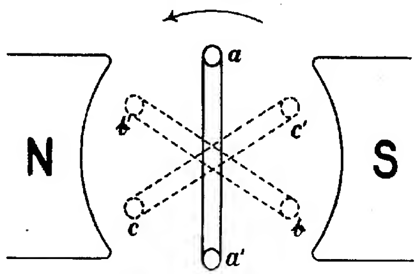

Fig. 97.—Diagram of Simple Three-Phase Alternator.

Thus, for a simple two-pole machine (Fig. 97), the coils aa', bb', and cc', would be at angles of 120° from

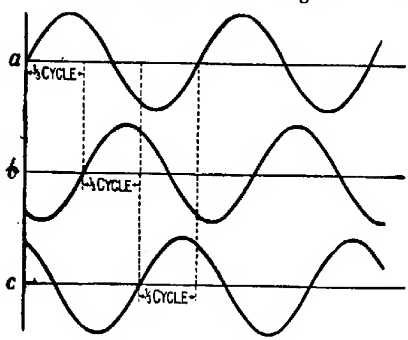

Fig. 98.—Three-Phase E.M.Fs. or Currents.

one another. In the positions shown, the e.m.f. in aa' is zero, that in bb' has nearly reached its maximum, whilst that in cc' is decreasing. Thus the three e.m.fs.,

and the currents set up by them, will vary as shown in Fig. 98.

If these three circuits were kept quite apart, as in Fig. 100, six conductors would obviously be required. But, as will be understood presently, it is not necessary to keep them apart; and only half the above number of conductors (*i.e.*, three) are necessary in most cases.

If the three curves in Fig. 98 are plotted on one

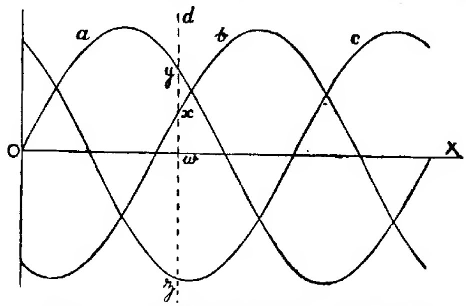

Fig. 99.—Showing that the Algebraic Sum of Three-Phase Currents is Zero.

time base, we obtain Fig. 99. If, at any instant along the time base OX, a vertical line be drawn through the diagram, it will be found that the sum of the lengths cut off by the waves above the line is always equal to the sum of the lengths below. Thus, at the dotted line d, the a and b currents are both +, while the c current is −; and if the curves be accurately drawn it will be found that—

$$wx + wy = wz,$$

these lengths indicating the relative strengths of the three currents at that particular instant. Hence it follows that *the algebraic sum of the currents in a*

three-phase circuit is zero at every instant. "Algebraic sum" in this case means that the "sign" or direction-of-flow of the currents — as well as their strengths—is taken into consideration.

On account of the above characteristic of three-phase currents, if the ends of the phases connected to the conductors A', B', C' in Fig. 100 are joined together as shown in Fig. 101, the algebraic sum of the currents at the centre is zero, and consequently there is no necessity even for a fourth conductor. In other words, the total current flowing towards the centre in one or two of the phases is always equal to the total current flowing away from the centre in the remaining phase or phases. Fig. 102 will make the meaning of this still clearer.

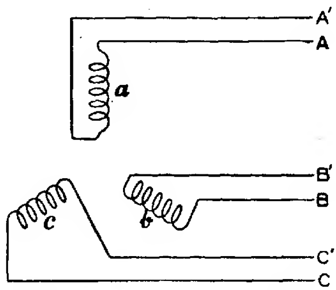

Fig. 100.—Connection of Three-Phase Windings to Six-Wire Mains.

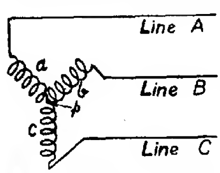

Fig. 101.—Star or Y-Connection of Three-Phase Windings.

The method of joining the phases together shown in Fig. 101 is known as the *star* or *Y-connection,* and the junction p of the three phases is called the *star point* or *neutral point.*

The action of three-phase currents in a complete

three-wire star-connected circuit will be made clear from the hydraulic analogy of an alternator connected to a motor, as depicted in Fig. 102. Here S is a shaft

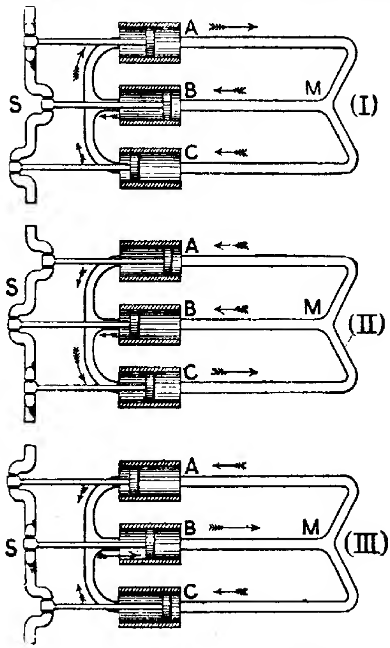

Fig. 102.—Flow of Current in a Three-Phase Circuit.

with three cranks set 120° apart, each crank being linked to a separate piston working in a cylinder. The three cylinders are connected together at front and back by pipes as shown. The pipes and cylinders have no external openings, and are supposed to be

filled-up with water. The crank-shaft stands for the driving engine; the three pistons and cylinders represent the three phase-sets of coils in a three-phase alternator; and the short pipes between the cranks and the cylinders represent the star-connection of the windings of the generator. The long pipes on the right of the cylinders stand for the transmitting cables, and the ends of these, where they join together at M, may be taken to represent the windings on the motor.

The crank-shaft is supposed to be rotating right-handedly, as viewed from the bottom end; and in the position shown at (I), the A piston is at the middle of its outward or positive stroke, the B piston has just commenced its inward or negative stroke, and the C piston is nearing the end of its inward stroke. The "inward" stroke is to be regarded as that made *away* from M.

Now the A piston is evidently moving at its maximum speed, so that we may say that the A current is at its maximum, and that it is flowing outwards or positively; this being indicated by the long arrow at each end of the A cylinder in (I). The pistons of phases B and C, which are respectively near the beginning and end of their inward strokes, are moving slowly. Thus the direction and strength of the currents in these two circuits is roughly indicated by the short arrows. Then we may say that the A current flowing outwards along the A cable splits up and returns *via* the B and C cables; the sum of the two currents in B and C being equal to that in A.

At position (II), the crank-shaft has moved through 120°, and the maximum positive current in C flowing outwards returns *via* the A and B cables; the sum of the currents in the latter two being equal to that in the former.

At position (III), the shaft has moved through another 120°, and here the B current has its maximum positive value, it being equal to the sum of the A and C currents, and returning *via* the A and C cables.

48. **MESH-CONNECTED THREE-PHASE CIRCUIT.**—Since the curves in Figs. 98 and 99 indicate the relationship between three-phase voltages as well as currents, it will be evident that the *algebraic sum of the e.m.fs. in a three-phase circuit is zero at every instant.*

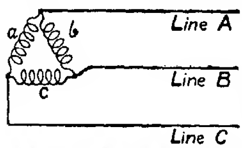

Fig. 103.—Triangle, Mesh, Delta, or Δ-Connection of Three-Phase Windings.

This fact enables us to connect the three phases in a different way from the "star" method, namely, that in Fig 103.

Let us start with the three phases isolated from one another as in Fig. 100. Now connect the end A' of the first phase to the beginning B of the second phase, the end B' of the latter to the beginning C of the third phase, and the end C' of that phase to the beginning A of the first phase. This will result in the arrangement shown in Fig. 103, which is known variously as *the mesh, delta,* Δ, *or triangle connection.*

If it were not true that the algebraic sum of the e.m.fs. in a three-phase circuit is zero, there would be an e.m.f. sending a circulating current round the windings a, b, and c, when connected in mesh (Fig. 103); but as no such circulating current is found to exist in practice when the e.m.f. waves are true *sine* waves, the above statement is evidently true.

49. **RELATION BETWEEN THE PHASE AND THE LINE VOLTAGES IN THE Y-CONNECTION.**—The voltage between the two ends of any one phase-winding (on a

generator or on a motor) is known as the *phase voltage*; and the current flowing in that phase is called the *phase current*. The voltage between any two of the three line conductors is known as the *line voltage*, while the current in each of these conductors is called the *line current*.

It will be evident that with the Y-connection (Fig. 101), the line current is the same as the phase current

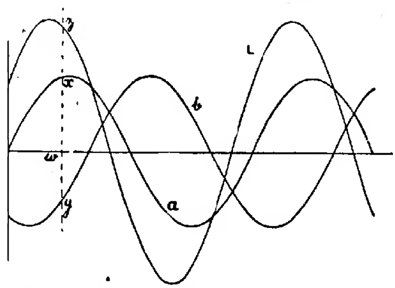

Fig. 104.—Curves showing Separate and Resultant Voltages in Two Phases of a Y-Winding.

The line voltage, however, is the algebraic sum of the voltages of two of the three phases.

Consider the voltages induced in windings *a* and *b* in Fig. 101. These two voltages vary with regard to each other in the manner shown in Fig. 104, which represents curves *a* and *b* of Fig. 98 plotted on one base. If, at a particular instant *w*, the voltage in the phase or winding *a* is *wx*, that in phase *b* will be *wy*. If the direction of the voltage *wx* in phase *a* is outwards from the neutral point, then the voltage *wy*, being negative, will be acting towards the neutral. In consequence, *wx* and *wy* will be trying to send current in the same direction so far as the distribution lines *A* and *B* in

Fig. 101 are concerned; so that the instantaneous voltage between these line conductors is given by wz in Fig. 104, where $wz = wx + wy$.

Proceeding in this way at various points along the time base, we obtain the curve L representing the variation of voltage between the conductors A and B in Fig. 101. Now it can be shown, both mathematically and graphically, that the virtual value of the line voltage between A and B is $\sqrt{3}$ ($=1{\cdot}73$) times that of the virtual voltage of either phase a or b. It can be similarly shown that the voltage between B and C, or between A and C, is also equal to 1·73 times the phase voltage.

Thus, if the voltage of each of the phases a, b, and c in Fig. 101 be 200, the voltage between any two of the line conductors A, B, and C, will be $200 \times 1{\cdot}73 = 346$ volts.

The whole matter may be summed up concisely as follows:—

In a three-phase STAR-CONNECTED CIRCUIT—

If:— E_P = phase voltage,
E_L = line voltage,
I_P = phase current,
I_L = line current,

Then— $I_P = I_L$, **(33)**

and— $E_L = 1{\cdot}73 \times E_P$.* **(34)**

It will be remembered that when we speak of a given e.m.f. or voltage, we mean the virtual value (§ 23). (* *See* Appendix C.)

50. **RELATION BETWEEN THE PHASE AND THE LINE CURRENTS IN THE Δ-CONNECTION.** — From Fig. 103, it is evident that with a Δ-connection the line voltage is exactly the same as the phase voltage. As regards the line currents, however, that in conductor A is the algebraic sum of the currents in phases a and b; that in B is the algebraic sum of the currents in b

and *c*; and that in *C* is the algebraic sum of the currents in *a* and *c*.

The currents in *a*, *b*, and *c* have a phase difference of 120°, as shown in Fig. 98; and the sum of any two can be obtained in a similar way to that explained in connection with voltages in Fig. 104. Here again, it can be shown mathematically or graphically that the current in each line is $\sqrt{3}$ or 1·73 times the current in each phase.

Hence, adopting the same notation as in the preceding section—

In a three-phase DELTA-CONNECTED CIRCUIT—

$$E_L = E_P, \tag{35}$$

and—

$$I_L = 1{\cdot}73\, I_P. \tag{36}$$

51. **RELATIVE ADVANTAGES OF THE Y- AND Δ-CONNECTIONS.**—It is shown in § 66 that the number of conductors per phase in the windings of an alternator or motor, for a certain frequency and flux, is directly proportional to the phase voltage. Hence, for a given line voltage, fewer turns-per-phase will be required with a Y-connection than with a Δ-connection; for in the former case the phase voltage need only be $\frac{1}{1{\cdot}73} = {\cdot}58$ times the line voltage, whereas in the latter case it must be equal to it. Consequently, a Y- or star-wound machine is the cheaper to wind. (See Formulæ 34 and 35.)

Another advantage of the star-connection is that the system of distribution mains may be arranged to suit both lighting and power circuits without the use of transformers, provided the voltage between the lines does not exceed 430. This is done, as shown in Fig. 105, by connecting a common wire to the "point" of the star of the generator *G*; and joining-up the two-wire

distribution boards *d*, *d*, *d*, between this common or neutral main and one or other of the three equal-sectioned outers *A*, *B*, and *C*.

These boards then get a lower voltage than if they were connected between *A* and *B*, or *B* and *C*, or *C* and *A*; and would be used for the feeding of lighting and heating circuits, whose pressure is limited to 250 volts. Small single-phase motors could also be connected

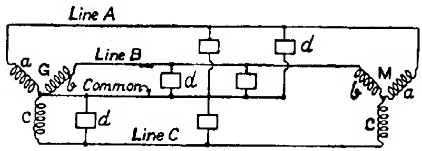

Fig. 105.—Three-Phase Four-Wire System.

thereto. Thus if $E_L = 430$, $E_P = 430 \times {\cdot}58 = 250$. Three-phase motors, advantageously served at greater pressure, would be connected to the three phases, as seen at *M*.

Yet another advantage of the Y-connection is that the neutral point of alternators can be (and generally is) connected to earth. When this is done, the potential-difference between each line conductor and earth is kept equal to the phase voltage, and is thus ·58 of the line voltage. The Δ-connection cannot be earthed in this way, so that should one of the line conductors become earthed through a fault, the voltage between each of the remaining two conductors and earth would be equal to the line voltage; and would thus be 1·73 times greater than the voltage to earth in the case of the Y-connection with earthed neutral. There would be a correspondingly higher stress in the insulating material, and greater liability of breakdown. This consideration is of importance with machines working at thousands of volts per phase.

The advantage of the Δ-connection is that transformers in general work more satisfactorily under such conditions. Furthermore, it is the only connection suitable for such machines as rotary converters; but the reasons cannot be given here.

52. **POWER IN THREE-PHASE CIRCUITS.**—In calculating the power transmitted by a three-phase circuit, the particulars given in sections 49 and 50 must be taken into consideration: and a brief explanation will show that, for a given value of line current and line voltage, *the power transmitted is the same whether the machines be connected in star or in mesh.*

For instance, suppose a mesh-connected motor is supplied with a current of I_L amperes at a pressure of E_L volts. The power absorbed by the motor (ignoring, for the moment, the reduction due to power factor) will be equal to *three times the power per phase, i.e.*, three times the product of the phase current and the phase voltage.

As the motor is mesh-connected, the phase voltage will be the same as the line voltage—*i.e.*, $E_L = E_P$ (by Formula 35). The line current I_L, however, will be equal to $\sqrt{3}$ times the phase current (by Formula 36), therefore the phase current will be equal to $\frac{I_L}{\sqrt{3}}$.

Consequently:—

The volt-amperes absorbed by the mesh-connected motor $= 3 \times$ phase current $\times$ phase voltage,

$$= 3I_P E_P = 3\frac{I_L}{\sqrt{3}} \times E_P$$

$$= \sqrt{3} I_L E_L$$

it being remembered that $E_P = E_L$ with the mesh connection.

If the motor be star-connected, the line current

equals the phase current—*i.e.*, $I_P = I_L$, and the phase volts $= \frac{E_L}{\sqrt{3}}$ (from Formulæ 33 and 34).

Therefore in this case :—

The volt-amperes absorbed by the star-connected motor

$$= 3I_P E_P = 3I_L \frac{E_L}{\sqrt{3}},$$

$$= \sqrt{3} I_L E_L.$$

Hence, whether the machine be mesh- or star-connected, the apparent power P absorbed in each case is the same, and is equal to

$$P\,(\text{app.}) = \sqrt{3} I_L E_L. \qquad \textbf{(37)}$$

If the power factor (p.f.) be taken into consideration,

$$P \text{ in watts} = I_L \times E_L \times 1{\cdot}73 \times \text{p.f.} \qquad \textbf{(38)}$$

53. **MEASUREMENT OF POWER IN A.C. CIRCUITS.** —It is evident, from what has already been said, that the power absorbed in an a.c. circuit depends not only upon the current and the voltage, but also upon the power factor, *i.e.* :—

$$\text{Actual watts} = EI \cos \phi. \qquad \text{(P. 140.)}$$

Hence all wattmeters for measuring alternating-current power must take account of these three quantities.

The most reliable type of instrument for this purpose is that acting on the dynamometer principle. In this there are two coils (or sets of coils), one of which is fixed, and the other movable. The moving coil (or coils) is connected in the current circuit, and the fixed coil (or coils) is connected so as to take the pressure, or *vice versa*.

Siemens' *dynamometer wattmeter* * was the pioneer

* This instrument, and the principle on which it (and others of its type) works, are described in the Author's *Electric Lighting and Power Distribution*, Vol. I.

of this class of apparatus, which—by the way—can also be used on continuous-current circuits. A modern dynamometer wattmeter is described in the next section.

In a dynamometer wattmeter, the turning force exerted by the coils at any instant is proportional to the *product of the values of the currents in them at that particular instant*, one of these currents, of course, being that due to the pressure. In other words, the turning force at any moment is proportional to the product of the instantaneous values of the current and the voltage, and so to the instantaneous watts. Consequently, the effective turning force is proportional to the watts.

It should be evident from Fig. 89 that when the pressure and current are in phase, the product of the instantaneous values of the two is always positive, *i.e.*, the moving system of the dynamometer would always be deflected in the same direction. Should the current and the voltage be 90° out of phase with each other, as in Fig. 90, the deflecting torque would be acting equally in alternate directions; and since the alternations follow each other in quick succession, the pointer would simply remain stationary and indicate no power. For a phase difference intermediate between the above two limits (Fig. 91), the deflecting torque would be greater in one direction than in the other, consequently the moving system would be deflected by an amount proportional to the real power. Thus it is that a dynamometer wattmeter takes account of power factor, and always indicates the true watts in the circuit.

It should also be evident that dynamometer wattmeters indicate truly whatever the frequency and form of the pressure and current waves.

54. **WESTON DYNAMOMETER WATTMETER.**—Figs. 106 and 106A give front and back views of the Weston dynamometer wattmeter, which may be used on either

alternating or continuous-current circuits; and which, as seen, is generally graduated in kilowatts. In Fig.

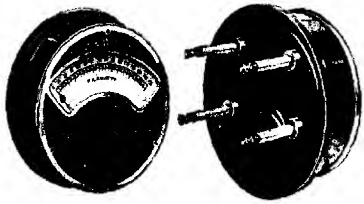

Fig. 106. Fig. 106A.
Weston Wattmeter (External Front and Back Views).

106A, the two top terminals are for the pressure and the two bottom ones for the current.

The internal arrangements are shown diagrammatically in Fig. 107. *C*, *C* are two fixed thick-wire current coils connected in series to the current terminals *I*, *I*; and *MC* is the movable fine-wire coil connected (in series with the non-inductive resistance *R*) to the voltage terminals *V*, *V*. The fixed and moving coils thus act as current and pressure coils respectively, and measure simultaneously the current and p.d., and hence the true watts in the circuit.

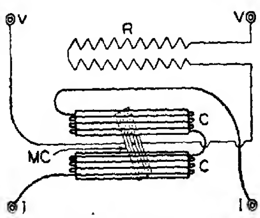

Fig. 107.—Internal Connections of Weston Wattmeter.

The actual arrangement of the interior is seen in Fig. 108. *C* are the two current coils connected to inside terminals at *t*, *t*. *MC* is the moving coil which—

together with the pointer p—is mounted on the delicately-pivoted spindle s. The ends of MC are connected to the spring-connectors b, b', in a manner described presently. w is a wire connecting spring b with one of the pressure terminals in Fig. 106A; the

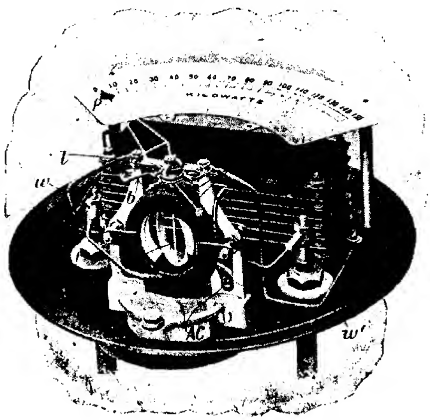

Fig. 108.—Interior of Weston Wattmeter.

wire w' connects spring b' to one end of the resistance R, the other end of which is joined-up to the other pressure terminal in Fig. 106A. R consists of series-connected sections of special resistance-wire wound on micanite sheets.

Fig. 109 shows the complete moving portion of the instrument, which may be described as follows: s is an aluminium-alloy spindle or staff with hardened-steel pivots p, p, which are supported in sapphire centres on the fixed frame. MC is the moving coil, p the pointer, w an adjustable counterweight, and V, V

aluminium damping vanes which are enclosed in air chambers *AC* (Figs. 108 and 110), and so render the instrument dead beat.

SS are two light spiral springs coiled in opposite directions. These springs fulfil a double purpose.

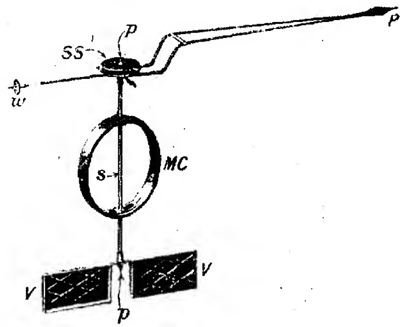

Fig. 109.—Moving Portion of Weston Wattmeter.

Firstly, they act as the controlling force of the instrument; that is to say, the electrical deflection of the moving coil takes place against the force of these springs, which tend to keep the coil and pointer in the zero position. The other function of the springs is to lead the pressure current into and out from *MC*; their outer ends being connected to lugs to which the spring-connectors *b*, *b′* (Fig. 108) are also joined. One of these lugs or "abutments" *l* has a small range of movement about the top bearing, to allow of the zero adjustment of the instrument by means of the screw seen just above the *N* near the bottom of the front of

the cover (Fig. 106). This screw operates a lever fitting in the fork on *l*.

Fig. 110 shows the frame support for the coils, the holder for the lower sapphire centre, and the air chambers *AC*, *AC*, with their covers removed. The vanes *V*, *V* (Fig. 109) fit these chambers very closely, without touching of course.

Fig. 110.—Portion of Frame of Weston Wattmeter showing Air-Damper Chambers.

55. **CONNECTION OF SINGLE-PHASE WATTMETERS IN CIRCUIT.** — The various methods of connecting single-phase wattmeters, described below, relate more particularly to Weston dynamometer wattmeters; but instruments of other makes are dealt with in very much the same manner.

The comparatively low-reading instrument in Fig. 106 may be connected directly in circuit, as seen in Fig. 111, provided the pressure and currents do not exceed 300 volts and 20 amperes respectively. The current range may be increased up to 100 amperes by providing a thicker winding on the fixed coils, and larger terminals than those seen in Fig. 106A. For pressures of from 300 to 750 volts, and currents not exceeding 100 amperes, the connections are as Fig. 112;

an external resistance R being connected in the pressure circuit.

If the current exceeds 100 amperes, and the pressure

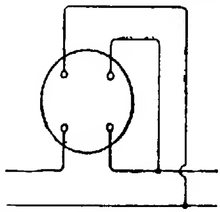

Fig. 111.—Direct Connection of Wattmeter in Circuit.

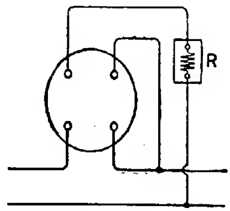

Fig. 112.—Connection of Wattmeter with External Resistance in Pressure Circuit.

is not above 750 volts, a current transformer (p. 259) is used, as shown at CT (Fig. 113).

If both the pressure and the current exceed the limits

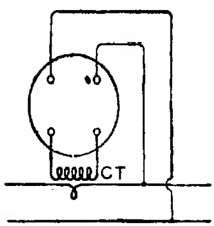

Fig. 113.—Connection of Wattmeter through Current Transformer.

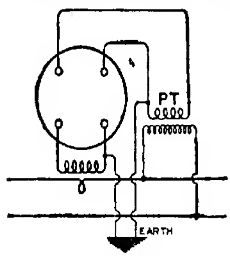

Fig. 114.—Connection of Wattmeter through Current and Potential Transformers.

mentioned above, besides the current transformer, either an external resistance or a potential (*i.e.*, step-down) transformer (p. 228) is inserted in the pressure circuit. The resistance is used for medium pressures, and the

transformer for high pressures. The latter case is illustrated in Fig. 114, where *PT* is the potential transformer.

The connection of one side of each circuit to earth is a safety precaution in case of leakage from the primary circuit of either transformer, and does not affect the proper working of the instrument.

It will be evident from the above diagrams, especially from Fig. 114, that there is practically no limit to the range in kilowatts for which a comparatively small and light instrument like that in Fig. 106, may be adjusted and graduated. But an instrument intended for connection through a resistance, or transformer, or transformers, will of course only read correctly when connected with the particular external apparatus sent out with it.

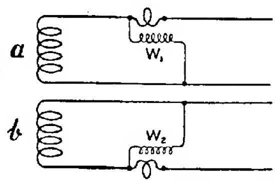

Fig. 115.—Connection of Wattmeters in a Two-Phase Circuit.

56. **MEASUREMENT OF POWER IN A TWO-PHASE CIRCUIT.**—Fig. 115 shows the use of two wattmeters, W_1 and W_2, in measuring the power in a two-phase circuit; *a* and *b* representing the two phases, which in this connection can be considered simply as two separate single-phase circuits. *a* and *b* may be the windings of a motor, or those of an alternator.

The sum of the readings of W_1 and W_2 will always give the total power in the two phases under all conditions of load.

If the two phases were balanced, *i.e.*, if the current, voltage, and power factor of the one were the same as those of the other, the total power could be obtained by doubling the reading of one of the wattmeters, and

dispensing with the second. But as the load on a two-phase circuit is generally more or less unbalanced, it is better, when accuracy is required, to use two instruments and add their readings.

In Fig. 115 the wattmeters are shown connected directly in circuit; but even if they were connected through transformers, the above explanation would still hold good. This applies to Figs. 116 and 117 also.

57. **MEASUREMENT OF POWER IN A THREE-PHASE CIRCUIT.**—The power in a star-connected three-

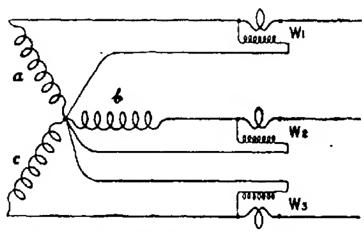

Fig. 116.—Three-Wattmeter Method of Measuring Power in a Three-Phase Circuit.

wire three-phase circuit can be measured with either three or two wattmeters; hence we have the *three-wattmeter method* and the *two-wattmeter method*. With a mesh-connected circuit, the two-wattmeter method is the more convenient.

Fig. 116 illustrates the three-wattmeter method; a, b, and c being the star-connected phases. The latter may be looked upon as three single-phase circuits, and the power in each is measured by the wattmeters W_1, W_2, and W_3, the pressure coils of which are connected across their respective phases. The sum of the readings of the three instruments gives the total power in the three-phase circuit.

If the circuit be balanced, it would suffice to use one wattmeter only, connected to one line and phase as at W_1 in Fig. 116. The reading so obtained, multiplied by three, would give the total power.

When, in the case of a star-connected circuit, the neutral point cannot be got at for making the necessary connections, or when the windings are in mesh, the two-wattmeter method is employed.

The connections for this method are depicted in Fig. 117, the mesh arrangement of windings being

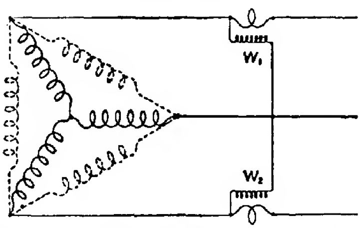

Fig. 117.—Two-Wattmeter Method of Measuring Power in a Three-Phase Circuit.

dotted. Here it will be seen that the current coils are connected in series with two of the three line conductors, the pressure coils being joined-up between these two conductors and the third. With this method of connection, it can be shown mathematically that the *algebraic sum* of the readings on W_1 and W_2 will give the total power under all conditions of load.

With a power factor below ·5, it will be found that the pointer of one of the wattmeters will tend to deflect the wrong side of zero. When this is so, the pressure connections of the reverse-reading instrument must be reversed. Its reading, however, must still be considered as negative, and must therefore be sub-

tracted from the reading of the other instrument; this giving the algebraic sum of the readings. The reasons for the above procedure cannot be explained here, as this would entail a certain amount of trigonometry.

When the p.f. is above ·5, both instruments will read in the same direction, *i.e.*, they will both be +; and their algebraic sum will be got by adding them together.

It may be mentioned that single instruments combining the two-wattmeter principle are available for two- or three-phase circuits; but it is better for the beginner to consider separate instruments only.

With a four-wire three-phase system, the three-wattmeter method only can be used.

CHAPTER III.—QUESTIONS.

In answering these Questions give Sketches wherever possible.

NOTE.—Questions marked * range slightly beyond the subject-matter of this Book. Those marked † can only be partly answered therefrom.

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1. Deduce a mathematical expression for the power in an alternating-current circuit in which a phase difference exists. (*Grade II., A.C.*, 1912.)

2. Define the terms, power factor, impedance, periodicity, and state if any of these affect the size and method of running cables. (*Wiremen's Final*, 1913.)

†3. Define the term *power factor* in relation to any alternating-current apparatus or circuit. Why is the power factor of a distribution system that works through feeders and step-down transformers not constant at all loads? How can such a system be worked so as to improve the power factor? What

disadvantage has a low power factor (*a*) to the consumer; (*b*) to the station engineer? (*Ord., A.C.*, 1908.)

4. The power factor on a three-phase public supply system is found to be very low, viz. 0·65: what steps can be taken to improve this? (*A.M.I.E.E. Exam.*, 1914.)

5. Show in a general way by a vector diagram that it is possible to improve the power factor of a circuit by means of an alternating synchronous motor. (*Grade II., A.C.*, 1913.)

6. A certain motor takes a current of 30 amperes at a pressure of 200 volts and 50 frequency. The power factor is 0·8, lagging. The motor is shunted by a liquid condenser of adjustable capacity. To what value must you adjust the capacity so that the current taken from the supply circuit shall be a minimum? (*Grade II., A.C.*, 1912.)

Ans. 287 mfds.

7. What brake horse power would be required in an engine to drive (*a*) a D.C. generator having an output of 230 volts 500 amperes at an efficiency of 85 per cent., and (*b*) an A.C. generator having the same output with an efficiency of 82 per cent. and a power factor of 85 per cent. (*Wiremen's Final*, 1913.)

Ans. (*a*) 181 b.h.p.; (*b*) 160 b.h.p.

8. An inductive coil takes a current of 10 amperes (effective or virtual value) when the potential difference between its terminals is 120 virtual volts at 50 frequency. Its resistance is 5 ohms. Calculate the power absorbed by the coil, its coefficient of self-induction, and the angle of phase difference between current and terminal voltage. (*Grade II., A.C.*, 1912.)

Ans. 500 watts; ·0347 henry; 65°.

†9. Show, by means of a vector diagram, the relations between the pressures and currents in a two-phase three-wire distributing system to which induction motors are connected. In what circumstances could a system of this character conceivably be employed with advantage? (*Final, 1st Paper*, 1912.)

†10. Explain the following terms:—Simple parallel circuit;

simple series circuit; series-parallel circuit; three-wire supply; three-phase supply; two-phase supply. Make a simple diagram of each. (*Wiremen's First*, 1911.)

11. What is the relation between the line volts and phase volts, and that between the line current and phase current, in a three-phase star-connected alternator? If one of the phase windings of a three-phase alternator is connected up wrongly, how would the line voltage be affected? (*Ord.*, *A.C.*, 1910.)

12. Each of the three circuits of a three-phase alternator gives a terminal potential difference of 1000 volts, and carries 60 amperes. What will be the voltage and current in the external circuit if the three circuits in the alternator are connected (*a*) in mesh, (*b*) in star? (*Ord.*, *A.C.*, 1911.)

Ans. (*a*) 1000 volts, 104 amps.; (*b*) 1730 volts, 60 amps.

13. It is proposed to distribute energy by a three-phase 4-wire system, and the declared pressure at the consumers' terminals is 220 volts. For what pressure should a three-phase motor be wound, and how should the whole installation of motors and lighting be connected so as to prevent any unbalancing of such a distribution system? (*A.M.I.E.E. Exam.*, 1914.)

Ans. 380 volts.

14. Discuss the value of the voltage to earth of the terminals on a three-phase star-connected generator with its star-point (*a*) earthed, (*b*) insulated. In case (*b*) what is the voltage to earth of each of the terminals when one of the terminals is grounded? How is your answer modified if the machine is mesh connected? (*Final*, *1st Paper*, 1914.)

15. The capacity measured by a ballistic galvanometer between two conductors of a three-phase cable is 1·5 microfarads. What charging current will the cable take when connected to three-phase terminals with a line pressure of 10,000 volts at 50 frequency? (*Grade II.*, *A.C.*, 1912.)

Ans. 8·15 amps.

16. Deduce an expression for the power in a single-phase alternating-current circuit in which a phase difference exists, and give the power in a three-phase circuit with balanced load

in terms of the line voltage and current. (*Grade II., A.C.*, 1914.)

17. The input to a three-phase star-connected line is 10,000 k.v.a., and the initial pressure is 30,000 volts between phases, the power factor being 0·95. The line is designed for a pressure-drop of 10 per cent of the initial pressure.

State—

(*a*) The current in each conductor.

(*b*) The voltage between each conductor and the neutral point at the receiving end.

(*c*) The total energy received at the end of the line. (*A.M.I.E.E. Exam.*, 1914.)

Ans. (*a*) 192 amps.; (*b*) 15,600 volts; (*c*) 8550 kelvins or B.o.T. units per hour.

18. Two identical installations, one supplied with three-phase alternating, and one with direct current, each take a steady load of 10 electrical H.P. at 200 volts. What is the line current in each case, and how many Board of Trade units would be consumed per hour, assuming the power factor of the A.C. circuit to be 0·85? (*Wiremen's Final*, 1914.)

Ans. (A.C.) 25·3 amps., (C.C.) 37·3 amps., 7·46 kelvins or B.o.T. units.

†19. Determine the relative weights of copper required in the transmission lines to supply a given power over a given distance at the same efficiency of line, when the transmission is made by (*a*) continuous current, (*b*) single-phase alternating current, (*c*) three-phase alternating current. The maximum voltage to earth of any one wire is to be the same in each case. How is the problem affected by the power factor if alternating current is used? (*Grade II., A.C.*, 1914.)

Ans. 2 : 4 : 1. Assuming 2-wire continuous current and 2-wire single-phase, each with one side earthed; and a three-phase star with neutral point earthed; and unity p.f.

*20. A certain factory is fed from a three-phase 50-cycle supply by means of a long aerial line. The kilovolt amperes measured at the factory are 1040 at a lagging power factor of 0·75, the voltage being 6000. Find the voltage at the

generating station if the resistance of each of the three-line wires is 1 ohm and the coefficient of self-induction 0·006 henry. If now there is installed a synchronous motor, which takes an input of 900 K.V.A. at 0·6 power-factor leading, in addition to the other load, find the voltage at the generating station. (*Final, 2nd Paper*, 1913.)

Ans. 6330 volts and 6210 volts.

21. Explain the principle of the electro-dynamometer, and show that if calibrated with a continuous current it will indicate the virtual value of an alternating current. (*Ord., A.C.*, 1911.)

22. Describe, giving diagram of essential parts and connections, some form of wattmeter suitable for an alternating-current circuit in which the power factor may be considerably less than unity. (*Grade II., A.C.*, 1912.)

†23. In a three-phase system the phases are unequally loaded. Show how by the use of two wattmeters you can determine the total power transmitted. (*Grade II., A.C.*, 1912.)

†24. Explain with diagram of connections the method of measuring power in a three-phase circuit by means of two wattmeters, assuming the loading to be unsymmetrical. (*Grade II., A.C.*, 1914.)

CHAPTER IV.

ALTERNATORS.

58. **ALTERNATORS.**—An *alternator*, or *alternating-current generator*, is a machine which supplies alternating e.m.fs. and currents.

Elementary two-pole single-, two-, and three-phase alternators and their circuits were described in §§ 4 and 47.

Alternators that are to be driven by ordinary (*i.e.*, reciprocating) engines, may have any even number of poles up to forty or fifty; the precise number depending upon the speed at which the machine is to be run, and upon the frequency required. As non-reciprocating engines (steam-turbines) usually run at much greater speeds than reciprocating engines, alternators for direct-coupling thereto (called *turbo-alternators*) have a correspondingly fewer number of poles; four and six being very usual numbers. Alternators driven by water-turbines do not run at such high speeds as those driven by steam-turbines, and so have a fairly large number of poles.

The connection between the frequency, the number of poles, and the speed of alternators was explained in § 8.

An alternator—unlike a dynamo—cannot be made self-exciting, *i.e.*, it cannot furnish the current for its own field-magnet. The continuous current necessary for the latter must consequently be derived from a small dynamo, the armature of which is in most cases

mounted on an extension of the alternator shaft, as seen in Fig. 127.

When a dynamo is used for the purpose of energizing the field system of an alternator or other machine, as just described, it is generally termed an *exciter.* Sometimes the exciter supplies the exciting current for a number of alternators, whose field circuits are connected in parallel thereto. It is then driven by a separate engine, or by an electric motor.

59. **CLASSIFICATION OF ALTERNATORS.**—As regards the kind of current that may be derived from them, alternators are of three principal types, viz.:—

(i.) Single-phase.
(ii.) Two-phase. } Polyphase.
(iii.) Three-phase. }

As will be explained presently, the essential difference between these three kinds of generator lies in the winding of the armature only; the form of the machine, and the arrangement of the field-magnet, being practically the same in all three cases.

Alternators, whether single-phase or polyphase, may also be classified in another way:—

(*a*) Those with fixed armature and rotating field-magnet (f.m.)

(*b*) Those with fixed f.m. and rotating armature.

The rotating part of an alternator (or of a motor) is often termed the *rotor*, and the fixed part the *stator.*

One essential difference between an alternator and a dynamo is that there is no commutator in the former; the armature windings having two, three, or four ends which pass direct to the terminals in class (*a*), and through slip-rings and brushes to the terminals in class (*b*).

All except very small alternators belong to class (*a*),

and examples of this class are shown in Figs. 125 to 131. As the e.m.fs. generated in large machines may in some cases be as high as 6000 volts or more, it is obviously of advantage to have the armature windings on the stator. With this arrangement, the mounting, insulation, and ventilation of the windings is rendered easier; and the difficulties of properly insulating the slip-rings and brush connections to withstand the high voltage are avoided. The pressure at which current is supplied to the field-magnet is only 100 volts or so; this fact and the simplicity of the winding being further reasons for making this portion of the machine the rotor.

In alternators of the (*b*) class, the ends of the armature windings terminate at two, three, or four insulated slip-rings on the shaft; and brushes pressing thereon connect with the terminals of the machine. From what has just been stated, this class of machine is clearly not suitable for high voltages, and it is never adopted nowadays except for very small outputs.

60. **SINGLE-PHASE ALTERNATORS.**—The field and armature circuits and the action of a multipolar single-phase alternator may be fairly-well grasped from Fig. 118. Here C_1, C_2, C, C represent the armature coils, and N_1, S, N, S the field coils and poles; the former being fixed and the latter rotating. The armature coils are placed end-on to the poles in the figure, so that their connection together may be clearly seen; but in reality they face the poles. In such a machine, the number of armature coils and of field-poles are equal, as shown.

Each coil in the figure has only two turns, in order to simplify the diagram; and if any one coil (say C_1) be considered, the e.m.f. is generated in the portions k, k, k, k, which may be termed the conductors. Other things being equal, the greater the total number of such

conductors in series, the greater the e.m.f. of the machine.

It will be clear that the e.m.fs. induced in those conductors passed by N. poles will be in the reverse direction to those induced in the conductors passed by S. poles. Therefore it is necessary for the coils to be

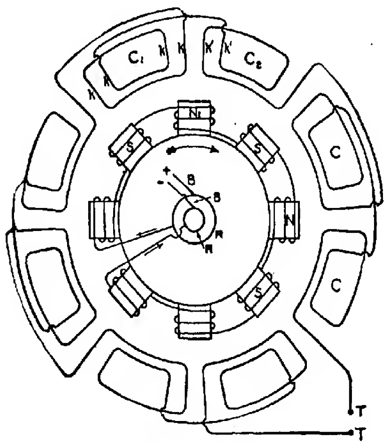

Fig. 118.—Connections of a Single-Phase Alternator.

connected together alternately right-handedly and left-handedly, as indicated in the figure; in order that the e.m.fs. being induced in their conductors at any given instant may all be in the same direction through the coil circuit. Consider, for example, the adjacent conductors $k\,k'$ of the neighbouring coils C_1 and C_2. Unless the coils were connected as above-mentioned, the e.m.fs. in k and k' would oppose each other. As the field poles move on in the direction shown by the curved arrow,

the conductors previously facing N. poles will now have S. poles in front of them, and *vice versa;* the e.m.f. in all of them being reversed simultaneously. Thus as the poles move round in front of the coils, a single-phase alternating e.m.f., similar to that described in § 5, will be generated in the machine; and single-phase currents will be set up in the circuits connected thereto.

The continuous current (exciting current) for the revolving field-magnet is led in through the brushes *B,B*, and the contact or slip-rings *R,R*. A simple perspective view of such brushes and rings was given in Fig. 13; and actual slip-rings are shown at *r* in Fig. 126.

The number of reversals of the e.m.f. during one revolution of the rotor will be equal to the number of poles on the field-magnet. Thus, while N_1 is in the position shown in Fig. 118, the e.m.fs. in the conductors *k,k'* will be in one direction, but will be reversed when the adjacent S. pole has reached this position. As there are two reversals of e.m.f. in each cycle (§ 7), the frequency of an alternator is equal to half the number of magnet-poles multiplied by the number of revolutions per second. In other words, the frequency is equal to the revolutions per minute divided by 60 and multiplied by the number of pairs of poles. (*See* Formula 1, page 34.)

The frequencies of alternators in British generating stations vary from about 25 to 100 cycles per second. (*See* § 7.)

Another and better way of representing the armature windings of a single-phase alternator is given in Fig. 119. This illustrates the inner cylindrical surface of the slotted laminated-iron core *C* of a 4-pole machine, cut across at the line *l* and flattened-out, with the windings *w,w* drawn diagrammatically. The whole

core-face is laminated, but only the portion p thereof is shown so in the figure. In reality, there are a number of conductors in each slot s, depending upon the voltage required and on the speed. The position of the field-poles is shown by the shaded rectangles N, S, N, S. The winding starts at S, and finishes at F; the ends marked a, b, c, d, at one side of the figure being con-

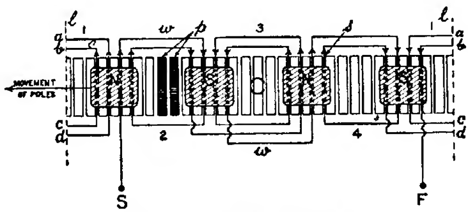

Fig. 119.—Single-Phase Winding.

nected to the corresponding ends on the other side; the conductors—like the core—being supposed to be cut at the line l. Thus in the actual machine there would be a continuous circuit from S to F. The pole faces N, S, N, S are supposed to be above the stator core C and its winding w, w; and the poles are shown in one of the positions where the maximum e.m.fs. are being generated in the conductors. Assume the poles to move from right to left; then the directions of the e.m.fs. in the various conductors (at the particular instant that the poles are in the positions shown) are as indicated by the small arrowheads*: and it should be noted that the connection of the four coils 1, 2, 3, 4 is such that all these e.m.fs. act in the same direction, viz. from S to F, through the winding.

* *See* "Left-Hand Rule for Induction of E.M.F." in the Author's *Electric Lighting and Power Distribution*, Vol. I., Seventh Edition.

61. DISTRIBUTION OF WINDING ON A SINGLE-PHASE ALTERNATOR.—In Fig. 119 it will be noticed that a number of the stator core slots have no conductors in them; in other words, a considerable portion of the core is unwound. This arrangement is preferable in a single-phase alternator, for though the winding of

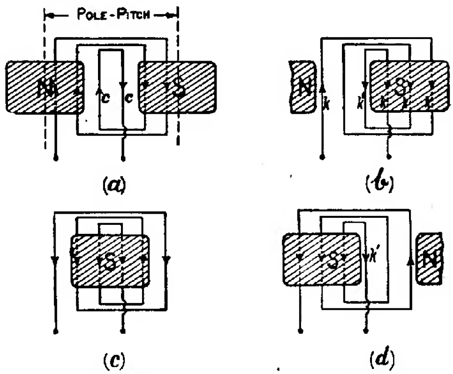

Fig. 120.—Bad Effect of Evenly-Distributed Single-Phase Winding.

all the slots would very slightly increase the output of the machine, the efficiency would be reduced.

Let us see why this is so. Figs. 120 (*a*), (*b*), (*c*) and (*d*) show one coil of a single-phase alternator with the turns *evenly* distributed over the whole of the portion of the stator core surface embraced by it. The shaded portions *N*, *S*, represent adjacent field poles and their positions relative to the coil; but the stator core and core-slots have been omitted to simplify the figure. The poles are supposed to be moving from right to left. In (*a*) the centre of the coil is midway between the two poles, so that the e.m.fs. induced in all the con-

ductors act in the same direction round the coil, as indicated by the arrowheads. In (*b*) the poles have shifted a quarter of a pole-pitch to the left; *pole-pitch* being the distance between the centres of two adjacent poles, as shown in (*a*). The e.m.f. in the conductor k' in (*b*) is now in opposition to that of its four neighbours k, k, k, k; so that the resultant e.m.f. is that due to three active conductors only. Thus k' and its nearest k neighbour are practically useless so far as the production of useful e.m.f. is concerned; and all they do is to increase the heating and therefore lower the output and the efficiency of the machine. Another quarter of a pole-pitch brings one of the poles opposite the centre of the coil, as at (*c*); and it will be noticed that the e.m.fs. in the three right-hand conductors are in opposition to those in the three left-hand ones, so that the coil as a whole gives no e.m.f. at all. In (*d*) we have the same effect as in (*b*), but on the other side of the S. pole. Another quarter-pitch movement of the poles would bring us back to a position similar to that in (*a*), except that a S. pole would be on the left and a N. pole on the right.

The above shows that, in a single-phase alternator with an evenly-distributed winding, the e.m.fs. in the inside conductors of all the coils would oppose each other during the greater part of the movement of the poles through a complete pole-pitch. This effect is known as *differential action*; and the increase of terminal voltage due to the use of the inside conductors c, c, when in their best position (Fig. 120 (*a*)), is not sufficient to justify the extra cost of material and of winding. In addition, these particular conductors increase the resistance and the reactance of the winding; so that the efficiency of the machine is decreased, and its regulation (§ 68) is impaired.

The general practice, therefore, is to open out the coils as in Fig. 119, so that a portion of the core inside each coil is left with vacant slots. In this figure it will be noticed that half the total number of slots are vacant; and in practice the actual number varies from one-third to one-half.

62. **TWO- AND THREE-PHASE ALTERNATORS.**— As stated in § 47, in a two-phase alternator there are two distinct sets of coils; and these are arranged alternately round the frame. Fig. 121 represents such a

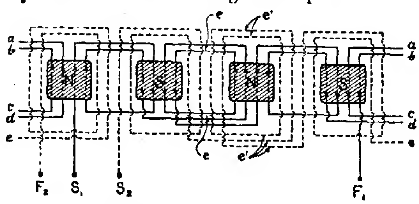

Fig. 121.—Two-Phase Winding.

winding for a four-pole machine, the core and core-slots shown in the kindred Fig. 119 being omitted in order to simplify the illustration.

The phase winding shown by a continuous or full line, starting at S_1 and finishing at F_1, is identically the same as that in Fig. 119. The second phase winding (shown dotted) begins at S_2 and ends at F_2, the only difference between it and the first phase being that it is wound half a pole-pitch to the right. It occupies, in fact, the slots which would be vacant in a single-phase machine.

The *end connections* or *end windings*, *e*, *e*, of one of the phases have to be made of a different shape from those *e′ e′* of the other, in order that they shall lie out of the way of the latter. This is accomplished by

bending the ends (*i.e.*, end-connections) of one of the sets of coils outwards, whilst the other set is kept flat. This arrangement will be understood from Figs 125 and 217.

Fig. 122 shows diagrammatically the manner in which three sets of coils, each similar to that in Fig. 119, can be arranged on a three-phase alternator; the different kinds of lines indicating the three different phase windings. For simplicity, each coil has been shown with only one turn; but in reality, of course,

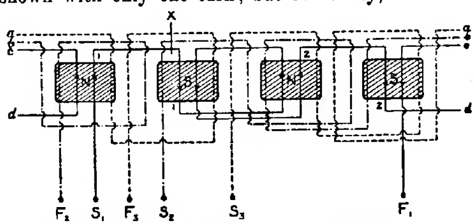

Fig. 122.—Three-Phase Winding with Three Different Shapes of Coils.

in this and in Figs. 119, 121, etc., there would be a large number of turns on each coil. Each phase is connected exactly as in Fig. 119; the second phase S_2 F_2 being wound *two-thirds of a pole-pitch* to the right of the first phase, while the third phase S_3 F_3 is wound still *another two-thirds of a pole-pitch* to the right. Thus all the core slots are occupied by conductors, one-third of the total number being allotted to each phase. It should be noted that this arrangement agrees with the relations between the three-phase coils shown in Fig. 97.

In Fig. 122, *1,1* are portions of one coil and *2,2* portions of another; and they would look more like coils if each had another turn: but if all three phases

were dealt with in this way, the extra number of lines would complicate the figure. However, if we take one phase only, and show it with two turns per coil, as in Fig. 123; the fact that there are as many coils per phase as there are poles in Fig. 122, *i.e.*, four coils in all, should be made quite clear.

From an examination of Fig. 122, it is evident that at several places, for example at *X*, the end connections or end windings of all the three phases overlap; and consequently these portions of each of the three sets

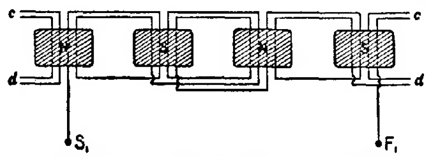

Fig. 123.—One Phase of Fig. 122 with Two Turns per Coil.

of coils would have to be different in shape in order to clear one another. Such coils would be troublesome and expensive to wind and place in position; and to avoid this, it is customary to arrange them in the manner shown in Fig. 124. Here, instead of having as many coils per phase as there are poles, *there are only half the number.* Thus, in the actual case under consideration, there are only two coils per phase, each with two complete turns. The advantage of this arrangement of the winding is that at no point do the end portions of all three phases overlap as they do at X in Fig. 122; so that the expense and difficulties already alluded to are avoided.

Now as there are only two coils per phase in Fig. 124 and four in Fig. 122, it might seem that there would be some difference in the e.m.f. generated in the two cases. The e.m.f. would be the same in each case,

however, for the simple reason that each method of winding gives the same number of conductors in series per phase, viz. eight. Thus in considering what e.m.f. an alternator will give, the *number of conductors in series per phase*, not the number of coils, is what matters.

Actual examples of this type of winding are shown in Figs. 125, 130, and 217.

When there are only a few conductors per slot, as in low-voltage machines, it may be more convenient to

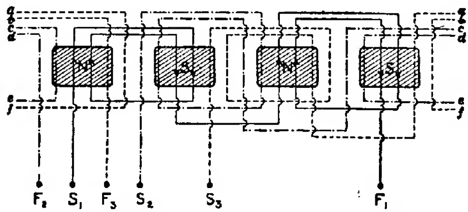

Fig. 124.—Three-Phase Winding with Two Different Shapes of Coils.

employ a different method of winding. Examples of the latter, with diagrams of the connections, are given in Figs. 219, 220, 223, and 224. These windings are arranged very similarly to those on the armatures of continuous-current machines; but on comparing them with Fig. 124, it will be seen that they give exactly the same results. Their chief advantage consists in the simplicity of the end-connections, especially where thick conductors have to be employed.

It has been explained that the windings of three-phase machines can be connected either in star or in mesh. For the star connection, three ends (one of each phase) are joined together to form the neutral point. In Fig. 124, for example, the leads S_1, S_2, and S_3 would

be connected to the three main terminals of the machine; while the ends F_1, F_2, and F_3 would be sweated together to form the neutral point. The latter is sometimes connected to a fourth terminal, which can then be used for earthing the neutral point, or for connecting the fourth cable in a four-wire system (Fig. 105).

If the windings in Fig. 123 were to be mesh-connected, F_1 would be joined to S_2, F_2 to S_3, and F_3 to S_1; the three points of connection being joined-up to the three terminals of the alternator.

63. **LANCASHIRE DYNAMO AND MOTOR CO.'S ALTERNATOR.**—In selecting actual alternators for illustration and description, it is difficult to make a choice from the numerous makes. Thus the particular machine shown in this section, as well as those in the next two, have been chosen very much at random. The alternator illustrated in Fig. 127 may be termed an ordinary form, because its construction adapts it for speeds midway between the slow-speed machine illustrated in Fig. 128, and the high-speed machine dealt with in § 65.

Fig. 125 illustrates the stator of a three-phase machine. The core C is built of annealed mild-steel stampings, with a number of ventilating ducts or spaces s to allow for efficient cooling. The winding is arranged on the method shown in Fig 124; the groups of coils marked e_1 belonging to one phase, those marked e_2 to the second phase, and those marked e_3 to the third phase. There are five groups of coils in each phase; and since, in Fig. 124, one group of turns per phase corresponds to two poles, it follows that the stator in Fig. 124 has been wound for a ten-pole field.

A view of the rotor is given in Fig. 126, where p, p, p are the faces or tips of three of the ten poles.

The pole cores are constructed either of cast-steel or of mild-steel laminations, are rectangular in shape, and are bolted round the periphery of a cast-steel flywheel

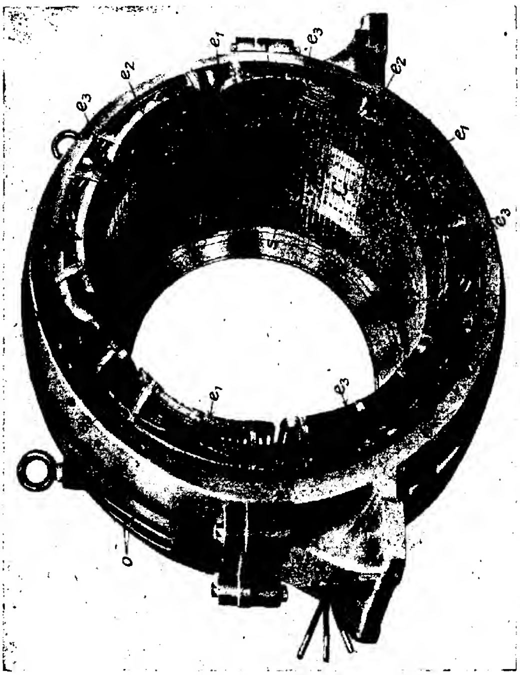

Fig. 125.—Alternator Stator (*Lancs. Dyn. and Motor Co.*).

F mounted on the shaft *S*. The field coils are generally wound with copper strip on edge, as with round wire there is a tendency for the outer turns to work out of position or "roll over" one another under the

action of centrifugal force. The sides of two of the coils are visible at k, k; and each coil is kept in place partly by the brass clamps f, and partly by the pole-

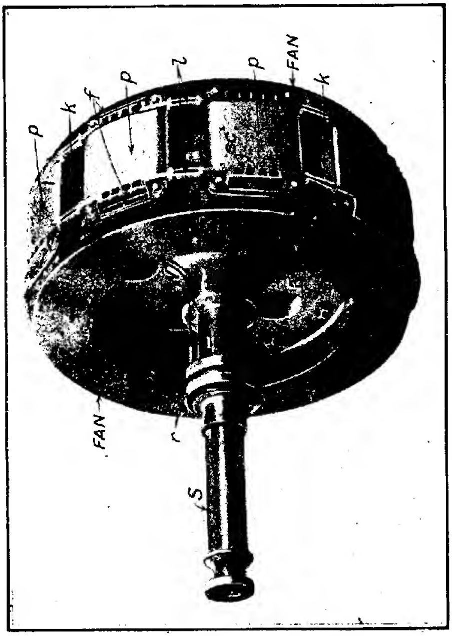

Fig. 126.—Alternator Field-Magnet Rotor. (*Lancs. Dyn. and Motor Co.*)

shoes or pole-tips p, which are shaped as shown at p in Fig 126A. The gaps in f are for saving weight and for ventilation, these clamps butting right onto the side of the pole. There are a number of copper rods c passing through the pole-shoes (Fig. 126A), and these are

riveted and soldered at each end to the coil clamps. The ends of these rods may be detected on the top two poles in Fig. 126. These rods go through the solid parts of the clamps *SC*, which afford a solid base for riveting against; and they are electrically connected together by the clamps, and to similar rods on adjacent poles through the copper links *l*, *l*. The purpose of these copper bars is to assist in the running of two or more alternators in parallel; and they are generally referred to as *damper windings or coils*. Their action is explained in § 91A. The clamps *f* thus perform the double duty of supporting the field coils, and of forming part of the electrical connection between the copper bars *c*.

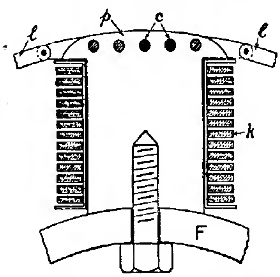

Fig. 126A.—Section of one Pole and its Winding. (*Lancs. Dyn. and Motor Co.*)

F, in Fig. 126A, is part of the flywheel on which the poles are mounted; and *k* are the strip pole-windings already referred to.

The exciting current is led into and out of the field coils by two mild-steel or bronze slip-rings *r* mounted on but insulated from the shaft; *W* being the wires or conductors connecting these rings with the field coils, which are all in series.

The rotor has two fans fitted to it, one on each side; and some of the blades can be seen in Fig. 126. The function of these fans is to draw in cold air and force it under and through the field coils and then through the stator core and end-connections; most of the air

eventually emerging through orifices *o* round the stator frame (Fig. 125). By thus "ventilating" the windings, or—in other words—cooling them, a greater output is obtained from a given machine for a given limit of temperature-rise.

The machine just illustrated is intended for direct coupling to its driving engine, so that it requires only one bearing, for which the shaft has been extended at *S*. The extreme end of *S* projects beyond the bearing, and the armature of the exciter is coupled to it.

Fig. 127 shows a three-phase alternator arranged for driving by belting or ropes, the pulley being missing from the right-hand end of the shaft. The machine has two pedestal bearings. The exciter is mounted on a raised extension of the bedframe, and its shaft is coupled to that of the alternator. The construction of the stator and rotor is very similar to that in Figs. 125 and 126. The field-circuit slip-rings may be seen at the exciter side of the alternator.

64. **VICKERS' FLY-WHEEL ALTERNATOR.**—Fig. 128 is an example of what is sometimes termed the "*fly-wheel*" form of alternator, because of its large diameter. This form is obviously suitable for low speeds only, and consequently has a large number of poles. The particular machine illustrated has 44 poles, is wound for three-phase work, and is rated at 3000 kW at 6600 volts and 60 cycles. Machines of this type and size have been supplied for direct-coupling to water turbines running at 164 r.p.m.

Fig. 184 has been selected as representative of the rotor and exciter armature of a synchronous motor, but it equally well represents the same portions of a fly-wheel alternator.

65. **PARSONS' TURBO-ALTERNATORS.**—A *turbo-generator* is a combination of a turbine (generally a steam turbine) and an alternator or dynamo; more specific

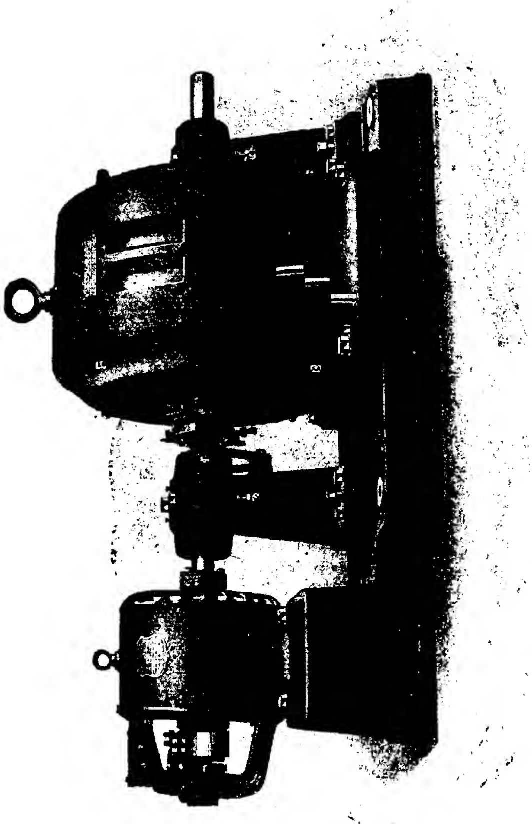

Fig. 127.—Three-Phase Alternator for Belt or Rope Driving. (*Lancs. Dyn. and Motor Co.*)

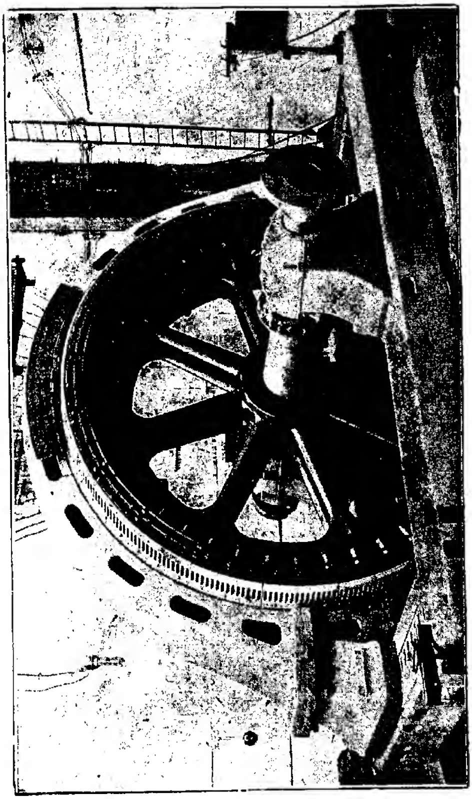

Fig. 128.—Slow-Speed Flywheel Alternator. (*Vickers Ltd.*)

names being *turbo-alternator* and *turbo-dynamo*. The turbine is coupled direct to the electric generator, so that the latter revolves at the same high speed as the former, and consequently has very few poles—generally 4 or 6.

The principle of the steam turbine is very similar to that of the turbine water-wheel. There are, in reality, a number of sets of turbine blades fixed alternately to the stationary casing and to a rotating drum ; the steam passing through the sets one after the other, and causing the drum and its shaft to revolve at a very high speed.

Fig. 129 illustrates a three-phase turbo-alternator for running at 1000 revolutions per minute (r.p.m.). It generates 6600 line volts and 750 line amperes, so that, by Formula 37, p. 162 (which can be applied to an alternator), its k.v.a. output is 8560. But as its load has a power factor of ·7, its true output is 6000 kW.*

The "set" in Fig. 129 really comprises a steam turbine (the two portions at the far end), an alternator, and an exciter. The exciter is at the extreme front end of the shaft, and the length of its commutator and the number of its brushes shows that the exciting current is of considerable strength. As a matter of fact this current is 560 amperes at 125 volts. One of the slip-rings for leading the exciting current into the rotor or field-magnet of the alternator can be seen at the front end of the alternator case, the other being at the other end. It is interesting to note that the set measures nearly 55 ft. from end to end, and stands 10 ft. 6 in. high from the floor level. No details of the construction of the turbine portion can be given here. The arrangements for lubricating such a set are of a very elaborate character, and most of the pipes seen in the figure relate to this work.

* This particular set was supplied to the Randfontein Mines, South Africa.

The stator of the alternator is shown in Fig. 130, and is constructed in a very similar manner to that de-

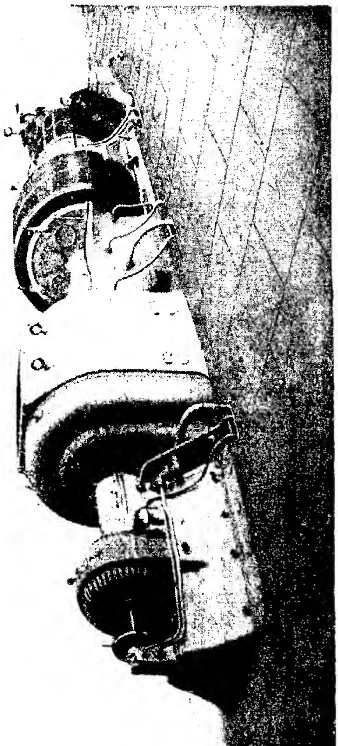

Fig. 129.—Turbo-Alternator, 6000 kW. Rating. (C. A. Parsons & Co.)

scribed in § 63, the chief difference being that in the present case the diameter is relatively less and the length

greater. This dimensioning lessens the mechanical stresses (due to centrifugal force) in the rotating portion. The end windings or connections are very firmly "anchored" to the stator frame by means of

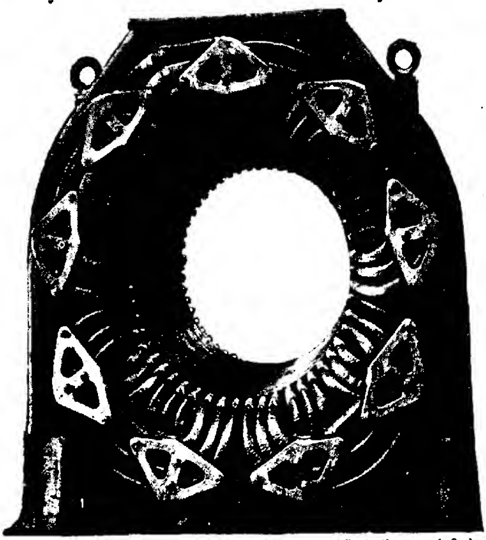

Fig. 130.—Stator of 6000 kW Turbo-Alternator. (*C. A. Parsons & Co.*)

massive clamps bolted thereto. This precaution is necessary to prevent the damage that might occur should the machine be accidentally short-circuited. The large currents that would flow under such a condition would produce a very powerful magnetic drag* between the rotating poles and the stator windings; and

* *See* the Author's *Electric Lighting and Power Distribution*, Vol. I.

this drag might be sufficient to bend and damage the latter if they were not adequately secured.

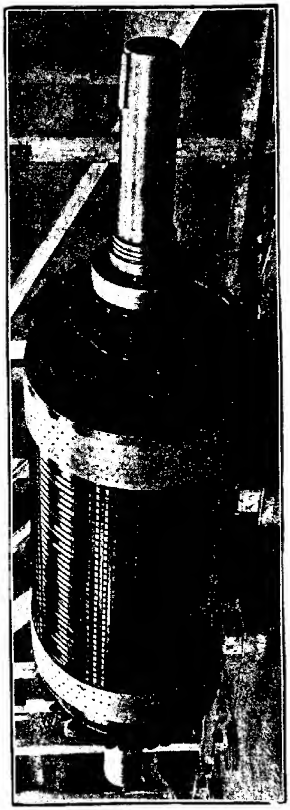

Fig. 131.—Rotor of 6000kW Turbo-Alternator. (C. A. Parsons & Co.)

This stator is wound with nine coils, arranged as in Fig. 124; each coil consisting of three groups of turns disposed in three pairs of slots. Thus there are three coils per phase, so that there are 6 rotor poles; and the frequency obtained at 1000 r.p.m. is 50 cycles per second (p. 34).

The rotor of the same machine is shown in Fig. 131, and is of very different construction from those in the previous two sections. Rotors with salient poles (*i.e.* poles projecting out as in Fig. 126 and 128) are not suitable for turbo work, on account of the high centrifugal stresses produced in the field

coils tending to destroy the insulation by causing compression between neighbouring turns, and also tending to throw some of the turns out of place. The rotor in Fig. 131 consists of a forged-steel shaft with a core formed of thick discs of steel boiler-plate rigidly mounted on it. The use of thick discs is permissible here, because the polarity of any given portion of the core does not change, and therefore there is no tendency for eddy currents to be generated.

Both axial and radial ducts are provided for ventilation of the core. In other words, there are air passages through the core from end to end parallel with the shaft; and numerous others at right angles to these between the core plates. The latter, which are the radial air ducts, are visible in the figure. At each end of the rotor can be seen a fan with a large number of blades. These fans force air into the axial air ducts from their respective ends, the air emerging from the surface of the core through the radial ducts, and then passing into and through similar ducts in the stator core.

The field-windings are placed in groups of slots running from end to end of the core, and are held securely in place by bronze keys or wedges as described presently. The end portions of the windings are strongly supported and held in position under the massive rings or collars adjacent to the fans in Fig. 131.

Two groups of slots and a portion of a third, and two portions of unslotted core showing the orifices of the radial ventilating ducts, will be seen in Fig. 131. The field-windings fill all the slots, and the unslotted portions—of which there are six—form the poles.

The arrangement of the windings will be better understood from Fig. 132, where a portion of the core

and the coil producing one pole $P \leftrightarrow P$ are shown, though not to scale. The area of the pole-face for instance should be relatively larger than is indicated. The shaft is omitted. $L\ L$ represent some of the steel discs separated from one another by the radial ventilating ducts $D\ D$. The windings W, each consisting of a number of turns, are first wound on blocks or "formers" to their correct shape, then inserted in the slots, and

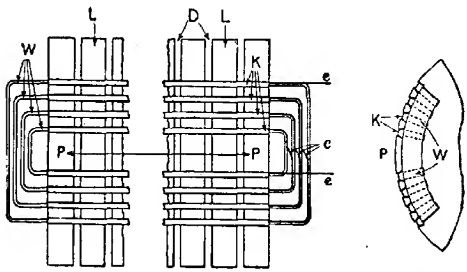

Fig. 132.—Arrangement of Rotor Winding of Turbo-Alternator.

fixed down by the bronze "keys" or wedge strips $K\ K$, each of these running the whole length of the core. The end of one set of turns and the beginning of the next are connected together at the points c, the whole forming one pole coil with ends at $e\ e$. These ends are connected to the adjacent coils, and the two ends remaining after all the coils have been joined together in series are connected to the slip-rings already referred to.

The coils must be so interconnected that the poles alternate in polarity, as shown diagrammatically in Fig. 133, which represents a four-pole winding flattened out, and with only two turns per coil. Here SR, SR are the slip-rings, and N, S, N, S, the poles. The core is

not shown. When in position, the conductors *c*, *c*, would lie in adjacent slots.

The makers of the above-described three-phase turbo-alternator built in 1913 what was then the largest machine of this kind in the world. This has an output of no less than 25,000 kW, and an overall length of 76 feet; its height from the floor level being over 15 feet. The condenser beneath the turbine extends to 14 feet below the floor. Considering that turbo-generators of one-fifth the above size were the largest built six or seven years ago, the progress made is wonderful. And it is interesting to know that the head of the above firm (the late Sir Chas A. Parsons) invented the first practical steam turbine thirty-one years ago (1884), and that this engine (of 6 h.p.) was coupled to a dynamo. This historical set is to be seen at the South Kensington Museum.

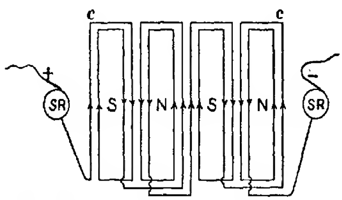

Fig. 133.—Connections of Rotor Winding.

The General Electric Co. of America has quite recently built a turbo-alternator with a capacity of 30,000 kW, *i.e.*, 40,200 e.h.p! It runs at a speed of 1500 r.p.m., and generates 9000 volts at 25 cycles.

66. **E.M.F. OF ALTERNATORS.**—If the flux entering the stator of an alternator from one pole of the rotor be F, and if the number of pairs of poles be p, the total number of lines of force cutting a conductor in one revolution will be the product of the flux per pole and

the number of poles; *i.e.*, the flux cutting one conductor in one revolution $= 2p\,\mathsf{F}$.

If the speed of the alternator is R revolutions per minute, and the frequency is f cycles per second, and as $f = \frac{Rp}{60}$ (p. 34),

then :—

Flux cutting one conductor in one second

$$= 2p\mathsf{F} \times \frac{R}{60} = 2\mathsf{F} \times \frac{Rp}{60} = 2f\mathsf{F}$$

Therefore, from what was said on p. 95, it follows that:—

the average e.m.f. induced in one conductor $= \frac{2f\mathsf{F}}{10^8}$ volts.

The meaning of *average e.m.f.* was explained on p. 79, and it was there shown that, for a sine wave, the virtual voltage $= 1{\cdot}11 \times$ average voltage.

$\therefore$ Virtual e.m.f. induced in each conductor $= 2{\cdot}22\frac{f\mathsf{F}}{10^8}$.

Let N be the number of conductors in series per phase on an alternator; then, if the e.m.fs. induced in all the conductors were in phase, that is to say, if the e.m.fs. in all the conductors reached their maxima at exactly the same time, the virtual e.m.f. induced in each phase would be :—

$$\frac{2{\cdot}22 N f \mathsf{F}}{10^8} \text{ volts.} \qquad \textbf{(39)}$$

But the above formula requires modifying slightly before it can be applied to an actual alternator, for the simple reason that the conductors on any one coil are not all in their best e.m.f. positions (opposite the centres of poles), or in their zero positions (midway between the centres of adjacent poles), at exactly the same time.

Consider Fig. 134 for example (showing one of the

coils in Fig. 124), and assume that the poles are moving in the direction indicated by the arrow. It will be evident that the e.m.fs. induced in conductors 1 and 3 will have their maximum and zero values a little later than the corresponding values in conductors 2 and 4. The resultant e.m.f. will consequently be less than if 1 and 2 were in one slot and 3 and 4 in another; and it can be calculated by a vector diagram as in Fig. 135, where OA is the sum of the e.m.fs. of 1 and 3, and OB that

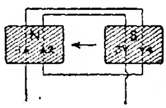

Fig. 134.—E.M.Fs. in an Alternator Coil.

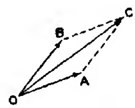

Fig. 135.—Vector Diagram of E.M.Fs. in an Alternator Coil.

of 2 and 4. The resultant e.m.f. is then given by OC, and is obviously less than the arithmetical sum of OA and OB.

Hence, in actual alternators, the voltage induced is less than the value given by Formula 39, the amount depending upon the distribution of the winding. The formula must therefore be modified by the addition of a factor k, which may be termed the *winding factor*, and which varies in practice from ·9 to unity.

Thus:—

$$\text{Virtual e.m.f. per phase} = \frac{2{\cdot}22kNfF}{10^8} \text{ volts.} \qquad \textbf{(40)}$$

The more distributed the winding, the less is the value of k.

67. **SYNCHRONIZING OF ALTERNATORS.**—When we think of the number of times the current from an alternator changes in direction in the course of a second, it is difficult to understand how such machines can be

made to feed in parallel into the bus-bars of a generating station. In early days this apparent difficulty was thought to be insurmountable, but it was eventually overcome, alternators being now practically always connected together in this way.

Suppose there are one or more machines already feeding into the bus-bars, and it is desired to switch-in another. Before doing this, the "*incoming*" machine must not only have the same voltage as those already working, but must also be *synchronized* with them: that is to say, it must be run-up to exactly the same frequency as, and its alternations got into step with those of the machines on the bus-bars. When once synchronized and switched-in with the others, all the machines tend to keep one another in step.

One method of synchronizing is as follows. The primaries p, p of two step-down transformers (§ 76) are connected, one to the bus-bars, and the other to the machine to be synchronized; while the secondaries s, s are joined-up in series and connected to an incandescent lamp and a voltmeter, as depicted in Fig. 136. The lamp should be a carbon-filament one, so as to be very susceptible to changes in the current through it; and it must be suited to the full voltage of $s+s$.

The windings of the transformers are so connected that when the e.m.fs. in the secondaries are in step, they work together; but if out of step, they oppose each other more or less. If the frequency (*i.e.*, the speed) of the alternator to be switched-in is not correct, the lamp lights up and goes out at rapid intervals. The speed of the machine must then be adjusted until the light of the lamp rises and falls only a few times a minute, this showing that the speed and therefore the frequency also are correct. The machine is then switched-in at the moment the lamp is at full in-

candescence, and the voltmeter reading is at its maximum; this indicating that the machine is then in synchronism with the others. Before closing the switch, the alternator voltage must of course be adjusted to correspond with that of the bus-bars.

A more recent arrangement than that shown in Fig. 136 indicates not only synchronism, but also if the incoming alternator is running too fast or too slow, or if it is out of

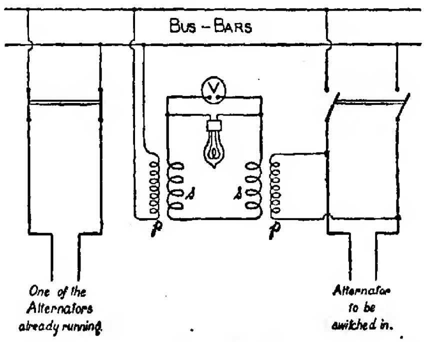

Fig. 136.—Synchronizing Arrangement for Alternators.

phase simply. This type, an example of which is given in Fig. 136A, is termed a *rotary synchronizer*, as it has a rotating pointer which moves in one direction or the other while the incoming machine is out of synchronism. If the pointer rotates clockwise, the incoming machine is too fast; if anti-clockwise, the machine is too slow.

The principle of this instrument is shown in Figs. 136B and 136C. The essential part of the apparatus consists of a miniature slip-ring motor. The stator and rotor are both wound for two phases (p. 185), and in series with these are connected non-inductive resistances R_1 R_2 (in the form of incandescent lamps) and choking coils C_1 C_2, the connections

to the rotor being made by means of three slip-rings SR. In § 107 it is explained that with such an arrangement of coils, resistances, and chokers, a single-phase current is split-up into two currents differing considerably in phase, and that a rotating magnetic field is set up in consequence. Thus in Fig. 136B, the windings A_1 and B_1 produce a rotating field, the speed of which depends upon the frequency of the bus-bar supply; and windings A_2 and B_2 also set up a rotating field, whose speed corresponds to the frequency of the incoming alternator. It is arranged that these two fields (indicated by N_1 S_1 and N_2 S_2 respectively in Fig. 136C) shall rotate in the same direction; and it will be evident that owing to magnetic attraction between N_1 S_2 and N_2 S_1, the rotor field N_2 S_2 will always endeavour to follow the stator field.

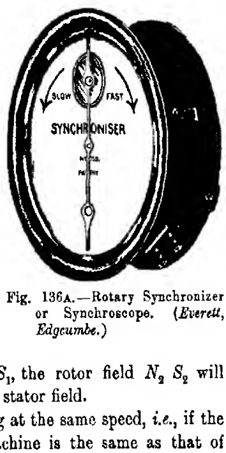

Fig. 136A.—Rotary Synchronizer or Synchroscope. (*Everett, Edgcumbe.*)

If the two fields are rotating at the same speed, *i.e.*, if the frequency of the incoming machine is the same as that of

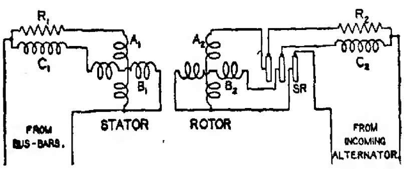

Fig. 136B.—The Principle of the Synchroscope.

the bus-bars, the rotor will remain stationary. But suppose the two frequencies are not the same, and that N_1 S_1 is rotating at, say, 1500 r.p.m., and N_2 S_2 at 1400 r.p.m., when the rotor is at a standstill; then, in order that N_2 S_2 may

still keep pace with $N_1 S_1$, the rotor must rotate in the same direction, (*i.e.*, counter-clockwise in Fig. 136C), at the rate of 100 r.p.m. The pointer which is attached to the rotor would thus travel round the dial in Fig. 136A, at 100 r p.m. in the direction marked "Slow." On the other hand, suppose $N_2 S_2$ is rotating at 1600 r.p.m. when the rotor is held stationary, due to the incoming alternator running at too high a frequency: then for $N_2 S_2$ still to keep in step with $N_1 S_1$, the rotor will rotate backwards (or clockwise) at a speed of 100 r.p.m. This would be indicated on the dial by the pointer travelling "Fast."

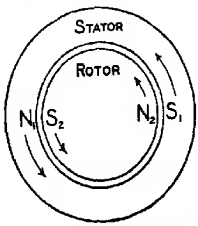

Fig. 136C.—The Rotating Magnetic Fields in the Synchroscope.

Now, in addition to equality of frequency, the voltage of the incoming alternator must also be in phase with that of the bus-bars. This is indicated on the synchroscope by the pointer (Fig. 136A) being vertical. If it is in any other position, the voltages are out of phase, the deflection of the pointer from the vertical being an indication of the phase difference.

As regards the actual process of synchronizing, the speed of the alternator is adjusted until the pointer of the synchroscope is rotating very slowly, say at 2 or 3 r.p.m. Then, after adjusting the voltage of the alternator to that of the bus-bars, the alternator switch is closed just as the pointer is coming to the vertical position.

In the pattern shown in Fig. 136A, a lamp is fitted inside the instrument behind the small circular window there seen. This lamp is alight as long as the synchronizer is connected to the bus-bars, and it fulfils the double function of acting as a non-inductive resistance in one of the instrument motor circuits and as a signalling lamp.

Between the lamp and the window is a shutter with a red and a green transparent disc. The red comes in front of the

lamp when the speed of the incoming machine is too high, and the green when the speed is too low. This coloured-light signalling arrangement serves as a convenient indicator to the distant engine-man regulating the speed of the alternator.

For high-voltage circuits, the synchroscope is used in conjunction with a small step-down transformer, the secondary voltage of which is generally arranged to be about 100 volts. A transformer of this kind, when used in conjunction with an indicating or registering instrument, is frequently referred to as a *potential transformer*, to distinguish it from a *current transformer* (§ 88).

Figs. 136 and 136B apparently relate to single-phase alternators only, since there are only two wires from the machine. But as a matter of fact, these arrangements are also applicable to two- and three-phase alternators, as connection has only to be made to one phase in these cases. Three-phase alternators, for instance, are connected to three bus-bars, and the two bus-bar wires of the synchronizer are joined-up to two of these bars. The other pair of wires from the synchronizer go to the *corresponding phase* of the incoming machine.

68. **"REGULATION" OF ALTERNATORS.**—If the exciting current of an alternator, running at normal speed, be adjusted to give the correct terminal voltage on full load, and if all the load be then taken off the machine, while the exciting current and the speed are kept constant, the terminal voltage will rise. This rise, when expressed as a percentage of the full-load voltage, gives the "*regulation*" of the alternator:—

i.e.,

$$\text{"Regulation"} = \frac{\text{Rise in terminal volts between full load and no load.}}{\text{Full-load voltage.}} \times 100 \qquad \textbf{(41)}$$

The less the value obtained for the "regulation" the greater is the ability of the machine to maintain its voltage steady as the load is varied; and the test for

"regulation," which is made before the machine leaves the makers, is obviously an important one.

The rise in terminal pressure for a given decrease of current depends upon:—

(1) Resistance of winding.

(2) Reactance of winding.

(3) Angle of lag or lead between the current and the voltage.

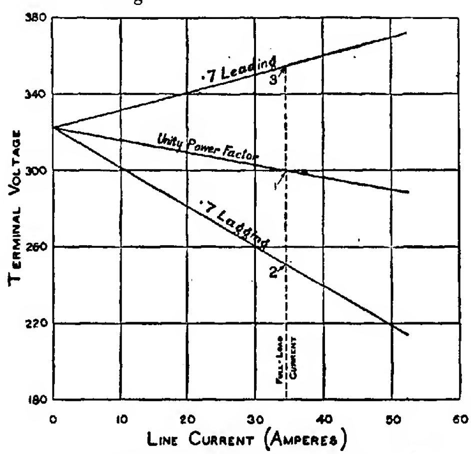

Fig. 137.—Variation of Alternator Terminal Volts under different Kinds of Load, with Constant Speed and Exciting Current.

The greater the resistance and reactance of the winding, the greater is its impedance; and therefore the greater will be the voltage required to send a given current through that winding. This means, in short, that the greater the impedance of the winding of an alternator, the worse is its "regulation."

It was stated in § 42 that a lagging current tends

to demagnetise the field-magnets; a leading current, on the other hand, assists in their magnetization. Thus the greater the lag of current behind the voltage, the less is the flux actually cutting the conductors, and consequently the less is the e.m.f. induced: hence, on a very inductive load, the terminal voltage rises rapidly as the load decreases. With a capacity load, however, the terminal voltage may remain nearly constant, or may even decrease, as the load is decreased.

Fig. 137 shows the actual variation in the terminal voltage of an 18 kVA, 300-volt, 3-phase alternator under different kinds of load, when the speed and the exciting current were kept constant. The behaviour of this small laboratory machine is typical of what occurs in practice with larger ones. From these curves it is seen that the correct terminal voltage (300 volts) has been obtained at full load (34·6 amps.) of unity power factor (point 1 on curve). With the same exciting current, the voltage obtained with the same full load, but inductive and having a power factor of ·7 lagging, is only 253 volts (point 2 on curve). On the other hand, a capacity load taking the same current, and possessing the same p.f., causes the pressure to increase to 357 volts (point 3 on curve).

When the alternator is on no-load, the terminal voltage obtained with the same speed and exciting current is 322, hence:—

$$\text{Regulation on non-inductive load} = \frac{322 - 300}{300} \times 100 = 7 \cdot 3\%.$$

To obtain the regulation on the inductive load, the exciting current would have to be increased until the terminal voltage is 300 when the line current is at its full-load value, *i.e.*, 34·6 amperes. The load would then be removed, and the increase in voltage noted. Similarly, the regulation on the capacity load would be

determined by decreasing the exciting current to give the correct pressure with the same line current as before; and then measuring the variation in voltage caused by taking off the load.

CHAPTER IV.—QUESTIONS.

In answering these Questions give Sketches wherever possible.

NOTE—Questions marked * range slightly beyond the subject-matter of this Book. Those marked † can only be partly answered therefrom.

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1. In what respects does a three-phase alternator differ from a single-phase alternator? What are the reasons that lead to the use of the former in preference to the latter? Give diagrams illustrating windings and connections of a multipolar generator of each kind. (*Ord., A.C.*, 1908.)

2. Give an armature winding diagram for a 4-pole three-phase and also for a two-phase A. C. generator having 24 slots. (*A.M.I.E.E. Exam.*, 1914.)

†3. A 1000-kW three-phase 50-period 6600-volt generator is to be designed to run at 500 r.p.m. Calculate :—

(*a*) The number of poles.

(*b*) The currents which will flow in the windings with a power factor of 0·8 with both delta and star connections.

(*c*) The true efficiency for the 3-phase winding at full load and 0·8 power factor, from the following data :—

Armature resistance between terminals: 0·6 ohm.

Field resistance: 0·5 ohm.

Field current: 150 amperes at full load 0·8 power factor.

Iron loss: 15 kW.

Friction and windage: 7 kW.

(*A.M.I.E.E. Exam.*, 1914.)

Ans. (*a*) 12 poles; (*b*) 63 amps. delta, 109 amps. star; (*c*) 95·8 per cent. efficiency.

*4. Describe clearly what is meant by the "hunting" of synchronous electrical machinery, and enumerate the various conditions which may bring it about. Discuss fully the nature of the cross currents which may pass between an engine driven alternator and a distant synchronous motor, and explain their effect upon the losses of the system. (*Honours*, 1*st Paper*, 1911.)

5. Deduce a formula for the virtual electromotive force of an alternator, in terms of the number of conductors, flux per pole, and frequency, on the assumption that the electromotive force follows a sine law. Show by a vector diagram the relation between the terminal voltage on load and the electromotive force developed in the armature. (*Ord.*, *A.C.*, 1910.)

*6. The equation for the electromotive force E, generated in one circuit of an alternator, may be written—

$$E = k \times f \times Z \times N \div 10^8;$$

where f is the frequency, Z the number of conductors in series in that circuit, N the flux of any pole (assumed all equal), and k a numerical coefficient, the value of which usually lies between the extreme values of 2 and 2·4. Discuss the origin of this coefficient k, and point out the features of the design which determine whether its value will be high or low. (*Honours*, 1*st Paper*, 1908.)

7. A circular coil is rotated about a diameter that is at right angles to a uniform magnetic field. Find the maximum, mean, and virtual (*i.e.* r.m.s.) values of the electromotive force generated if the coil has 400 convolutions of mean radius 12 ins., and rotates at 500 r.p.m. in a field of strength $H=$ 100 lines per square centimetre. (*Based on Ord.*, *A.C.*, 1908.)

Ans. 61 max., 38·9 mean, and 43·2 virtual volts.

8. Describe, with accompanying sketch, some form of

synchronizing gear for paralleling alternators. (*A.M.I.E.E. Exam.*, 1914.)

†9. Describe the construction of a modern form of synchronizer, and explain in detail the principles upon which it is based. Indicate how such synchronizers can be arranged for automatically switching-in the incoming alternator at the right moment; and state in what circumstances it would be profitable to employ such automatic paralleling arrangements. (*Honours, 2nd Paper*, 1909.)

†10. Make a diagram of connections for a switchboard suitable for two single-phase alternators, which are to be switched to the same bus-bars, and show the arrangements which are necessary to synchronise the machines before switching on to the bars. (*Final, 1st Paper*, 1914.).

*11. Two similar alternators, separately driven by two good engines, are working in parallel on the same 'bus-bars. It is observed that they are not sharing the load equally. In what way or ways can they be made to take equal loads? In what way or ways can the whole of the load be thrown over from one to the other while they are still connected to the 'bus-bars? What phase changes, if any, occur during the last-mentioned operation? (*Ord., A.C.*, 1908.)

12. What are the effects of a low power factor on the economical running of a power house? (*A.M.I.E.E. Exam.*, 1914.)

†13. Explain why the terminal voltage of an alternating current generator varies with the amount and character of the load. Give numerical values for the voltage drop which may in practice be permitted. (*Grade II., A.C.*, 1913.)

*14. Give some account of the armature reactions in alternators, and explain in what way the cross-magnetizing and the demagnetizing effects depend on the nature, as well as on the amount, of the load. (*Ord., A.C.*, 1910.)

CHAPTER V.

TRANSFORMERS AND CHOKING COILS.

69. **CHOKING COILS.**—It has already been shown (§§ 9, 11, and 27) that there is a very important difference in the kind of obstruction offered to an alternating current by ordinary resistance and by reactance respectively. Resistance obstructs the current by absorbing the power and converting it into heat. Reactance, on the other hand, obstructs the current by setting-up an alternating counter e.m.f., and so reduces the current in the circuit without wasting power.

The possession of this reactive effect may be regarded as one of the advantages of alternating over continuous currents; for by introducing reactance into a circuit, any required reduction of the current can be effected with comparatively little loss of power. This is generally done by increasing the inductance in a circuit, and consequently also its reactance and impedance, by means of a winding on an iron core. The device is made with as little resistance as possible, and what little loss of power does take place in it is due chiefly to this resistance. It is called variously a *reactance coil*, *impedance coil*, *choking coil*, *choke coil*, or *choker*.

Fig. 138 illustrates the principle of choking coils. *C* is a solenoid or coil of thick wire, with a laminated iron core, *IC*, which may be either fixed or movable. In the

first case, the inductance of the coil is invariable, and so also is its reactance—with a given frequency. In the second case, the inductance and consequently the reactance may be respectively increased or diminished by inserting the core farther within the coil or by withdrawing it.

In some small forms of choking coil with a fixed core, the inductance is varied by varying the number of turns of winding in circuit. (See Figs. 141 and 142.)

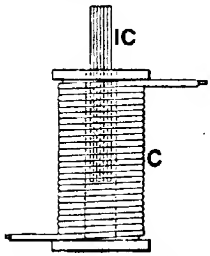

Fig. 138.—Simple Choking Coil.

In § 31 it was shown how to calculate the inductance required in a choker to effect a certain reduction of pressure.

The resistance of the winding of a choker is kept very low, while the iron losses (due to eddy currents and hysteresis*) are made very small by constructing the core of laminations of transformer iron (pp. 226 and 243). In this way the power absorbed by the choking coil is reduced to a negligible quantity.

70. **USES OF CHOKING COILS.**—Choking coils are largely employed for reducing and regulating the pressure in arc-lighting circuits; for varying or "dimming" the light given by any number of glow lamps in such buildings as theatres, music-halls, churches, etc.; and for controlling the voltages across electric furnaces, and welding and kindred apparatus.

Another common use for choking coils, which in this case have no iron core, is to protect from lightning electrical machinery in generating or sub-stations feeding (or fed by) overhead lines. The coil is inserted between

* *See* the Author's *Electric Lighting and Power Distribution*, Vol. I.

the transmission line and the apparatus to be protected, as shown at C in Fig. 139. On account of its small inductance, the effect of C upon the ordinary supply is negligible; but it offers extremely powerful opposition to the passage of lightning, the exceedingly high frequency of the latter tremendously increasing the reactance of the coil. The result is that the current produced by the high voltage in the line due to the lightning finds an easier path across

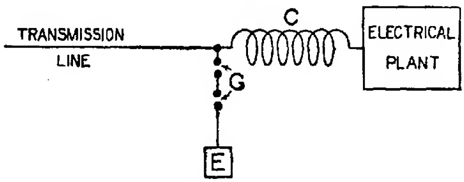

Fig. 139.—Arrangement of Choking Coil and Spark Gaps as Lightning Arrester.

the spark gaps G, of which there are sometimes several in series, and which are connected to earth on the other side. Thus the line is relieved of the lightning charge without any injury being done to the electrical plant. The combination of a choking coil and spark gaps for the above purpose is termed a *lightning arrester*; and there are many interesting points about the construction of these accessories—especially of spark gaps, but these cannot be dealt with here.

Choking coils are sometimes put in series with large alternators to prevent the flow of an excessive current due to accidental short-circuiting. Without a choker, the impedance of the alternator itself would tend to put some limit upon the short-circuit current; but the presence of a choker is a great additional help.

Choking coils (with very few turns) may also be used

to limit the primary voltage when a large transformer is suddenly switched onto the mains; for it is found that the whole voltage for the first instant or two is heaped-up—so

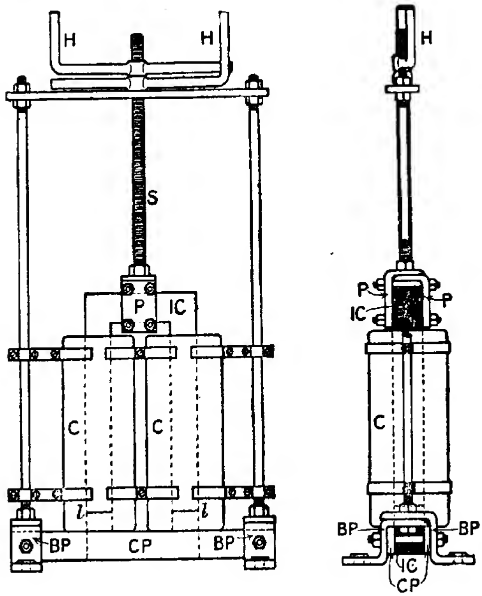

Fig. 140.—Adjustable Choking Coil. (*Foster Engineering Co.*)

to speak—on the end turns of the primary, and is liable to break-down its insulation. If a choker is not used, these outside turns of the transformer must be extra-heavily insulated.

71. **PRACTICAL FORMS OF CHOKING COIL.**—Fig. 140 illustrates the construction of one form of adjustable choking

coil, suitable for regulating large alternating currents for various purposes.

An iron core *IC* is built-up of thin rectangular sheets of a transformer iron known as "Stalloy" (p. 243), with a rectangular opening in the centre. This built-up core is then sawn in two near the bottom (at the dotted lines *ll*), the lower part being rigidly bolted between the base plates *BP* and cross-pieces *CP*; whilst the upper portion (forming an inverted *U*) is clamped tightly by the plates *P*, attached to a screwed shaft *S*. The latter can be raised or lowered by handles *H*,*H*, so arranged that they can lock the shaft in any desired position. The winding consists of two coils, *CC*, connected in series.

When the movable iron core is in the position shown in Fig. 140, there are no air gaps between its extremities and those of the fixed portion, so that the maximum flux is produced in the core by a given current. In other words, the coils *CC* then have their maximum inductance, and —with a given frequency—the choker exerts its maximum reactance and impedance. By raising the shaft *S*, air gaps of increasing length are formed between the two portions of the core, and the flux produced by the same current is decreased. Thus the inductance of the choker may be gradually decreased until, when *S* has been raised as high as possible, it is very small; and the apparatus then exerts very little choking effect.

By varying the dimensions of the apparatus, and the size and number of turns of conductor in its winding, its inductance and current-capacity can be made to suit any requirements.

Another form of variable choker, for small currents, is illustrated in Fig. 141 with and without its cover. This is called a *dimming switch* by its makers, as it is intended for varying the candle-power of a lamp or small group of lamps, by inserting more or less impedance in their circuit.

The connections of this apparatus are given in Fig. 142. *IC* is a laminated iron core with two series-connected coils *CC*. Tappings *TT* are taken off at different places, and connected to the contact-studs 1 to 6 of the multiple-way switch *S*. The latter enables the whole or various portions of the choker winding to be put in series with

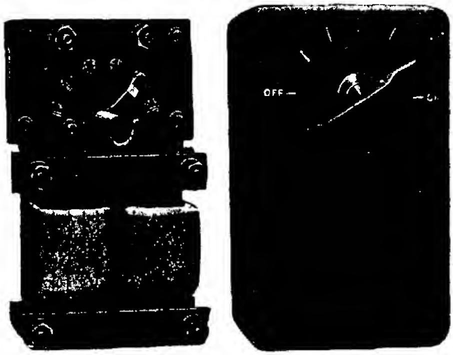

Fig. 141.—Dimming Switch. (*Foster Engineering Co.*)

the lamp circuit. Thus when the switch-arm is on stud 6, all the turns are in circuit, and the lamps then give only a dim light; when the switch-arm is on stud 1, the choker windings are cut out altogether, and the lamp or lamps give their full candle-power.

It should be obvious that choking coils (like that in Fig. 141) whose effect is varied by means of a multiple-way switch, are only suitable for comparatively small currents and low-voltage circuits. With higher voltages and large currents the switch portion would become very large and expensive, and would be subject to considerable *sparkwear*.

72. **TRANSFORMERS.**—A *transformer* is an apparatus without moving parts, which—by electromagnetic induction

—transforms alternating voltages into higher or lower alternating voltages at the same frequency. *Step-up* and *step-down transformers* respectively increase and diminish the voltage of the supply.

Transformers are constructed for either single-, two-, or three-phase systems; and some forms enable two-phase circuits to be fed from three-phase supplies, or the reverse.

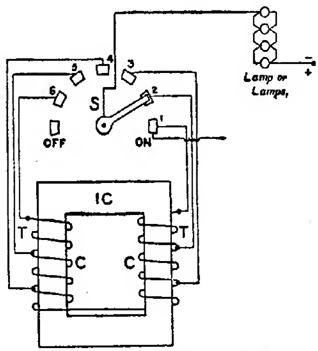

Fig. 142.—Connections of Dimming Switch.
(*Foster Engineering Co.*)

73. PRINCIPLE OF THE TRANSFORMER.—The single-phase transformer depends for its action upon the induction between two neighbouring but distinct electric circuits, which are interlinked with a magnetic circuit common to both.

With apparatus arranged as in Fig. 143, when the key *K* is depressed, a current is sent along *a b*, and this current creates a magnetic field, the lines of which will practically all be linked with, *i.e.* will pass round, *c d*. Then, by the action of electromagnetic induction (p. 36), an e.m.f. will be induced in *c d*, causing a momentary current through the galvanometer *G*. When the current in *a b* is stopped, the magnetic field will collapse, so that another momentary current will be induced in *c d*, in the opposite direction to the first induced current, as indicated by the deflection of the galvanometer *G* in the opposite direction. Thus, if the key be continually depressed and released, causing an

intermittent current to flow in $a\ b$, an alternating current will be induced in $c\ d$.

Since the e.m.f. induced depends upon the total flux linked-with or surrounding each conductor, the above-mentioned effect will be much increased if the wires are coiled-up as in Fig. 144; and still more increased if the

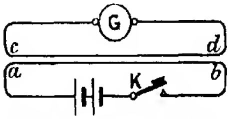

Fig. 143.—Induction of E.M.F. and Current.

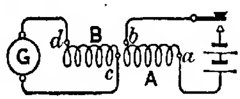

Fig. 144.

coils A and B be inserted one within the other, and provided with an iron core in order to increase the number of lines of force, as shown in Fig. 145. This is the principle of the ordinary *induction coil*, which is an apparatus for transforming an intermittent continuous into an alternating current.*

The simplest form of transformer (Fig. 146) is similar

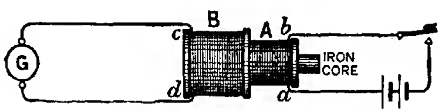

Fig. 145.—Induction of E.M.F. and Current.

in construction to an induction coil, in that it consists of two separate coils wound upon an iron core; but it differs from it in having no contact-breaker or condenser. In Fig. 146, IC is an iron core on which are wound two distinct coils P and S. One of these, P, is called the *primary coil*, because it carries the *inducing current*;

* *See* the Author's *First Book of Electricity and Magnetism*. (Fourth Edition.)

while the other, S, is termed the *secondary coil*, because it is the one which has currents (or more correctly speaking—electromotive forces) induced in it.

It should be perfectly clear from what has just been said, that if an intermittent current be sent through the primary coil P, an alternating e.m.f. will be induced in the secondary coil S. It will now be shown that if an alternating current be sent through P, an alternating current will be induced in S, provided its ends be joined through some kind of circuit. Referring again to Fig. 143, if a current is sent from a to b, a momentary current will flow from d to c; and when the current in a b is stopped, another momentary current will flow from c to d. But if the current in a b, instead of being stopped, is immediately reversed, the second induced current in c d will still flow from c to d, but will be much greater. Thus it is evident that an alternating current passing through the primary coil P (Fig. 146), will induce an alternating current in the secondary coil S.

Fig. 146.—Simple Transformer.

Dr. S. P. Thompson has pointed out that the transformer may be regarded as a separately-excited alternating-current generator in which there are no moving parts; the necessary fluctuations of the magnetic field being brought about by the use of an alternating exciting current. Regarded in this way, the primary represents the field winding, and the secondary the armature winding.

74. **CONSTRUCTION OF TRANSFORMERS.**—When a straight iron core like that shown in Fig. 146 is used, the lines of force emerging from one pole have to return to the other pole through the air, and the inducing flux is consequently comparatively small. In order to get a very strong flux or magnetic field inside its coils, a practical

transformer is made with a closed magnetic circuit, that is to say, the latter is formed wholly of iron.

The first transformer, constructed by Faraday, was of this type, and consisted of a solid iron ring *R* (Fig. 147) on separate portions of which the primary and secondary coils *P* and *S* were wound. Later experience showed two grave drawbacks in this form.

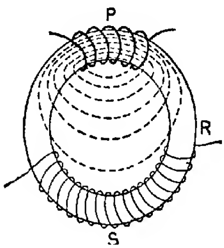

Fig. 147.—Diagram of Faraday's Original Transformer.

The *first of these drawbacks* is the inevitable leakage of magnetism, due to the windings of the coils being on separate parts of the ring. Because of this leakage, only a portion of the magnetic lines set up by the primary pass through the secondary; some of them finding a return path through the air, as shown by the dotted lines in Fig. 147. This *leakage flux* consequently does not assist in the production of e.m.f. in the secondary coil; and the greater it is, the less will be the flux threading the secondary, and the secondary voltage will be correspondingly affected.

The magnetic leakage is reduced by winding the primary and secondary coils over the whole available portions of the core, the two windings being then adjacent to each other throughout their whole length. This ensures that most of the flux produced by the primary shall cut the secondary winding.

The leakage flux is still further reduced by making the magnetic circuit of a long rectangular shape, and covering the long sides of the rectangle with the windings, as seen in Fig. 148. This design increases the length of the leakage path, which is chiefly from one end to the other of each coil. The lengthening of the coils and of their leakage

paths reduces the leakage, because the reluctance or magnetic resistance of the paths is increased.

The second drawback of the form of transformer depicted in Fig. 147 is the great waste of energy due to eddy currents in the solid iron. It has already been explained (p. 223) how an alternating e.m.f. is induced in a secondary winding by an alternating current in the primary. Remembering this, let us see how eddy currents are induced in the Fig. 147 construction.

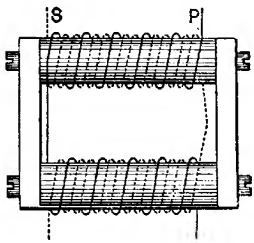

Fig. 148.—Principle of Construction of Most Transformers.

Suppose, to start with, that the iron ring was made up of a large number of concentric tubes insulated from each other; the section of the ring would then be something like Fig. 149. If now an alternating current were sent through the winding *AB*, an alternating magnetic field would be set up in the core: and since this alternating flux would cut the iron tubes in an exactly similar way to that in which it would cut a secondary winding, were one present, alternating e.m.fs. would be induced in the tubes in the directions of the double-headed arrows. And these e.m.fs. would give rise to currents circulating round the tubes. The same generation of e.m.fs. occurs when, as in the Fig. 147 construction, the core is solid; but the paths of the currents in the iron are less definite. These currents, which are comparatively large on account of the low resistance opposed to their e.m.fs., take more or less the circular paths indicated in Fig. 149, and they are therefore termed *eddy currents.*

Eddy currents cause a great waste of power, and so assist in heating-up the transformer; consequently they

have to be prevented as far as possible. This is done by building the transformer cores of very thin sheet stampings of iron or mild steel. These core sheets or plates are usually from about ·012 to ·02 inch (·03 to ·05 cm.) in thickness, and are separated from each other by a very thin layer of insulating material, such as paper or varnish. This *lamination* of the core must be so arranged that the metal affords a continuous path for the lines of force, but is discontinuous in directions at right angles to this, that is, in the directions in which eddy currents would tend to be set up.

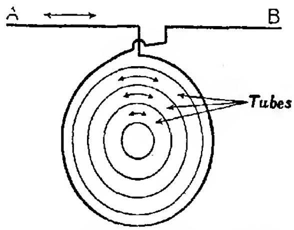

Fig. 149.—Generation of Eddy Currents in a Core consisting of Concentric Tubes.

It is very necessary that the laminations of a transformer core should be tightly clamped together, else the apparatus will emit a humming noise when excited. This *humming* is due to the alternation in the magnetic flux. Thus at one instant, when the flux is maximum, all the laminations are of the same polarity (N. at one end and S. at the other), and they will consequently repel one another. When the flux decreases to zero, the repelling force disappears, and the laminations come together; immediately after they are magnetised in the reverse direction, and again repel one another. It is this continual vibration of the laminations that gives rise to humming, and the only way to overcome this effect is that mentioned above.

Excessive humming is an indication of bad construction.

75. E.M.F. EQUATION OF A TRANSFORMER.—When an alternating current is sent through the primary of a

transformer, such as that represented in Fig. 148, an alternating e.m.f. is induced not only in the secondary but also in the primary winding; for the alternating flux set up in the core by the primary current cuts the primary as well as the secondary winding. The e.m.f. induced in the primary is a reactive or back e.m.f.

By assuming that the leakage flux is negligible, so that the same flux cuts both the primary and the secondary windings, a simple equation for the e.m.f. of a transformer can be deduced as follows.

If E_P be the back e.m.f. induced in the primary circuit, f the frequency, T_P the number of turns in the primary, and F the maximum total flux passing through the core; then (as already explained in § 28A), during one complete cycle, the total change in the number of lines of force passing through the transformer windings will be $4\mathsf{F}$. Hence, during one second, the total change in the lines of force will be $4\mathsf{F}f$ and:—

Average e.m.f. induced in one turn $= \dfrac{4\mathsf{F}f}{10^8}$ volts (p. 95).

$\therefore$ Total of the average e.m.fs. induced in the primary winding $= \dfrac{4\mathsf{F}fT_P}{10^8}$ volts.

For a sine wave, virtual voltage $= 1{\cdot}11 \times$ average voltage (p. 79).

$$\therefore \text{Virtual voltage induced in the primary} = E_P = \frac{4{\cdot}44 T_P f \mathsf{F}}{10^8} \qquad \textbf{(42)}$$

Similarly, if T_S be the number of turns in the secondary,

$$\text{Virtual voltage induced in the secondary} = E_S = \frac{4{\cdot}44 T_S f \mathsf{F}}{10^8} \qquad \textbf{(43)}$$

The numerical value of any one of the quantities just given can be found if others be known; one example of the use of the formulæ being as follows.

EXAMPLE.—*The net sectional area of the core of a transformer is* 16 *square inches, and the maximum value of the alternating flux-density due to the primary current is* 60,000 *lines per square inch. If the frequency is* 50 *cycles per sec., how many turns must the secondary winding have in order that it may generate an e.m.f. of* 255 *volts on open circuit?*

Taking Formula 43 overleaf:—

$\mathsf{F}=16\times 60{,}000$, $f=50$, $E_S=255$, and T_S is required.

Transposing the formula we get:—

$$T_S=\frac{E_S\times 10^8}{4{\cdot}44 f\mathsf{F}}$$

$$\textit{i.e.},\ T_S=\frac{255\times 10^8}{4{\cdot}44\times 50\times 960{,}000}=120 \text{ turns.}$$

76. STEP-UP AND STEP-DOWN TRANSFORMERS.—If the primary and secondary coils had an exactly equal number of turns, the pressure in the secondary would be practically the same as that in the primary. If the secondary has a *greater* number of turns than the primary, as in a *step-up transformer*, the pressure in the secondary will be greater than the pressure in the primary. On the other hand, if the secondary has a *less* number of turns than the primary, as in a *step-down transformer*, the pressure in the secondary will be less than that in the primary. In short, the voltage induced depends upon the number of magnetic lines set up by the primary and the number of turns of wire which they cut in the secondary.

If Formula (42) be divided by Formula (43), we have

$$\frac{E_P}{E_S}=\frac{T_P}{T_S}=k \qquad \textbf{(44)}$$

Thus if we divide the primary voltage * by the secondary voltage, or the primary turns by the secondary turns, for any given case we shall get the same quotient k. This quantity k is termed the *ratio of transformation*. It is below unity in a step-up transformer, and above unity in a step-down transformer.

In practice, it is usual to speak of the ratio of transformation as 10, 20, or 30 to 1; or 1 to 10, 20, or 30, as the case may be. Thus a 20 to 1 transformer is one in which the secondary turns or volts are $\frac{1}{20}$th of the primary turns or volts: and a 1 to 20 transformer is one in which the secondary turns are 20 times greater than the primary turns, *i.e.*, one in which the secondary pressure is 20 times as great as that in the primary.

The current of course is not the same in the primary and secondary circuits. In a step-up transformer, a current at a given pressure is converted into a smaller current at a higher pressure. In a step-down transformer it is just the reverse, a current at a given pressure being transformed into a larger current at lower pressure.

The reason of this is the impossibility of getting more watts out of the secondary than are put into the primary: and remembering that the watts in a circuit depend upon the volts and amperes, it is clear that if the volts (say) are increased, the amperes must diminish, and *vice versâ*. As a matter of fact, the power given out by the secondary circuit is always slightly less than that put into the primary circuit; because a certain amount is absorbed in the process of transformation, due to the heating of the coils, to the setting up of eddy currents,

* When the transformer is supplying a load, on account of the resistance and reactance of its windings, the primary terminal voltage will be slightly higher than E_P, and the secondary terminal voltage will be slightly lower than E_S. In well-designed transformers, however, these differences are very small; so that the ratio of the primary terminal pressure to the secondary ditto can be regarded as approximately equivalent to the ratio of the turns in all circumstances.

and to hysteresis, as evidenced by the heating of the iron core (§ 82).

Practically speaking, the power absorbed by the primary circuit of a transformer is proportional to that drawn from the secondary circuit. In other words, the current in the primary increases and decreases with every increase and decrease of the current in the secondary.

The frequency of the secondary e.m.f. is exactly the same as that of the primary e.m.f.

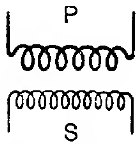

Fig. 150.—Step-Up Transformer.

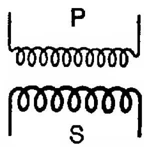

Fig. 151.—Step-Down Transformer.

Transformers are represented diagrammatically as in Figs. 150 and 151, P being the primary and S the secondary in each case. The first represents a step-up and the second a step-down transformer, the difference in the number of turns in P and S indicating this. These diagrams also indicate that the conductor in the primary is thicker than that in the secondary in a step-up transformer; and that the reverse is the case in a step-down transformer.

77. **CORE AND SHELL TRANSFORMERS.** — Transformers are divisible into two classes, viz., *core-type* and *shell-type*, according to the relative disposition of the core and windings.

A *core-type transformer* is one in which the iron core is nearly enclosed by the windings. An example is given in Fig. 152, which is similar to Fig. 148 in principle. In Fig. 152, IC is the laminated core, PP the primary winding, and S the secondary winding.

There is a comparatively long magnetic circuit, owing to the presence of the yokes or unwound portions YY; and a comparatively large magnetising current is neces-

sary. In other words, the longer a magnetic circuit of a given cross-section the greater its reluctance, and the greater the number of ampere-turns necessary for a given flux.* The core type, however, possesses the advantages that the parts are easily assembled or put together, and that it has a good cooling surface, since so much of the outer winding is exposed. Actual transformers of this type are illustrated and described in the next two sections.

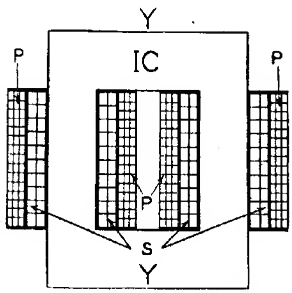

Fig. 152.—Section of Core-Type Transformer.

A *shell-type transformer* may be likened to a core-type transformer turned inside out, for in this type the windings are practically surrounded by the iron core, instead of the iron core being more or less covered by the windings. A simple example is given in Figs. 153 and 154. The core-plates *CP* (Fig. 153) are of rectangular shape, with two holes punched through. One side is cut through at *c*, so that the middle piece *m* may be bent-up like a flap to facilitate the placing over the coils *P* and *S*; *m* being bent back flat again when the plate is in position. Fig. 154 gives a perspective view, where *CP* are the core-plates, and *P* and *S* the primary and secondary coils, these being separately wound and bound-up with insulation before the "shell" is built-up round them.

The advantage of the shell over the core type is that it

* *See* the Author's *Electric Lighting and Power Distribution*, Vol. I.

possesses a shorter magnetic circuit, and therefore takes a less magnetising current. The coils, however, are not readily accessible; and in the form of construction shown in Figs. 153 and 154, they do not possess so much cooling surface as in the core type.

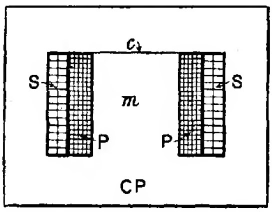

Fig. 153.—Section of Simple Shell-Type Transformer.

The only transformer of the shell-type that is in general use is the Berry transformer, detailed particulars of which are given in § 80. As will be there seen, the construction is very different from that illustrated here, and the cooling difficulty just alluded-to is non-existent.

In Figs. 152 and 153 the sections of the windings are indicated by a sort of lattice-work. This is the usual way of showing a section of winding in a diagram, it being much more convenient than drawing a number of small circles to represent the wires. The winding with the most turns is indicated by a finer "mesh"; and it is evident from this that Figs. 152 and 153 both represent step-down transformers.

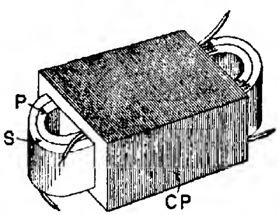

Fig. 154.—Simple Shell Transformer.

78. **SIMPLE CORE TRANSFORMER.** — Fig. 155 illustrates a small-capacity transformer for use on low-voltage circuits; and the illustration gives a very simple and clear idea of the core-principle.

The rectangular laminated iron circuit is held in a cast-iron base-frame, with holes to enable it to be screwed to a bench or other support. The primary and secondary windings have each only one coil, and the two are bound-up together and mounted on one limb of the core. The terminals are fixed on flanges on the base frame, but are, of course, insulated therefrom. The magnetic circuit is obviously a long one in this particular case; but the consequent low efficiency and power factor are of little moment in the uses for which this form and size of transformer is intended, and are compensated for by the simplicity of construction.

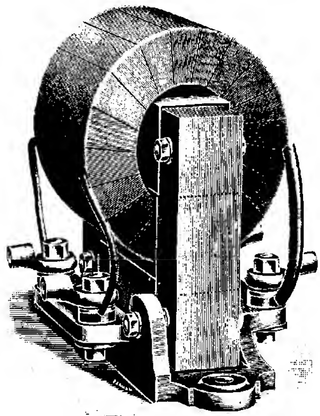

Fig. 155.—Simple Core Transformer. (*Griffin.*)

79. **JOHNSON AND PHILLIPS' TRANSFORMER.**—There are many makes of large core-type transformer, but as the main features of these are very much the same, it is only necessary in a book like this to select one make for description.

Fig. 156 illustrates a 50-kVA, 13,000/2400 volts, 60-cycle, core-type transformer; the parts of the same being displayed in Figs. 157 and 158.

The magnetic circuit consists of two laminated cores *CC*, and laminated yoke-pieces *YY'*, built up of varnished sheets of "Stalloy" iron (p. 243). These sheets are compressed

tightly between pairs of clamps tt of T-section girder steel, the whole being held together by bolts.

In the smaller sizes of these transformers, the core plates of the limbs CC are separate from and are interleaved with those of the yokes YY'. In the larger sizes, as in Fig. 157, the portions CC and Y are built of U-shaped laminæ; and the contacts between the limbs and the top yoke are butt-joints machined so as to secure good magnetic connection between them. This arrangement renders the assembling and the replacement of the coils a very simple matter. It will be noticed that the cores are of peculiar "stepped" section.

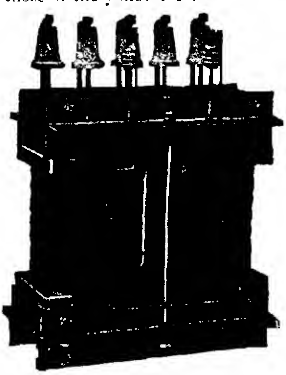

Fig. 156.—A 50-kVA Transformer. (*Johnson and Phillips.*)

Because of this, when the circular former-wound low-voltage coils are put on, a number of oil* channels are left between them and the cores (see Fig. 158), these channels permitting the necessary circulation of the oil and cooling of the core.

Before the low-voltage winding L is put on, the cores have sheets of press-spahn and varnished cloth bound round them. Between the low-voltage and the high-voltage coils L and H is placed a cylinder I of insulating material. The free ends of the windings are connected to porcelain terminals,

* This transformer is of the oil-cooled type (p. 253), and is therefore intended for enclosure in a steel tank, somewhat similar to that seen in Fig. 167.

mounted upon the transformer framework. Very often extra terminals are connected to tappings on one of the

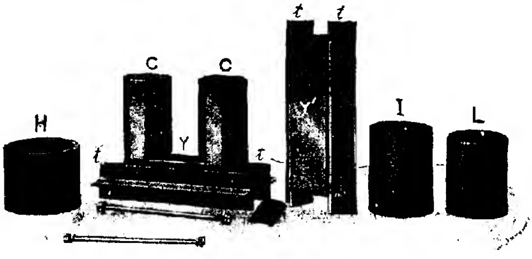

Fig. 157.—Transformer in Parts. (*Johnson and Phillips.*)

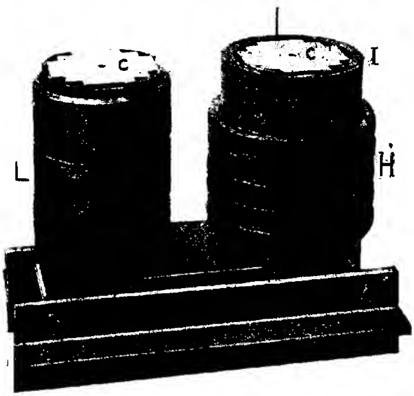

Fig. 158.—Transformer in Course of Assembly. (*Johnson and Phillips.*)

windings. Thus, in Fig. 156, the primary is supplied with three such tappings and terminals, to enable the transformer to work on circuit pressures of 12,500, 12,000, or 11,500

volts, if necessary, instead of the maximum pressure of 13,000 volts. These tappings also enable the secondary pressure to be varied with a fixed primary pressure.

It will be noticed in Fig. 158 that the windings are divided into sections, and that the number of sections is greater for the high-voltage winding than for the low-voltage one. This sub-division decreases the voltage between adjacent layers, and thus reduces the possibility of a breakdown of the insulation. Suppose that, instead of being in sections, one of the coils was wound round the whole length of one limb of the core. There would then be a very large number of turns between, say, the beginning of the first layer and the end of the second, and therefore a proportionately large voltage between those two points, with a serious risk of breakdown of the insulation between the wires at that place. When the coils are wound in sections, the maximum voltage between each layer on any coil is comparatively low. The H coils in Fig. 158 are wound in more sections than the L coils on account of the higher voltage at which they have to work.

In a step-down transformer, the H windings would be in the primary circuit and the L windings in the secondary circuit. In a step-up transformer it would be the reverse.

80. **BERRY TRANSFORMER.**—The Berry Transformer, so-called from the name of its designer, is of the shell type, and is cylindrical in form. A complete transformer is shown in Fig. 159, the terminals being on the top. The pair to the right are the low-voltage terminals, the high-voltage ones being mounted on corrugated porcelain insulators as seen.

The transformer core consists of iron laminations II, arranged in groups which radiate from the centre, as seen in the section in Fig. 160. The inner laminations are built-up around a steel centre shaft which is afterwards removed. The core also encloses the windings at

top and bottom. Thus the iron circuit is formed in two separate parts, one chiefly inside and one chiefly outside the windings. All the sheets are L-shaped, those of the

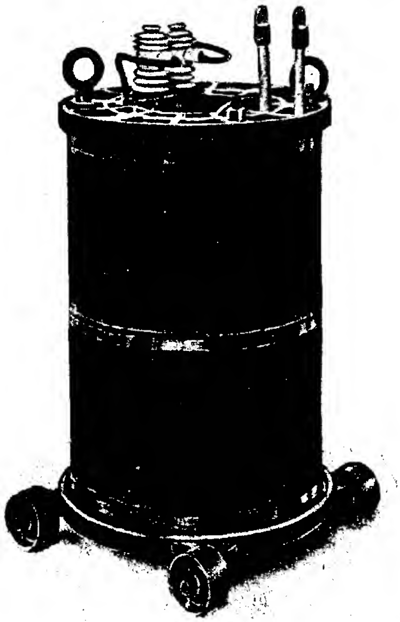

Fig. 159.—Berry Transformer. (*British Elec. Transformer Co.*)

centre core also forming the outside base. The L sheets of the outer core are inverted, and thus form the outside and top of the transformer.

When the inner portion of the core has been built-up

and well-insulated with mica sheets, the inner half of the low-voltage winding *S* is put on, and bound with layers of mica. Next to this come the two portions of the high-voltage winding, these forming two cylinders *P* (Fig. 160)

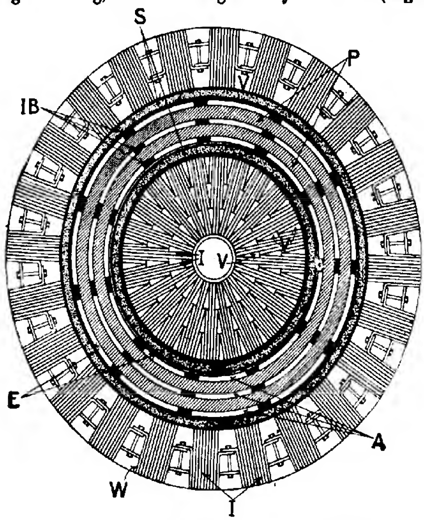

Fig. 160 —Horizontal Section of Berry Transformer.

each consisting of sections one above the other. Surrounding this is the second half of the low-voltage winding *S*. *IB* are insulating separator blocks between the windings, there being several sets round the frame. These blocks serve not only to keep the windings apart, but also to form spaces *A* through which the air can circulate. Similar ventilating spaces *V*, *V*, *V*, are formed between the radial arms of the iron core. Sometimes the complete transformer is mounted in an oil tank. (See p. 253.)

In some cases, *earth shields E* are interposed between the high and low-voltage windings. These consist of finely-woven copper-gauze sheets, built-up to form cylindrical screens. Being connected to earth through the transformer frame, they effectually protect the low-voltage winding from any leakage from the high-voltage circuit.

After the windings have been completed, the outer core plates are put in position, and the groups of plates are held apart by the wedge-shaped spacing blocks *W* (Fig. 160). As seen in Fig. 159, the transformer, as a whole, is first bound round in two or three places with steel bands or wires, and then bolted rigidly between circular cast-iron grids at top and bottom. It is then frequently mounted on rollers or small wheels as shown.

Fig. 161 shows the inner half of the core with the windings in position, ready for the placing of the outer half of the core. In this particular case it is evident, from the size of the terminal lugs, that the low-voltage winding is intended to carry a heavy current. This winding, in fact, usually consists of a number of copper strips connected in parallel.

81. **RELATION BETWEEN THE PRIMARY AND SECONDARY CURRENTS.**—When the secondary of a transformer is on open circuit, *i.e.*, when there is no current in it, a small current is taken by the primary sufficient to produce the necessary flux in the core, and also to supply the watts absorbed by hysteresis and eddy currents and in the winding itself. In other words, when the secondary circuit is open, the primary acts just as if it were a powerful choking coil.

When a current is taken from the secondary, it tends to set up a flux of its own in the opposite direction to the main flux (in accordance with Lenz's Law). This "counter flux" weakens the flux passing through the primary, thereby decreasing the back e.m.f. induced in that circuit; and

since the voltage applied to the primary is constant, more current flows from the mains into the transformer. This

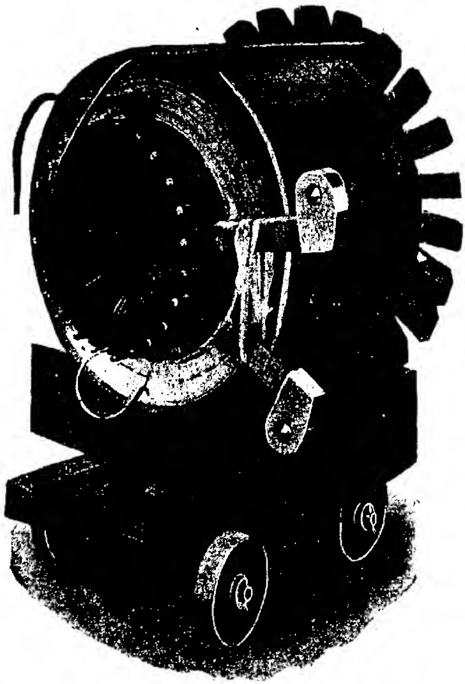

Fig. 161.—Windings and Inner Core of Berry Transformer.

current increases until the increase of flux due to it is equal to the decrease of flux due to the secondary current. Thus it is that the actual flux in the core, and the e.m.f.

induced in the secondary, are both maintained practically constant at all loads.

The flux-producing action of a current depends upon the product of the current and the number of turns through which it flows, that is, upon the ampere-turns. Thus when, as just seen, the secondary ampere-turns try to *decrease* the flux by a certain amount, they will be balanced by an increase in the primary ampere-turns trying to *increase* the flux by the same amount. Consequently, neglecting the small current taken by the primary winding when the secondary is on open circuit, we have—

$$\text{Primary ampere-turns} = \text{Secondary ampere-turns}$$

or,

$$I_P T_P = I_S T_S$$

From the above and Formula 44, p. 228, we have :—

$$\frac{I_S}{I_P} = \frac{T_P}{T_S} = \frac{E_P}{E_S} = k \qquad \textbf{(45)}$$

$$\therefore \; I_S = k I_P.$$

The above explains and proves the statement made on page 230 that any increase or decrease in the current drawn from the secondary causes a proportionate increase or decrease in the current in the primary.

It was stated above that the voltage at the secondary terminals of a transformer was maintained nearly constant; so that apparently the less the resistance of the circuit connected to the secondary, the greater the current that can be drawn from the latter. There is a limit, however, since the maximum possible current (with the secondary short-circuited) cannot exceed that given by dividing the secondary induced voltage by the impedance of the secondary winding. In practice, of course, the secondary is never short-circuited; and if this were done unintentionally, the primary and secondary windings would probably be burnt out if there were no fuses in these circuits.

82. **LOSSES IN AND EFFICIENCY OF TRANSFORMERS.**—The full-load efficiency of transformers ranges from about 85 per cent. in very small sizes to 99 per cent. in very large ones.

The efficiency of transformers is thus very high compared with that of other electrical apparatus such as generators or motors; and the error incurred by neglecting the losses in a transformer is consequently very small. Even when it is not working up to its full load, *i.e.*, up to its full capacity, the efficiency of a transformer is comparatively high.

The losses (of power) in a transformer are of two kinds viz. (1) those in the two electric circuits, and (2) those in the magnetic circuit. These may be detailed as follows:—

(1) *Losses in the primary and secondary windings due to their resistance.* These are generally referred to as the *copper losses,* and can be determined as below after measuring the respective resistances R_p and R_s of the primary and the secondary windings.

Thus if I_p and I_s be the respective currents:—

The copper losses $= I_p^2 R_p + I_s^2 R_s$ (watts).

(2) *Losses in the iron core due to* (a) *hysteresis, and* (b) *eddy currents.* These losses are generally grouped together, and are known as the *iron losses,* and may be expressed in watts.

The hysteresis * loss is the power absorbed in maintaining the alternating flux in the iron. The eddy-current loss is due to e.m.fs. induced in the iron by the alternations of the flux, and is minimised by laminating the core, as already explained (§ 74).

The hysteresis loss in the early transformers was found to increase if the iron core was allowed repeatedly to heat-up past a certain degree, this increase being said to be due to

* *See* the Author's *Electric Lighting and Power Distribution*, Vol. I.

the *ageing* of the iron. So far, the cause of this does not seem to have been discovered; but it is known that the phenomenon occurs with some qualities of iron and not with others.

During recent years, much progress has been made in the manufacture of iron suitable for building the cores of transformers and other alternating-current apparatus, mainly by the addition of a small percentage of silicon. The alloy

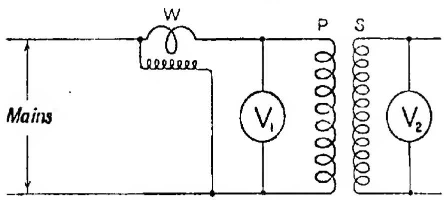

Fig. 162.—No-Load Test for the Iron Losses in a Transformer.

known as "Stalloy," for example, contains about 3 per cent. of silicon. Iron alloyed like this possesses a much higher specific resistance than ordinary iron, so that the eddy currents are greatly reduced. It is also claimed that the hysteresis loss is much smaller, and that trouble from ageing is eliminated.

The connections for the simplest method of determining the total iron losses in a transformer are given in Fig. 162. The primary is connected through a wattmeter to alternating-current mains of the correct voltage (indicated on V_1) and frequency, while the secondary is left on open circuit. The voltmeter V_2 is not used in the present test, but is referred to later (p. 267). The current taken by the primary will only be a small fraction of the full-load current, so that the copper loss in the primary will be negligible.

It was explained in the preceding section that the actual flux in the core of a transformer remains practically constant at all loads; and since the iron losses

depend upon the value of the flux, these will also remain constant from no load to full load. Hence *the reading on the wattmeter W in Fig.* **162** *will give the iron losses (in watts) in the transformer.*

Now as regards efficiency:

$$\text{Efficiency} = \frac{\text{Output (in watts).}}{\text{Input (in watts).}}$$

But:— $\text{Input} = \text{Output} + \text{Total losses.}$

$$= \text{Output} + \text{Copper losses} + \text{Iron losses.}$$

$$\therefore \text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Copper losses} + \text{Iron losses.}} \qquad \textbf{(46)}$$

The above would give the efficiency value as a fraction, but if multiplied by 100 would give it as a percentage.

EXAMPLE.—*Calculate the efficiency at full load of a* 400-*kVA transformer, when the power factor is* 0·8, *from the following data:—Primary volts* = 4000. *Secondary volts* = 500. *Resistance of primary and secondary windings* = 0·2 *and* 0·0025 *ohm respectively. Iron losses (determined by the no-load test)* = 2500 *watts.* [*The* 400-*kVA is, of course, the output from the secondary circuit of the transformer.*]

$$\text{Secondary current} = \frac{\overbrace{\text{kVA} \times 1000}^{\substack{\text{secondary} \\ \text{volt-amperes}}}}{\text{secondary voltage}}$$

$$= \frac{400 \times 1000}{500} = 800 \text{ amperes.}$$

$$\text{Approximate primary current} = \frac{I_s \times E_s}{E_p} \text{ (by Formula 45)}$$

$$= \frac{800 \times 500}{4000} = 100 \text{ amperes.}$$

Watts lost in primary winding $=100^2\times\cdot2=2000$

" " secondary " $=800^2\times\cdot0025=1600$

$\therefore$ Total copper losses $=3600$ watts.

Output $=400$ kVA

$=400\times1000$ volt-amperes

$=400\times1000\times\cdot8=320000$ watts.

Hence from Formula 46 we have:—

$$\text{Efficiency}=\frac{320000}{320000+3600+2500}\times100$$

$$=98\cdot1 \text{ per cent.}$$

From what was said in §§ 43 and 44, it should be clear that the volts and amperes in the secondary circuit primarily represent volt-amperes, and that the actual power in watts which these represent depends upon the power factor of the circuit (or circuits) connected to the secondary. In the present case this p.f. value is ·8.

In the above example, the primary current of 100 amperes was obtained on the assumption that the efficiency was 100 per cent. But since the actual efficiency is 98·1 per cent., the primary current will be $\frac{100}{98\cdot1}\times100=102$ amperes. However, the neglect of the slight increase in the copper loss due to this extra current is to some extent compensated for by the fact that a small copper loss is actually included with the iron losses when the latter are determined by the Fig. 162 test.

83. **LOSSES IN AND EFFICIENCY OF TRANSFORMERS** (*cont.*)—In the previous section it was possible to get the copper losses by calculation, as the resistances of the windings, as well as their currents, were assumed to be known. But when the resistances are not known, or when their correct values (allowing for heating-up and skin effect) are not easy to determine, the copper losses in a transformer may be got by test, with apparatus and connections as shown in Fig. 163.

P and S are the primary and secondary windings. A wattmeter, ammeter, and voltmeter are connected in the primary circuit; and the terminals of the secondary are short-circuited through another ammeter. Hence the name *short-circuit test.*

A *low* voltage at the correct frequency is applied to the primary-circuit terminals T, T; and this is increased until the secondary current indicated by A_2 has reached its full-load value. The ammeter A_1 will consequently be registering the full-load primary current, because the ratio between the

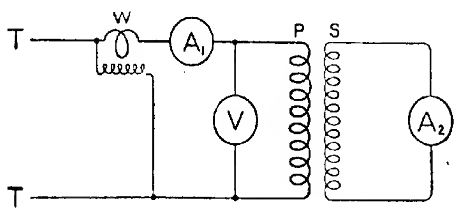

Fig. 163.—Short-Circuit Test for the Copper Losses in a Transformer.

two currents is always constant (p. 241); and the I^2R losses in both windings will therefore also be at their full-load values. *The watts indicated by W* (in Fig. 163), *will then represent the full-load copper losses in both the primary and the secondary windings*; and the value may be inserted at once in Formula 46 (p. 244) when calculating the efficiency, instead of working out these losses as on the preceding page.

There are two points about the above test which need explanation:—(*a*) How is it possible to get the full-load currents by applying a much lower voltage at the primary terminals than that at which the transformer is normally intended to work? (*b*) How is it that some of the watts indicated by W do not represent iron losses?

Although, in the above test, the full-load current is got through the secondary, the latter is giving practically no output in watts. The reason is that as the resistance of the ammeter A_2 is exceedingly low, the voltage at the secondary terminals, and therefore the output of the transformer, will be virtually zero. It follows then that practically all the watts supplied to the primary are dissipated in the transformer windings.

The only e.m.f. that has to be induced in the secondary winding is that necessary to send the full-load current against the resistance and the reactance, *i.e.*, against the impedance of that winding. This e.m.f. will therefore be only a small fraction of the normal secondary e.m.f. And as the e.m.fs. *induced* in the primary and secondary windings are always proportional to each other (Formula 44, p. 228), the e.m.f. induced in the primary, which is a back e.m.f., will also be very small.

The voltage applied to the primary terminals has only to send the full-load current against the impedance of the primary winding, and to neutralise the above-mentioned back e.m.f. And since these values are very small, the voltage required on the primary is also very small. In other words, the primary voltage necessary for setting-up the full-load current in a short-circuited transformer is a small fraction of the normal input voltage. This answers question (*a*) on the opposite page.

The answer to question (*b*) on the same page is as follows:—

Formula 43 (p. 227) for the e.m.f. (E_S) in the secondary circuit is:—

$$E_S = \frac{4{\cdot}44\ T_S\, f\, \mathsf{F}}{10^8}$$

As the secondary terminals are short-circuited in the test (Fig. 163), the voltage E_S required to be induced in the secondary winding to produce the full-load current

therein will be very small. And since the number of turns and the frequency are the same as when the transformer is working under normal conditions (see equation), the value of the flux F must also be very small. On account of this, the counter e.m.f. induced in the primary winding will also be small; and this further explains why only a low voltage has to be applied to the primary winding when performing the test. Now, as the iron losses depend on the magnitude of the flux, and since—as just explained—the flux is very small when the transformer is on short circuit, therefore, in this test (Fig. 163), the iron losses will be negligible compared with the copper losses.

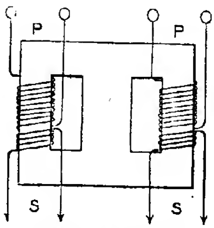

Fig. 164.—Diagram of Two-Phase Transformer.

The advantages of the no-load and short-circuit tests (Figs. 162 and 163) are that they are very easily performed, that they absorb but little power, and that the *regulation* of the transformer can be determined from them in the manner described in § 89c. The particular tests described are only applicable to single-phase transformers; but similar tests may be arranged for two- and three-phase transformers.

84. **TWO- AND THREE-PHASE TRANSFORMERS.**—The pressure on two- or three-phase circuits may be decreased or increased by connecting single-phase transformers in each phase of the circuit; and this is often done. But instead of employing separate transformers, it is more economical, as far as iron and the space occupied is concerned, to combine them by winding each phase on a separate core and using a common yoke for all the cores. Thus Figs. 164 and 165 show diagrammatically the arrangement of two- and three-phase transformers respectively.

An actual three-phase transformer is illustrated in Fig. 166; and Fig. 167 shows a corrugated sheet-steel tank in which it is placed, immersed in oil. This oil acts not only as a cooling medium (§ 86), but also as an excellent insulator.

The four low-pressure terminals are seen on the near side of the transformer in Fig. 166. The nearest terminal in the figure is connected to the junction of the three leads seen at the bottom of the transformer, this junction forming the star point.

The ends of the high-tension windings, as well as a number of tappings therefrom, terminate at the porcelain-insulated terminals seen on the far side of the transformer.

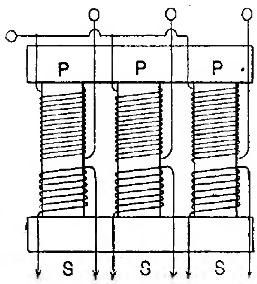

Fig. 165.—Diagram of Three-Phase Transformer.

Berry three-phase transformers are built-up by simply mounting three single-phase transformers (Fig. 159) one above the other, and fixing on the top the necessary terminals for the set.

On a three-phase system, the question as to whether the transformers shall be grouped single-phase, or the combined type, is often a mere question of expense, since the arrangements are equally good electrically. For example, in a small three-phase power-station it is generally more economical to instal groups of three single-phase transformers instead of single three-phase transformers. The reason is that the always-necessary spare plant in the first case need never exceed say one single-phase transformer per group; whereas in the other case, one or more three-phase transformers (each costing between two and three times as much)

would be required. But when there are a large number of transformers in the station, it is more economical to instal three-phase transformers.

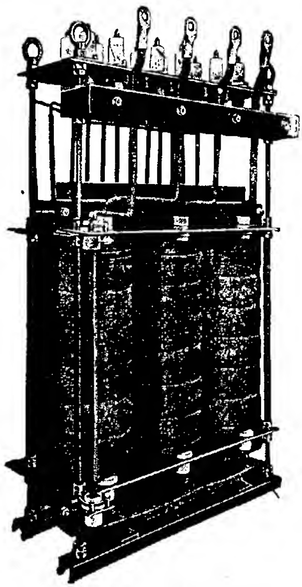

Fig. 166.—Three-Phase Transformer. (*Ferranti.*)

Suppose a 110-kVA, 50-cycle, single-phase transformer costs £130, and a 330-kVA three-phase transformer for the same frequency costs £320.

For a small station of say 330-kVA capacity:—

Cost of 4 single-phase transformers = £520.

Cost of 2 three-phase transformers ≐ £640.

Thus in this case single-phase transformers involve the lesser outlay.

Now consider a station of say 1980-kVA capacity:—

No. of 110-kVA single-phase transformers required

$$=\frac{1980}{110}=18.$$

No. of 330-kVA three-phase transformers required

$$=\frac{1980}{330}=6.$$

Assuming that three spare single-phase transformers be

considered sufficient in the former case, *i.e.*, one to each two

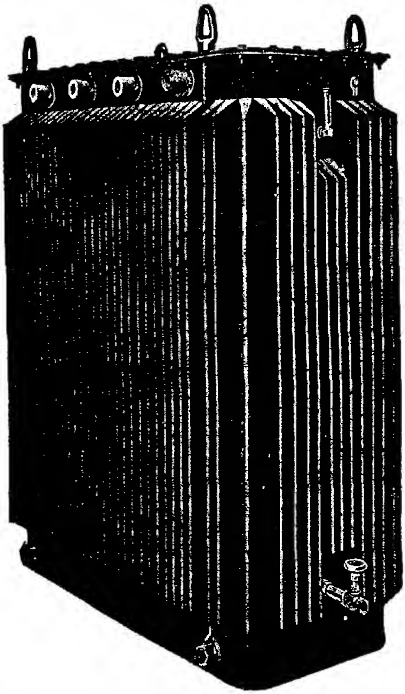

Fig. 167.—Steel Tank for Oil-Immersed Transformer. (*Ferranti.*)

groups, then in the latter case two three-phase transformers will suffice. We then get :—

Total cost of 21 single-phase transformers $= 21 \times 130$
$= £2730.$

Total cost of 8 three-phase transformers $= 8 \times 320 = £2560.$

Thus it is clear that for large stations three-phase transformers are cheaper, even after allowing for more spare plant.

The use of single-phase transformers has the advantage that in the event of a breakdown of one phase, it is comparatively easy to remove the single-phase transformer concerned, and substitute another. But such an occurrence in a three-phase transformer would obviously entail the substitution of a complete fresh transformer.

On the other hand, when generating stations (or—more often—sub-stations) are situated in the centre of a thickly-populated district, the question of floor-space has to be considered very carefully. In such cases, the greater compactness of three-phase transformers renders them much more advantageous than single-phase transformers.

85. **STAR- AND MESH-CONNECTED THREE-PHASE TRANSFORMERS.**—Whether a three-phase transformer be composed of three single-phase transformers, or be of the composite type (Fig. 166), it is clear that its windings may be connected either in star or in mesh; and it may be represented as in Figs. 168 or 169.

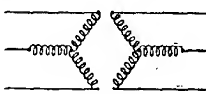

Fig. 168.—Star-Star Transformer.

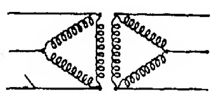

Fig. 169.—Mesh-Mesh Transformer.

It is also frequently the case that the primary is connected star, and the secondary mesh, as in Fig. 170; or *vice versa.*

The actual transformer in Fig. 166 is star-connected, and the terminal joined-up to the star-point enables the transformer to feed into a four-wire system, such as that diagrammed in Fig. 105.

It is useful to know of the above various methods of

connecting three-phase transformers, but their respective advantages are too involved for consideration here.

86. **COOLING OF TRANSFORMERS.**—The *capacity* or *rating* of a transformer (or in other words, its *output*) is partly limited by the necessity of preventing the overheating of its core and windings, and the damage that would result to the insulation of the latter. It follows then, that if means are provided for getting rid of the heat quickly, the output will be sensibly increased, especially in large sizes.

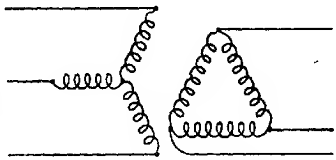

Fig. 170.—Star-Mesh Transformer.

A transformer is generally said to be *air-cooled* when it is installed with its core and windings fully exposed to the air inside the transformer house. In *oil-cooled transformers*, the case is filled with a special kind of oil which serves the double purpose of conducting the heat from the transformer to the sides of the case, and of improving and maintaining the insulation of the windings. The sides of the case are generally corrugated as seen in Fig. 167, in order to present a larger surface to the outer air.

Water-cooled transformers are oil-cooled transformers with the addition of a pipe-coil immersed in the oil and connected externally with a water-supply. The flow of initially cold water through the pipe-coil is very effective in carrying off the heat from the oil. Water-cooling naturally complicates the arrangements for a transforming plant, and is usually only adopted in very large stations, where the transformers may be 10 feet or more in height.

In *air-blast transformers*, means are provided for forcing cool air through the iron case in which the transformer is

"housed." This method is not so effective as oil or water cooling; and it is not suitable for high-voltage transformers, as the accumulation of dust which takes place tends to breakdown of the insulation.

87. **AUTO-TRANSFORMERS.**—An *auto-transformer* is a kind of transformer with one winding only, part of this being common to both the primary and the secondary circuits. The latter are consequently in metallic connection.

The ordinary auto-transformer is used for reducing the primary pressure for the secondary circuit; but in some cases it is arranged to increase the pressure slightly, and may then be termed a *reversed auto-transformer.* As the two circuits are in connection, the apparatus is not suitable for use on high-voltage circuits. It is generally employed for motor-starting work, and also for reducing the ordinary supply pressures of 250 volts or under to suit arc lamps, low-voltage metal-filament lamps, and other circuits. The "inverted" type is used when a number of arc lamps in series require a voltage slightly higher than the supply voltage.

The auto-transformer cannot do more than the ordinary transformer does; and the fact that the primary and secondary circuits of the latter are isolated from each other is a distinct advantage, especially when the transformer is of the step-down type. But when the isolation of the circuits is not of first importance, the auto-transformer has the advantage of lesser size and cost, as the separate secondary winding of the ordinary transformer is dispensed with. This is one reason why the auto-transformer is sometimes termed an *economy coil.*

The principle of the auto-transformer is as follows:—In Fig. 171, C represents the winding of a choking coil, suitable for connection to a supply pressure of (say) 200 volts, and V is a voltmeter with one side permanently connected

to one terminal T of the choker. Now it is obvious that if we connect the wire w from the other side of V to the terminal T' of the choker, V will indicate the full supply pressure; and that if we make connection with w at some intermediate point, such as at t or t', we shall get something less than the full pressure.

It follows from the above that if we connect a lamp-circuit L (or a distribution box) to one terminal T and the midway point t of the winding, as in Fig. 172, the secondary circuit S will be fed at half the primary voltage, in this case 100 volts. If the connection were made a quarter the distance (measured by the number of turns) from T, *i.e.*, at t', the voltage on S would be one-fourth the supply-voltage, viz. 50 volts. It is thus a very simple matter, in constructing an auto-transformer, to arrange for the connection of the terminals of the secondary circuit so as to get therein any required fraction of the primary voltage: and some auto-transformers have more than one tapping, so that the secondary voltage may be adjusted to suit different requirements.

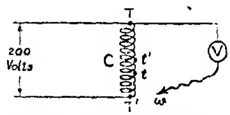

Fig. 171.—Principle of the Auto-Transformer.

In Fig. 172, the whole winding T to T' may be said to act as the primary, and the portion T to t as the secondary.

Now, assuming that the losses in the auto-transformer are negligible, the power in the secondary circuit is equal to that taken from the mains. Thus if I_P and E_P = primary current and voltage respectively, and I_S and E_S = secondary current and voltage respectively,

We have:—

$$I_P E_P = I_S E_S.$$

$$\therefore \quad I_S = \frac{I_P \times E_P}{E_S}.$$

But it has been shown that E_s depends entirely upon the proportion of the total turns TT' included between Tt,

Hence:—

$$I_s = I_P \times \frac{\text{total turns between } TT'}{\text{no. of turns between } Tt}. \qquad \textbf{(47)}$$

If the primary current was, say, 10 amperes at 200 volts; then, when the tapping t is midway between TT', the output on the secondary side would be 20 amperes at 100 volts. Or if t was a tenth of the distance (measured by number of turns) from T, the secondary current would be 100 amperes at 20 volts.

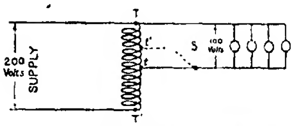

Fig. 172.—Principle of the Auto-Transformer.

These figures show very distinctly the usual function of an auto-transformer, namely, the conversion of a small current at a high voltage to a large current at a low voltage. This function is thus the same as that of a step-down ordinary transformer.

Let us next consider why the auto-transformer is more economical than an ordinary transformer. For this purpose we will take the case when t is midway between T and T', with the currents and voltages of the values just given. Referring to Fig. 173, at a certain instant the conductor A will be + and B−; and a current of 10 amperes will be flowing in the primary in the direction of the full line arrows. At the same instant, the pressure of the lead C in the secondary circuit will be 100 volts above that of D; consequently the direction of the 20 amperes in that circuit will be as indicated by the dotted arrows. It follows that in the portion PS, common to the primary and secondary circuits, there will be a current of 10 amperes flowing from t towards

T, as indicated by the double-headed arrow. This current will unite at T with the 10 amperes from the mains to give the secondary current of 20 amperes in conductors C and D. When this 20 amperes returns to t, it divides, one half flowing through PS and the other half through P. In other words, the main or primary current of 10 amperes flows *via*

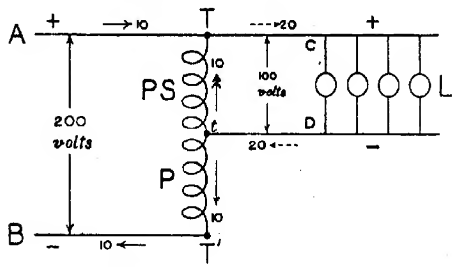

Fig. 173.—Distribution of Currents in an Auto-Transformer.

A, T, C, L, D, t, T', B; and the current of 10 amperes from the portion PS flows *via* t, T, C, L, D, t.

If an ordinary transformer were employed for attaining the same purpose, the arrangement and the distribution of the currents at the instant considered above would be as shown in Fig. 174. In comparing the two diagrams and the currents carried by the different portions of the circuits, it will be evident that the whole of the winding P, PS in Fig. 173 need only be the same size and length as the primary P alone of Fig. 174. Thus, the copper required for the secondary winding S in the latter figure is entirely saved by using an auto-transformer, this effecting an economy in copper of 50 per cent. in the particular case considered, since in ordinary transformers the weights of copper in the primary and secondary windings are practically equal.

The 50 per cent. figure for the economy in copper effected by using an auto-transformer relates to a transformation-ratio of *two* (see page 229). The actual saving, however, varies with the ratio of transformation; the smaller the ratio the greater the economy. Thus, if the pressure on the secondary is to be one-fourth that of the primary, the

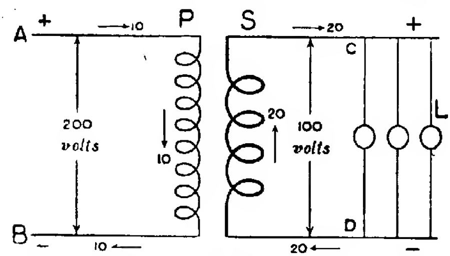

Fig. 174.—Distribution of Currents in an Ordinary Transformer.

transformation ratio will be 4, and the saving of copper only 25 per cent.

A disadvantage of the auto-transformer, alluded to at the beginning of this section, is that the secondary is connected with the primary circuit; and it is therefore possible to get shocks at higher voltages than the normal secondary pressure under certain conditions. But when one of the supply mains is earthed, this drawback is minimised by connecting one of the secondary ends of the "auto." to the earthed main. Thus, in Fig. 173, *AT* should be the earthed main, and if this is so, the p.d. in the secondary circuit cannot rise higher than the normal voltage therein.

Fig. 175 illustrates an auto-transformer in which the secondary voltage is higher than that of the supply, and one use for such an arrangement would be to enable say five 50-volt arc lamps to be run in series off 200-volt supply

mains. It will be seen that the arrangement is the reverse of the ordinary, the secondary circuit taking its supply off the whole of the winding, while the supply mains are connected to a portion only. In this case we may say that the portion *S* is virtually a secondary winding connected in series with the primary *P*, and that the extra pressure in the secondary circuit is due to the extra volts induced in *S*.

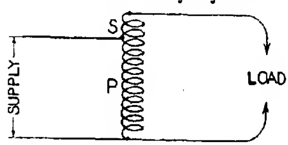

Fig. 175.—Reversed Auto-Transformer.

Fig. 176 shows a type of auto-transformer with a number of secondary wires connected to it at equidistant points. It is termed a *balancing auto-transformer*, and one of its uses is to feed low-voltage circuits through special distribution boards.* The arrangement may also be applied to the secondary winding of an ordinary transformer.

88. **CURRENT TRANSFORMER.**—It has been found difficult to construct instruments to carry alternating currents greater than about 100 amperes; and, for circuits working at or above 1000 volts, it is necessary to have the instrument circuits isolated from the high-pressure conductors. For these reasons, transformers are very often employed in conjunction with ammeters, wattmeters, and energy meters; the primary in each case being in series with the conductor carrying the current to be measured, while the secondary

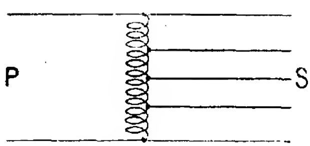

Fig. 176.—Balancing Auto-Transformer.

* *See* the Author's *Electric Wiring, Fittings, Switches, and Lamps*

is connected directly to the instrument circuit, as shown in Figs. 113 and 114.

In Fig. 177 we have one of various forms of *current transformer*, and Fig. 178 illustrates the same diagrammatically. The latter figure shows that the primary consists of a straight heavy conductor PP, round which is an iron core C built up of annular ring laminations. The secondary S consists of a large number of turns (1000 turns in Fig. 177) of fine wire wound round C, and its two ends TT are connected to the indicating instrument. The primary and secondary windings are insulated from each other by a porcelain tube.

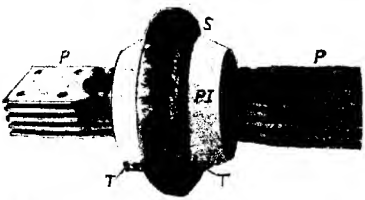

Fig. 177.—Current Transformer. (*British Thomson-Houston Co.*)

When an alternating current passes along PP, it sets up an alternating flux in the iron ring C, causing an alternating e.m.f. to be induced in the windings S in the way already explained in § 73. The value of the e.m.f. so induced is practically proportional to the current in the primary, consequently the current sent through the indicating instrument will vary directly as the current in the main circuit.

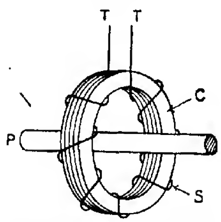

Fig. 178.—Diagram of Current Transformer.

The actual transformer shown in Fig. 177 is suitable for

measuring currents up to 5000 amperes; and since the transformation ratio is $\frac{1}{1000}$, the maximum current through the indicating instrument will not exceed 5 amperes.* The primary conductor *PP* is built up of massive copper plates, and the bus-bars (or other parts of the main circuit into which the transformer is to be inserted) are formed of similar plates which fit between and can be bolted through to those on the transformer. The secondary terminals are seen at *TT*, and *PI* is the porcelain insulator already mentioned.

In order to ensure accuracy of readings under all conditions, the iron cores of current transformers are constructed of laminations possessing the minimum hysteresis and eddy-current losses; and they are worked at very low flux-densities. Their leakage flux (§ 74) is negligible.

89. "**REGULATION**" **OF TRANSFORMERS.**—When the voltage applied to the primary circuit of a transformer is kept constant, the terminal voltage on the secondary side almost invariably rises as the load is decreased. The ratio of the rise in voltage between full load and no load to the full-load voltage is termed the *regulation* of the transformer. Thus—

$$\begin{array}{c}\text{Percentage}\\ \text{Regulation}\end{array} = \frac{\begin{array}{c}\text{Rise in secondary volts}\\ \text{between full load and no load}\end{array}}{\text{Secondary volts at full load}} \times 100 \qquad \textbf{(48)}$$

The less this value the better the transformer, for clearly, a good transformer should keep its secondary pressure as constant as possible under all conditions of load.

The regulation value of a transformer depends upon its design and upon the character of the secondary load: the less inductive the load, the better the regulation of a given transformer.

The most direct method of determining the regulation

* See Formula 45, p. 241.

would be to take a reading of the voltage at full load, and another at no load, and therefrom obtain the rise in the secondary voltage.

This procedure would involve loading the transformer:—

(*a*) On an artificial inductive load formed of choking coils and resistances.

or (*b*) on an artificial non-inductive load such as a water trough.

An *artificial load* is one that can be rigged-up in the testing department.

Neither of the above methods is adopted to any extent in practice, one reason being that a large amount of power would be necessary, especially with large transformers. Further, the (*a*) method would necessitate expensive adjustable choking coils and resistances, while the (*b*) method would only give the regulation at unity power factor.

When it is possible to adopt the (*a*) method, a set of curves can be got out for each transformer, showing the regulation at different loads and at different power factors.

89A. **DETERMINATION OF THE REGULATION OF SINGLE-PHASE TRANSFORMERS.**—The regulation of a transformer is generally worked-out with the help of values obtained from the no-load and short-circuit tests (Figs. 162 and 163); but before detailing the method, it is necessary to investigate one or two preliminary matters.

It was explained in § 83 that when the secondary of a transformer is short-circuited, the voltage required on the primary to cause the full-load current to circulate in the two windings is very small in comparison with the normal working voltage.* For the primary voltage in Fig. 163 has

* It should be evident from what was said in § 83 that if the normal voltage were applied when the secondary terminals were short-circuited, the primary and secondary currents produced would be many times their full-load values, and would probably be sufficient to burn out the windings.

only to send the full-load current through the impedance of the primary winding, and to induce in the secondary winding a low e.m.f. sufficient to send the full-load current against or through the impedance of that winding.

It is therefore possible (on our working diagram) to substitute for the primary and secondary windings a single winding having the same impedance as the sum of the impedances of the two transformer coils. Thus, in Fig. 179, the winding C (consisting of the resistance R and inductance L) can be assumed to be such that the same primary current I is produced in it by the same terminal volts V, and with the same phase-angle between them, as in the case of the short-circuit test of the actual transformer.

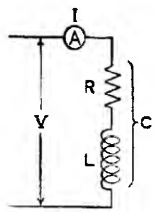

Fig. 179.—Circuit equivalent to a Short-Circuited Transformer.

Since this *equivalent circuit* consists only of resistance and inductance in series, its vector diagram is akin to Fig. 72, in so far that OI in that figure represents the current in the circuit, OC the applied voltage, and ϕ the lag of the current behind the voltage.

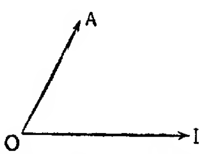

Fig. 180.—Relation between the Voltage and Current in the Equivalent Circuit in Fig. 179.

Thus in Fig. 180, OI represents the full-load current through C (Fig. 179), OA the voltage required to produce this current, and AOI the phase-angle or angle of lag between them. This angle is got from the readings taken on the short-circuit test (Fig. 163), since by Formula 29, p. 142,

$$\frac{\text{wattmeter reading}}{\text{primary current} \times \text{primary voltage}} = \text{power factor} = \cos AOI.$$

Fig. 179 leads us to Fig. 181, which is an imaginary circuit diagram on which our final calculations of the regulation will be based. This figure is Fig. 179 with two transformer windings added to it, but the transformer is to be considered as an "ideal transformer" *IT*, *i.e.*, one which has neither resistance nor inductance, these two quantities being assumed to be transferred to the coil *C*.

Further, we are first going to deal with a transformer

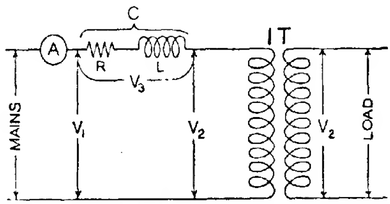

Fig. 181.—Impedance Coil and Ideal Transformer equivalent (for calculation purposes) to an Actual Transformer.

with unity transformation ratio, *i.e.*, one with an equal number of turns on each winding; and *IT* in Fig. 181 is assumed to be likewise. Now, in an actual transformer with a 1 : 1 ratio, it is the resistance and the inductance of the windings that produce a difference between the primary and secondary terminal voltages. Since *IT*, however, is to be regarded as having neither resistance nor inductance, its primary and secondary terminal voltages can be assumed to be equal at *all* loads.

89B. **DETERMINATION OF THE REGULATION OF SINGLE-PHASE TRANSFORMERS** (*continued*). — Having now examined the preliminaries, we can proceed to investigate the method in which the no-load and short-circuit tests are utilised, and the further calculations involved.

It should be remembered that we are at present dealing

with a transformer having unity transformation ratio, so that the effect of stepping the voltage up or down does not yet have to be taken into consideration.

We have first to calculate the secondary voltage that the real transformer will give at a full load of a certain power factor, when the correct working voltage is applied to its primary. The no-load secondary voltage of such a transformer is practically the same as that of the primary, so that the calculation of the regulation by Formula 48 (p. 261) is then a matter of simple arithmetic.

Let us now get to our calculations.

Let us assume that the load is partly inductive, and that its power factor is known.

When the full-load currents are flowing in the circuits (Fig. 181) with a constant supply voltage V_1, the voltage V_3 absorbed by the coil C will be the same voltage as OA in Fig. 180, since R and L in Fig. 181 are assumed to have the same values as in Fig. 179. The value of OA is the same as that of V in the short-circuit test (Fig. 163), and the angle AOI is obtained from the results of that test in the manner explained on p. 263.

As already stated, the secondary voltage of the ideal transformer IT (Fig. 181) is exactly equal at all loads to that of the primary, so that, *if we can determine the voltage* V_2 *across the primary of such a transformer, that will also be the voltage across the load.* This explains why the imaginary circuit in Fig. 181 is used.

The value of V_2 can be found by additional working upon Fig. 180 as follows :—

Let OI (Fig. 182), be the current vector; then OA (corresponding with OA in Fig. 180, where it represented the voltage at the terminals of C) now represents the voltage drop in C, in Fig. 181; the angle AOI (as in Fig. 180) being made equal to the phase difference between the current and the voltage across C, and being determined from the power

factor as explained on p. 263. Next draw OB at an angle BOI, such that cos BOI gives the power factor of the load on the secondary circuit.

It should be noted that the lag AOI in Figs. 180 and 182 is deduced from the p.f. calculated from the short-circuit test, and is due entirely to the inductance of the transformer windings. On the other hand, BOI (Fig. 182) is the angle of lag produced by the inductance of the secondary load; that is to say, BOI will be the phase difference between the current and the voltage in the secondary of our ideal transformer.

Now, since IT has neither resistance nor inductance, the phase difference between the current and the voltage may be assumed to be exactly the same in its primary and secondary windings, so that BOI will also be the angle of lag in the primary of IT.

With centre O (Fig. 182), and radius equal to the supply voltage V_1 in Fig. 181 (to the same scale as OA represents V_3), draw arc D. The end of the vector representing the applied or supply voltage must lie on this arc. AE is next drawn parallel to OB and EF parallel to AO. Since the resultant of OF and OA is given by the diagonal OE (§ 25) representing the supply voltage, then OF represents the voltage V_2 across the primary of the ideal transformer, and therefore that across the load.

In other words, as V_3 and V_2 in Fig. 181 are due to V_1, and as in Fig. 182 we started with one component OA ($=V_3$) and the resultant OE ($=V_1$), the other component OF must be equal to V_2.

The length of OF (measured-off to the same scale as that to which OA and the radius of the arc D were drawn) will thus give the full-load secondary voltage of the transformer to which Fig. 181 relates.

On no load, the secondary voltage would be practically the same as the input voltage. Thus in Fig. 182, OE

represents the secondary voltage on no load, and OF the secondary voltage on full load.

Hence, by Formula 48, p. 261,

$$\text{Percentage Regulation of the transformer} = \frac{OE - OF}{OF} \times 100.$$

So far, the transformation ratio has been assumed to be unity. The further calculation required, however, if the

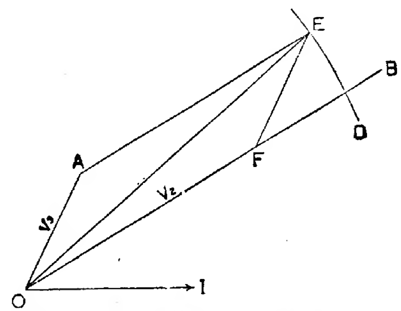

Fig. 182.—Vector Diagram for Circuit shown in Fig. 181.

ratio is other than unity—as is practically always the case —is very simple. Thus if the transformer has a ratio of 1 to 5, the actual no-load and full-load secondary voltages will be five times the above values of V_1 or OE and V_2 or OF respectively. Or if the ratio is 8 to 1, the no-load and full-load terminal voltages on the secondary side will be $\frac{1}{8}$ of the above values. The transformation ratio can be determined experimentally from the no-load test (Fig. 162).

It should be evident from the foregoing that a transformer may be regarded as being partly an inductive coil, and partly a "voltage-changer," the latter function being performed without any volts being absorbed.

89C. **DETERMINATION OF THE REGULATION OF SINGLE-PHASE AND POLYPHASE TRANSFORMERS.**— The following is an actual example of the calculation of the regulation of a single-phase transformer:—

EXAMPLE.—*The following readings were obtained on a* 10-*kVA transformer:—On the no-load test* (*Fig.* 162) *the primary volts were* 2000, *and the secondary volts* 500. *On the short-circuit test* (*Fig.* 163), *the primary volts were* 135 *when the secondary current was* 20 *amperes, the primary current being* 5·1 *amperes, and the primary watts* 283. *Determine the regulation of the transformer, and the actual secondary voltage at a full load having a power factor of* ·8.

From the no-load test readings and Formula 45 (p. 241), we have:—

$$\text{Transformation ratio} = \frac{2000}{500} = 4.$$

From the short-circuit test readings and by Formula 29, p. 142, we have:—

$$\text{Power factor on short-circuit} = \frac{283}{135 \times 5{\cdot}1} = {\cdot}411.$$

∴ angle of lag of current behind voltage, of which ·411 is the cosine value, may be seen from the Table on p. 271 to be between 65° and 66°. The actual value is 65° 44′.

In Fig. 183, OI is the primary current vector, and OA represents *to scale* the primary voltage on the short-circuit test (= 135), leading 65° 44′ in front of the current.

Since we are told that the power factor of the secondary load will be ·8, and as ·8 is the cosine of an angle of 36° 52′, the latter will be the phase difference between the secondary current and the secondary terminal voltage. Hence we next draw OB such that the angle BOI is 36° 52′. Then, with centre O and radius (to same scale as before) equal to 2000 (the primary impressed voltage), an arc D is drawn. On this arc must lie the end of the vector representing the

applied voltage. AE with E on the arc D is next drawn parallel to OB, and EF parallel to AO.

It should be now evident that the parallelogram $OFEA$ is very similar—except in dimensions—to that in Fig. 182; so that if the transformation ratio were unity, OF would represent the secondary voltage on the full load, and OE the secondary voltage on no load. Actually, however, the

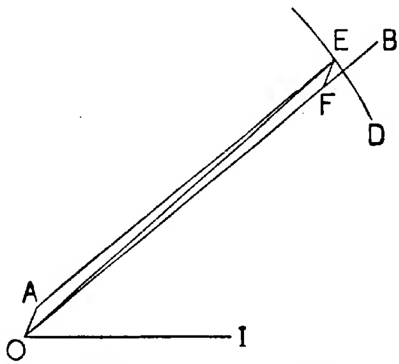

Fig. 183.—Diagram for determining the Regulation of a Transformer.

ratio of transformation has already been found to be 4, so that we must first ascertain by scale measurement what voltage OF represents ($=1880$), and then divide it by the transformation ratio (4) to get its true value, thus $\frac{1880}{4}=470$ volts. We already know that the secondary voltage on no load is 500.

Hence:—

$$\text{Regulation}=\frac{500-470}{470}\times 100$$
$$=6{\cdot}38 \text{ per cent.}$$

If the value of the regulation was required for a secondary load of a different power factor, this could easily be determined in the same manner after altering the angle BOI to correspond.

The *regulation of a polyphase transformer* can be ascertained by working out Fig. 182 for one phase only. There is an additional point, however, which must be taken care of, namely, the calculation of the power factor from the short-circuit test. The power factor for a polyphase transformer is given by :—

$$\frac{\text{Watts per phase}}{\text{Primary phase current} \times \text{primary phase voltage}} \quad \textbf{(48A)}$$

This formula will be self-evident from the definition of power factor in § 44.

89D. Table of Natural Sines, Tangents, etc.

N.B.—*Take the left-hand column of degrees for sines and tangents, and the right-hand column for co-sines and co-tangents.*

Deg.	Sine.	Tangent.		Deg.	Sine.	Tangent.		Deg.	Sine.	Tangent.	
0	0·000	0·000	90								
1	·017	·017	89	31	·515	·601	59	81	·875	1·80	29
2	·035	·035	88	32	·530	·625	58	62	·883	1·88	28
3	·052	·052	87	33	·544	·649	57	63	·891	1·96	27
4	·070	·070	86	34	·559	·674	56	64	·899	2·05	26
5	·087	·087	85	35	·573	·700	55	65	·906	2·14	25
6	·104	·105	84	36	·588	·726	54	66	·913	2·25	24
7	·122	·123	83	37	·602	·753	53	67	·920	2·35	23
8	·139	·140	82	38	·616	·781	52	68	·927	2·47	22
9	·156	·158	81	39	·629	·810	51	69	·933	2·60	21
10	·174	·176	80	40	·643	·839	50	70	·940	2·75	20
11	·191	·194	79	41	·656	·869	49	71	·945	2·90	19
12	·208	·212	78	42	·669	·900	48	72	·951	3·08	18
13	·225	·231	77	43	·682	·932	47	73	·956	3·27	17
14	·242	·249	76	44	·695	·966	46	74	·961	3·49	16
15	·259	·268	75	45	·707	1·00	45	75	·966	3·73	15
16	·276	·287	74	46	·719	1·03	44	76	·970	4·01	14
17	·292	·306	73	47	·731	1·07	43	77	·974	4·33	13
18	·309	·325	72	48	·743	1·11	42	78	·978	4·70	12
19	·325	·344	71	49	·755	1·15	41	79	·982	5·14	11
20	·342	·364	70	50	·766	1·19	40	80	·985	5·67	10
21	·358	·384	69	51	·777	1·23	39	81	·988	6·31	9
22	·375	·404	68	52	·788	1·28	38	82	·990	7·11	8
23	·391	·424	67	53	·799	1·33	37	83	·992	8·14	7
24	·407	·445	66	54	·809	1·38	36	84	.994	9·51	6
25	·423	·466	65	55	·819	1·43	35	85	·996	11·43	5
26	·438	·488	64	56	·829	1·48	34	86	·997	14·30	4
27	·454	·509	63	57	·839	1·54	33	87	·998	19·08	3
28	·469	·532	62	58	·848	1·60	32	88	·999	28·64	2
29	·485	·554	61	59	·857	1·66	31	89	·999	57·29	1
30	·500	·577	60	60	·866	1·73	30	90	1·000	Infinity	0
	Co-sine.	Co-tangent.	Deg.		Co-sine.	Co-tangent.	Deg.		Co-sine.	Co-tangent.	Deg.

CHAPTER V.—QUESTIONS.

In answering these Questions, give Sketches wherever possible.

NOTE.—Questions marked * range slightly beyond the subject-matter of this Book. Those marked † can only be partly answered therefrom.

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†1. Why is it advisable to use iron for the magnetic circuit of a choking coil, and why should there be an air-gap? Give a proof of the law that the maximum power-factor is obtained if iron and copper losses are equal. (*Ord.*, *A.C.*, 1909.)

2. Explain the general principles of action of a transformer, and illustrate by a sketch, showing the different parts approximately to scale, stating the material of which each part is made. (*Grade II.*, *A.C.*, 1913.)

3. Deduce an expression for the virtual value of the electromotive force in volts in a coil of n turns, due to a magnetic flux with crest value of $N \times 10^6$ lines interlinking the coil and varying as a sine function of the time at a frequency of f cycles per second. (*Ord.*, *A.C.*, 1911.)

Ans. ·0444 nfN.

4. What does the term "magnetic leakage" mean, as applied to an alternating-current transformer? Give sketches to show how it can occur (*a*) in a core-transformer with "sandwiched" windings, (*b*) in a core-transformer with concentric (cylindrical) windings. What has the leakage to do with the voltage-drop? (*Ord.*, *A.C.*, 1910.)

*5. What is meant by the *magnetising current* of a transformer, and what is its phase-relation to the supply voltage? Find the value of the magnetizing current of a transformer with closed iron circuit, and of which the data are as follows:—

Primary voltage	2200
Primary turns	320
Area of core	130 square ins.

Mean magnetic length of core	. .	40 ins.
Permeability of iron	. . . .	2000
Frequency of supply	. . . .	50 periods per second.

(*Ord., A.C.*, 1908.)

Ans. ·33 ampere.

AUXILIARY NOTE.—The maximum value F of the flux is calculated from Formula 42, p. 227. Then if l be the length of the magnetic path in cms., a the cross-section of the core in square cms., and μ the permeability * of the iron: the maximum value of the ampere-turns in the primary necessary to produce the flux F is $\frac{\cdot 8 l F}{\mu a}$.

Dividing the ampere-turns (thus determined) by the primary turns gives the maximum value of the magnetising current; and the r.m.s. value of that current can then be obtained (page 78).

6. In a single-phase transformer intended for use on constant pressure mains, explain (*a*) why the flux through the iron core is very nearly independent of the load; (*b*) why the applied potential difference on the primary and the electromotive force in the secondary are nearly 180 degrees out of phase; (*c*) why magnetic leakage produces a greater effect on the regulation with inductive than with non-inductive loads. (*Ord., A.C.*, 1911.)

7. An installation has been wired for a pressure of 200 volts, a drop of 2 per cent. being allowed from the point of supply to the farthest lamp. If a transformer is subsequently used to reduce the pressure to 50 volts, what is the percentage pressure drop, assuming the same number of watts to be used in each case? (*Wiremen's Final*, 1914.)

Ans. 32 per cent.

*8. A transformer of 10-kW capacity runs 6 hours a day at full load and 5 hours a day at one-half load, while for the remaining time it runs light. What is the difference in yearly running cost between one transformer having at full load 1 per cent. iron loss and 2·5 per cent. copper loss and one having $1\frac{3}{4}$ per cent. iron loss and $1\frac{3}{4}$ per cent. copper loss, when the cost per unit (kw.-hour) is $1\frac{1}{4}$d.? (*A.M.I.E.E. Exam.*, 1914.)

* *See* the Author's *Electric Lighting and Power Distribution*, Vol. I.

Ans. £2, 8s. The copper loss has to be assumed to vary as the square of the load.

9. In three-phase work one occasionally finds three single-phase transformers used instead of a single three-phase transformer. Discuss the relative advantages of these alternatives for the different cases that may arise. (*Ord.*, *A.C.*, 1910.)

10. What is an auto-transformer? Are there any advantages in using an auto-transformer for house-lighting? In what circumstances would auto-transformers be inadmissible? (*Ord.*, *A.C.*, 1908.)

11. Make a diagram of the connections of an auto-transformer connected up on the same principle as an economy coil. Is this arrangement more economical than a plain transformer with two separate windings? (*Wiremen's Final*, 1910.)

*12. What was the object of using an auto-transformer for a metal filament lamp installation? Make a diagram showing how it should be connected up on an installation supplied from one side of a three-wire system. (*Wiremen's Grade I.*, 1912.)

13. Describe what is meant by an auto-transformer (or compensator) and indicate the currents in the windings of a 100-kw. 3-phase star-connected auto-transformer having a ratio of 1000 to 400 volts. (*A.M.I.E.E. Exam.*, 1914.)

Ans. 57·7 and 86·5 amperes approximately, assuming the efficiency to be 100 per cent.

NOTE.—The use of the term "compensator" as an alternative for "auto-transformer" is not to be recommended, for it is not clear what the apparatus compensates for.

14. Describe a simple test, requiring little power, for obtaining such data that you can pre-determine the voltage-drop of a transformer at any load and any power factor. (*Ord.*, *A.C.*, 1909.)

15. What tests requiring little power will you make to obtain the information necessary for determining the voltage drop of a transformer under any conditions of load? Give the theory of this determination. (*Ord.*, *A.C.*, 1911.)

16. The resistance of the windings of a transformer being given, explain a method by which, from the short circuit test

and the open circuit test of the transformer, you can predict the voltage drop at any load and any power factor. (*Grade II., A.C.*, 1912.)

17. Explain how the efficiency of a transformer may be estimated by a full voltage no-load test, and a test on short-circuit with full-load current in the secondary. How may the pressure regulation of the transformer on non-inductive load be found from these tests? (*Grade II., A.C.*, 1913.)

18. Explain how the voltage drop of a transformer under any conditions of load can be determined from two tests which require only little power. What is the object of dividing the primary and secondary windings of a transformer into a number of coils, and how far should this division be carried? (*Grade II., A.C.*, 1914.)

19. What do you understand by the term "regulation" of a transformer? Describe a test involving small expenditure of power by which the regulation of a large transformer can be predicted for any load and any power factor. (*Final, 1st Paper*, 1913.)

*20. What points in the design of a transformer require special attention in order to get good "regulation" and high efficiency? (*Final, 2nd Paper*, 1912.)

*21. A certain oil-cooled single-phase transformer rated at 75 kVA 10,500/3500 volts and 60 cycles requires the application of 700 volts on the primary winding to cause a current equal to the full load current to flow through the secondary winding, when the latter is short-circuited, the power absorbed by the primary under those conditions being 1·8 kilowatts. Indicate by means of a vector diagram how the full-load pressure regulation of this transformer at power-factors of 70 per cent. and 100 per cent. respectively can be approximately ascertained, and give the necessary data for such a diagram in the present case. Discuss the accuracy of the diagram you employ. (*Honours, 2nd Paper*, 1908.)

Ans. The diagram will be similar to Fig. 182, but drawn to the following size:—$AOI = 69°$; $BOI = (a)\ 45·6°,\ (b)\ 0°$; $OA = 700$; $OE = 10,500$; transformation ratio $= 3$.

CHAPTER VI.

MOTORS.

90. **ALTERNATING-CURRENT MOTORS AND THEIR CLASSIFICATION.** — *Alternating - Current Motors* are machines which convert alternating-current energy into mechanical energy: and the electrical energy may be single-phase or polyphase. In other words, there are *single-phase, two-phase,* and *three-phase motors.*

In most cases, the motor stator is similar in construction to that of the corresponding type of alternator. For example, the stator of a three-phase motor, in every respect except size, is similar to that of a three-phase alternator. Thus the particulars concerning windings, etc., in §§ 60 and 62 apply equally well to the stators of most a.c. motors.

As regards their rotors, on the other hand, a.c. motors usually differ from alternators; and they can be divided into three groups, according to the type of rotor employed, viz. :—

$$\left.\begin{array}{l}\text{(a) Synchronous motors.}\\ \text{(b) Asynchronous or induction motors.}\\ \text{(c) Commutator motors.}\end{array}\right\}\begin{array}{c}\text{Single-phase}\\ \text{or}\\ \text{polyphase.}\end{array}$$

Each of the above types will be considered in the following sections.

91. **SYNCHRONOUS MOTORS.**—Think of two similar alternators, *A* and *B*, each driven by a steam engine, and connected in parallel with one another, as described in § 67. Now if the steam were shut off from *A*'s engine, the latter would take current from alternator *B*, and would run

as a motor and drive its own engine at *exactly* the same speed as the latter ran when it was taking steam.

Alternator *A* would then be acting as a *synchronous motor*, that is to say, it would be running *in synchronism* or in step with the generator to which it was connected.

The above two machines would run at the same speed because they are supposed to be similar, and therefore to have the same number of poles. But it is possible for two machines to run in synchronism and yet be at different speeds. Thus, suppose we take some other alternator (call it *C*) with a different number of poles, and connect it as a motor to alternator *B*. The actual speed of *C* will depend upon the number of its poles, and upon the frequency supplied by *B*, which would be proportional to *B*'s speed. Thus, if *B*'s frequency is 50 cycles per second, and if *C* has 12 poles, the speed of *C* would be given by Formula I (p. 34). Thus:—

$$f = \frac{Rp}{60}$$

$$\therefore \quad R = \frac{60f}{p} = \frac{60 \times 50}{6} = 500 \text{ r.p.m.}$$

A synchronous motor, such as *A* or *C* in the above examples, maintains a constant speed under all normal conditions of load. This characteristic is of importance in some applications.

It will now be evident that synchronous motors are simply alternators used as motors; so that their constructions are identical.

The number of poles on an alternator depends upon the speed at which it is to be driven, and upon the frequency required. The number of poles on a synchronous motor depends upon the supply frequency, and upon the speed required. With a given frequency, the lower the speed the greater the number of poles. All this is evident from the above formula.

Like alternators, synchronous motors require continuous current for the excitation of their rotor poles. When a machine is used as an alternator, the magnetic lines of these poles sweep the stator windings and generate the e.m.fs. therein. When a machine is used as a motor, the alternating current passed through the windings sets up a rotating magnetic field (as will be explained presently), and this field acts on the poles of the rotor and maintains the rotation of the latter. As will be understood in due course, a synchronous motor is not self-starting, unless it is wound for two- or three-phase current, and is fitted with a *damper winding*. Even then, it can only start under a very light load (pp. 192, 281, and 282).

Fig. 184 shows the rotor and exciter armature of a synchronous motor. The rotor consists of a number of poles bolted onto the rim of a massive cast-iron or cast-steel wheel, and the pole-coils are connected in series to the slip-rings, as seen in the figure. It will be noticed that there is a damper winding fitted onto the pole-shoes. The object of this is to prevent unsteady running or *hunting* of the machine; and there is then much less liability of its falling out of step with the supply current. The arrangement and action of the damper winding is explained in the next section.

The exciter armature seen in Fig. 184 forms part of a dynamo for supplying current to the winding on the rotor poles. Sometimes, as with alternators, the exciting current is derived from a separate machine.

Fig. 185 shows a synchronous motor (on the right-hand side), driving a 600-kW continuous-current generator. The current for the excitation of the rotor of the motor is in this case supplied from a separate source.

A *synchronous motor-generator* (as in Fig. 185) is sometimes used when continuous current is required, and the main supply is alternating. A *rotary converter* performs

the same function, but it differs in construction, the synchronous motor and the continuous-current generator being combined into a single machine.

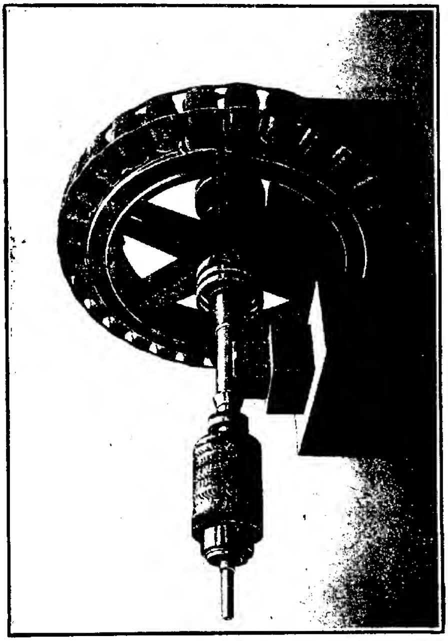

Fig. 184.—Rotor of a Synchronous Motor. (*Electric Construction Co.*)

In addition to giving absolutely constant speeds at all loads up to the maximum, synchronous motors possess the additional advantage that if their fields be strongly excited, they act as condensers, in so far that they take a leading current. Thus they tend to compensate for lagging currents

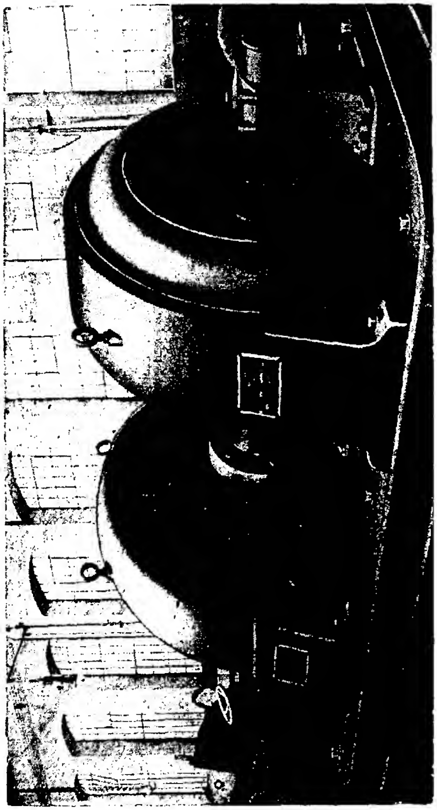

Fig. 185.—Synchronous Motor-Generator Set. (Electric Construction Co.)

due to inductive loads on other parts of the supply system.

On the other hand, motors of this class as a rule have the serious drawback of not being self-starting, and additional apparatus must be provided for that purpose. The usual method is to fit a small continuous-current motor, or an induction motor (§ 92), on an extension of the shaft; this auxiliary motor being used only for running the synchronous motor up to its proper speed. The latter is then synchronised onto the mains in the same manner as an alternator (§ 67). Another method of starting, applicable only to three- or two-phase machines, utilises the damper winding already referred to. The latter, during starting, acts like the squirrel-cage rotor of an induction motor (§ 97), and the currents induced therein may be sufficient to start the machine on light load.

When the reader has learnt something about rotating fields, it may appear to him that a polyphase synchronous motor without a damper winding ought to be self-starting. The reason it is not is that the rotating field of the stator starts off at full speed directly the current is switched on, this speed being too great to enable the field to overcome the inertia of the rotor. If means could be provided for starting with a stator current of very low frequency, and for gradually increasing the frequency, the poles of the rotating field (§§ 93 and 94) would move round very slowly to start with, and would—by their magnetic attraction of the rotor poles—be able to set the latter in motion.

The fact that continuous current is necessary for excitation constitutes another disadvantage of the synchronous motor; and yet another is that the motor will stop altogether if overloaded sufficiently to prevent the rotor keeping pace with the stator field.

On account of the above disadvantages, synchronous motors are only employed to a limited extent.

91A. "**HUNTING" AND DAMPER WINDINGS.**—The action of the damper winding of a synchronous motor may be understood by considering the relative motion between a two-pole field-magnet *NS* (Fig. 185A) and the rotating magnetic field of the stator (§§ 93 and 94). The direction of rotation of both will be the same, and under normal conditions they will rotate at exactly the same speed. Let *d*, *d* (Fig. 185A) be the damper winding; this consisting of copper bars passing through the pole faces and connected together at each end by copper strips, as seen in Fig. 184.

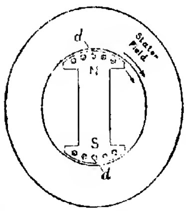

Fig. 185A.—Action of Damper Windings.

Now, if a load be thrown suddenly on the synchronous motor, the rotor will start oscillating or "swinging" relative to the stator field: *i.e.*, it will first slow down a little, then it will accelerate and rotate a little faster than the rotating flux, next it will slow down again, and so on. This action, known as *hunting*, will be indicated by the oscillation of the needle of the ammeter in the exciting circuit, for variable counter e.m.fs. will be induced in that circuit: and if the "hunting" is not checked, it often increases to a sufficient extent to cause the rotor to fall out of step with the rotating field, and then stop.

When a damper winding is fitted, the relative motion due to "hunting," *i.e.*, the different speeds of the rotor and of the stator flux, induces e.m.fs. in the bars *d d*; and the currents set up therein exert a magnetic drag which "damps out" the "swinging" action.

The relative motion between the rotor poles and the stator field due to "hunting" may be better understood by considering two trains, *A* and *B* (Fig. 185B), travelling on adjacent lines in the same direction.

Now, if A represents the rotating field of the stator, and B the rotor, the "swinging" or "oscillation" or "hunting" of B would be represented by its alternately going a little faster and a little slower than A.

Suppose the two trains were of equal length, and that a strong helical spring, SS, of the same length, was fixed to the front of A and the rear of B, and was suitably supported at intermediate points. The action of this spring would be

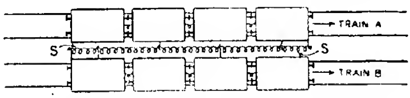

Fig. 185B.—Train Analogy illustrating the Hunting and Damping of Rotors.

analogous to that of the damping winding, for it would obviously tend to prevent any difference in speed between the trains A and B. The compression of SS would tend to prevent B going faster, and the elongation of SS would tend to prevent B going slower than A.

It is for a similar reason, *i.e.*, the maintenance of an even rotation, that damper windings are fitted on alternator rotors (see Figs. 126 and 126A); the cause of "hunting" being then chiefly due to the uneven torque of most reciprocating engines, especially oil and gas engines.

92. **POLYPHASE INDUCTION MOTORS AND THEIR ROTATING FIELDS.**—The *polyphase* (generally three-phase) *induction motor* is by far the most common type of a.c. motor, since it is exceedingly simple in construction, is very reliable, and can be arranged to start against a heavy load. The single-phase induction motor, which will be dealt with in §§ 107 and 108, is not so good in these and other respects.

The principle of action of polyphase induction motors may be gathered from the following.

When a dynamo or alternator is supplying current, a considerable torque is required to rotate the armature or rotor against the magnetic drag of the field. It follows, then, that if the part that is usually fixed were mounted so that it were free to turn about the same axis as the rotor, it would follow the latter in its rotation.

Consider, for example, a simple 4-pole continuous-current machine, and suppose that the field-frame is capable of rotation concentrically with the armature; and that the latter is connected-up through an external circuit, the field-windings being energised from some independent source through brushes and slip-rings. If, now, the armature were mechanically driven, e.m.fs. would be induced therein, causing current to flow round it and the external circuit in the usual way. And the magnetic drag that would consequently be produced would gradually set the field-frame in motion, until at length its speed of rotation would be considerable.

It would then be found that very little pressure or power was being generated in the armature; for obviously there would be very little cutting of the lines of force of the field, since the latter would now be rotating as well as the armature, though not at the same rate. If the armature commutator brushes were short-circuited, or if the segments of the commutator itself were short-circuited (say, by tightly winding a layer of bare copper wire round them) the field-frame would rotate at a higher speed. The reason of this is that the current in the armature due to a given induced e.m.f. would then be greater, and the magnetic drag would consequently also be greater.

After the above, it is easy to understand that if the short-circuited armature were belted or otherwise geared to a machine, and if the field-frame were rotated by means of an engine, the armature would revolve in consequence of its being in a rotating field, and would drive the machine.

The foregoing furnishes the key to the action of a polyphase induction motor, wherein there is a *rotating field* in spite of the fact that the field-frame or stator is fixed. The rotation of the field is due to the polyphase current; and the rotor of such a motor is roughly comparable with a continuous-current armature from which the commutator has been removed, and which has had the free ends of its windings all connected together. The rotor might be simply a conducting cylinder, but by having insulated conducting strips or "windings," the induced currents are confined to well-defined paths, and the torque or turning effort is thereby rendered greater. The simplest form of such rotor is the *squirrel-cage rotor*, which is described in §§ 97 and 105.

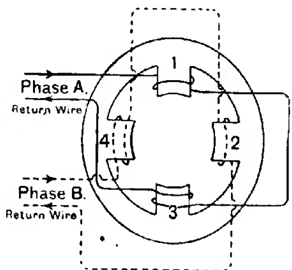

Fig. 186.—Winding of a Two-Pole Two-Phase Motor Field.

93. **PRODUCTION OF A ROTATING FIELD BY TWO-PHASE CURRENT.**—The magnetic field set up by a two-phase current circulating in the coils of a suitably-wound stator will be understood on reference to Fig. 186. This represents a laminated 4-pole magnet; the windings of poles 1 and 3 being connected in phase *A* circuit, and those of poles 2 and 4 in phase *B* circuit. As explained in § 47, and illustrated in Fig. 94, the current in phase *B* lags 90° or a quarter-period behind the current in phase *A*. This is also shown in Fig. 187, which differs from Fig. 94 in that the two current waves are drawn on the same, instead of on separate base lines. The relation between the two

currents is such that when one is at a maximum, the other is passing through zero.

As the 4-pole magnet (Fig. 186) has two poles—a north and a south—formed in each phase, it is termed a 2-pole two-phase magnet or stator. Now the magnetising forces due to the currents shown in Fig. 187 will tend to set up two fields in the rotor space. And these two fields will unite to

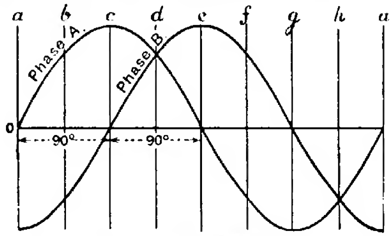

Fig. 187.—Two-Phase Current.

form a resultant field, though at certain momentary periods in each cycle, the one or the other field will exist alone. As will presently be proved, the rising, falling, and reversing of the currents in the two phases will cause their resultant field to rotate uniformly around the face of the stator core.

It should be understood that the poles of an induction motor do not really project from the rest of the stator core as seen in Fig. 186. They are shown so in the figure in order to simplify the latter. (See § 96.)

In Fig. 186, alternate poles are joined-up to phases *A* and *B* respectively, and the two coils in each phase are wound alternately right-handedly and left-handedly.* In other

* The terms "right-hand winding" and "left-hand winding" have to be used for want of anything more explicit. Strictly speaking, in such a case any winding is alternately right-handed and left-handed, as the current in it is alternating. Similarly, in Fig 186 one wire in each phase is marked "return wire," but since the current is alternating, each of the two wires in each phase becomes the return wire alternately.

words, they are so connected that when one is N. the other will be S. Thus poles 1 and 3 are connected in phase A circuit, and poles 2 and 4 in phase B circuit: poles 3 and 4 being wound or connected right-handedly, and poles 1 and 2 left-handedly.

For each pole a *polarity curve* may be drawn, that is, a curve which represents the change in its polarity throughout one cycle of the current circulating in the winding. Such curves are shown in Fig. 188. The full line curve $ABCDE$ represents the polarity wave for pole 1 in phase A circuit: and as—at any instant—the polarity of pole 3 will be equal and opposite to this, the wave for pole 3 is represented by the dotted curve $AB'CD'E$. The parts of the curves above the horizontal base line AE denote N. polarity, and the parts below this line S. polarity. Similarly, the polarity waves for poles 2 and 4 in phase B circuit are indicated by the curves $FGHIJ$ and $F'GH'IJ'$; these being 90° (or a quarter period) behind the corresponding phase A curves. The curves are divided-up by ordinates a, b, c, d, etc., one-eight of a cycle or 45° apart.

Now the instantaneous polarity of each pole at the first ordinate a may be stated as follows:—

Polarity of pole $1=0$.
" " $2=l\ m$, and is S.
" " $3=0$.
" " $4=l\ n$, and is N.

Here, since the magnetising force of phase A is at zero value, the field, being then due to phase B alone, clearly lies between poles 2 and 4, as shown in Fig. 189 at (I).

At ordinate b in Fig. 188:—

Polarity of pole 1 is N.
" " 2 is S.
" " 3 is S.
" " 4 is N.

and they are all of equal strength. Hence there are now two fields set up, as shown by the dotted arrows in Fig. 189 (II). These combine to form a resultant field, the direction of which is indicated by the firm-line arrow passing midway between poles 4 and 1, and between poles 2 and 3.

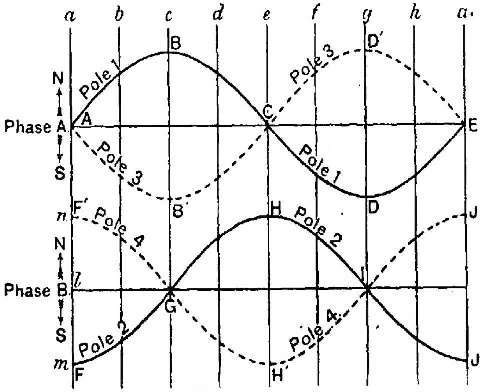

Fig. 188.—Polarity Curves of a Two-Pole Two-Phase Motor Field.

Thus in one-eighth of a cycle, the resultant field has moved through 45° or one-eighth of a revolution.

At ordinate *c* (Fig. 188):—

Polarity of pole 1 is N.
" " 2 is 0.
" " 3 is S.
" 4 is 0.

Hence the field, being due to phase *A* alone, now lies between poles 1 and 3, as shown in Fig. 189 (III); so that in this second one-eighth of a cycle, the resultant field has made a further one-eighth of a revolution.

At ordinate *d* (Fig. 188) :—

Polarity of pole 1 is N.
" " 2 is N.
" " 3 is S.
" " 4 is S.

Thus the resultant field is now due to the two north poles 1 and 2 and the two south poles 3 and 4; its direction

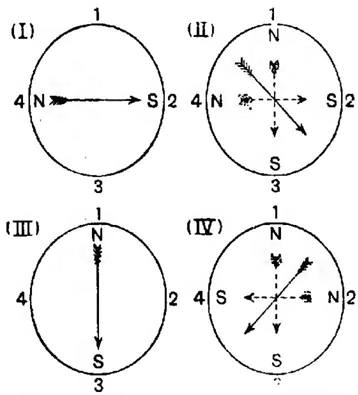

Fig. 189.—Two-Phase Clock-Face Polarity Diagrams.

being indicated by the firm-line arrow in Fig. 189 (IV). In Fig. 186, the directions of the currents in the two phases and the resulting polarities correspond with this instant.

In the same manner, if the other ordinates in Fig. 188 be considered, it will be found that during each further one-eighth of a cycle the resultant field makes one-eighth part of a revolution. Hence it may be concluded that the rotation of the field is uniform, and in a clockwise direction.

Had poles 1 and 4 been wound right-handedly (looking at their faces) instead of poles 3 and 4, the rotation would

still have been uniform, but in the opposite or counter-clockwise direction. This the reader may prove for himself by drawing curves and clockface diagrams after the manner of those in Figs. 188 and 189.

It follows then that the direction of rotation of such a

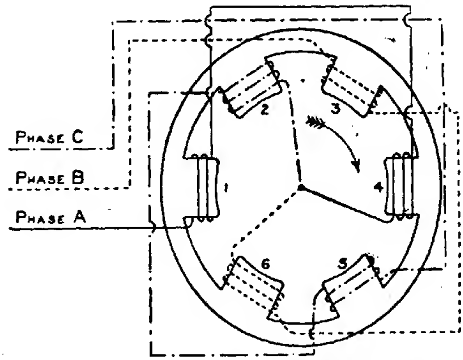

FIG. 190.—Winding of a Two-Pole Three-Phase Motor Field.

motor may be reversed by reversing the connections to one of the phases. This fact is further alluded to in § 95.

94. **PRODUCTION OF A ROTATING FIELD BY THREE-PHASE CURRENT.**—In a three-phase stator, the coils in the three windings carry currents differing in phase by 120° (Fig. 98). Some care is therefore needed to ensure that they are connected in the right sense, so that the direction of the three magnetising forces in the rotor space may be such as to produce the desired effect.

Fig. 190 represents the stator of a 2-pole three-phase motor, with imaginary polar projections. Here poles 1 and 4 are in phase *A* circuit, poles 3 and 6 in phase *B* circuit, and poles 5 and 2 in phase *C* circuit. And it will be noticed

that poles 1, 3, and 5 are wound left-handedly, and 4, 6, and 2 right-handedly.

Fig. 191 gives the polarity curves for this arrangement of the three-phase circuits; and, as in the case of the two-phase

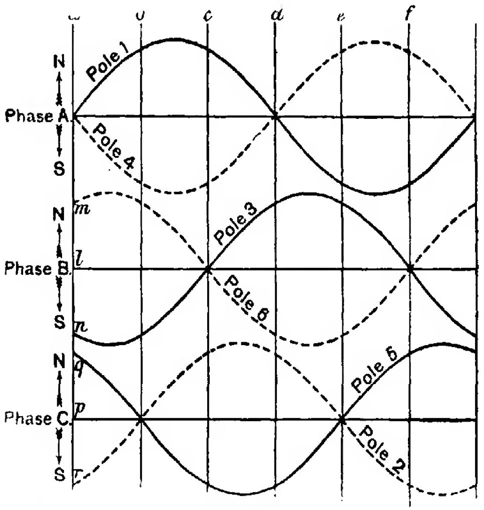

Fig. 191.—Polarity Curves of a Two-Pole Three-Phase Motor Field.

stator, we shall draw diagrams therefrom to show that the rotation of the resultant field is uniform. Taking the left-handedly wound poles, the wave for pole 3 in phase *B* is 120° behind that for pole 1 in phase *A*; and the wave for pole 5 in phase *C* is 120° behind that for pole 3 in phase *B*.

The curves are divided-up by ordinates *a*, *b*, *c*, *d*, etc., one-sixth of a cycle or 60° apart.

Now the instantaneous polarity of each pole at the first ordinate a may be stated as follows :—

Polarity of pole $1 = 0$.
" " $2 = p\,r$ and is S.
" " $3 = l\,n$ and is S.
" " $4 = 0$.
" " $5 = p\,q$ and is N.
" " $6 = l\,m$ and is N.

There are thus two fields set up in the stator, and these will complete their paths through the core of the rotor as shown by the dotted lines in Fig. 192 at (I). As $p\,r$, $l\,n$, $p\,q$, and $l\,m$ are all equal, these two fields are also equal in strength, and their resultant field may be represented by the arrow passing midway between them through the centre of the rotor.

At ordinate b in Fig. 191 the state of things is as shown in Fig. 192 (II), poles 1 and 6 being N., and poles 3 and 4 S. The N. polarity of the resultant field is therefore situated between poles 1 and 6, and the direction of that field across the rotor space is as shown by the arrow. From this figure it is evident that the field (Fig. 190) has moved round one-sixth of a revolution in a clockwise direction, and that this distance has been traversed in the one-sixth part of a cycle between the ordinates a and b in Fig. 191.

At ordinate c, the polarity conditions are as shown in Fig. 192 (III). Here poles 1 and 2 are N., and poles 4 and 5 S., the direction of the resultant field being indicated by the arrow. Thus the N. polarity of the field may now be said to be situated between poles 1 and 2; so that, as before, the field has moved through one-sixth of a revolution in one-sixth of a cycle.

Similarly, the positions of the field at ordinates d, e, f, and a', are as shown in Figs. 192 (IV), (V), (VI), and (VII);

(VII) being similar to (I), as the cycle has then been completed.

These figures indicate that the field (Fig. 190) passes through one-sixth of a revolution in each one-sixth of a cycle, and it may therefore be concluded that its rotation is uniform. Further, the rotation is in the clockwise direction.

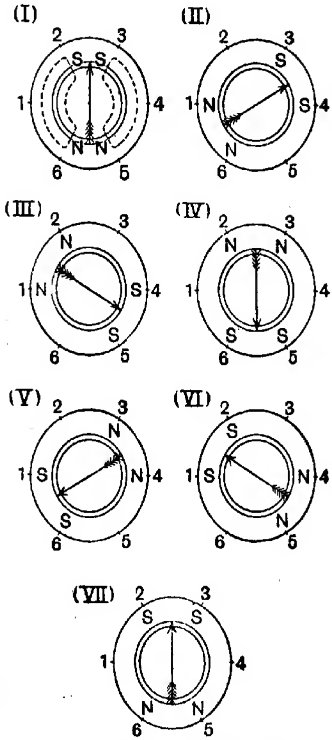

Fig. 192.—Three-Phase Clock-Face Polarity Diagrams.

A winding such as that in Fig. 193, where the connections for phase *C* are the reverse of those in Fig. 190, would not give a uniformly rotating field, assuming that the difference in phase between the currents was 120°. This the reader may prove for himself by analysing the winding in a manner similar to that pursued in the case of Fig. 190.

Why is it that the first winding (Fig. 190) produces a uniform field rotation, whereas the second winding (Fig. 193) does not? Referring to Fig. 190, it will be seen that poles 1, 3, and 5 in the three phases, *A*, *B*, and *C* respectively, are all wound in the same sense, viz. left-handedly. Further, these poles are 120° apart, *i.e.*, their axes intersect at angles equal to the phase-difference between the currents

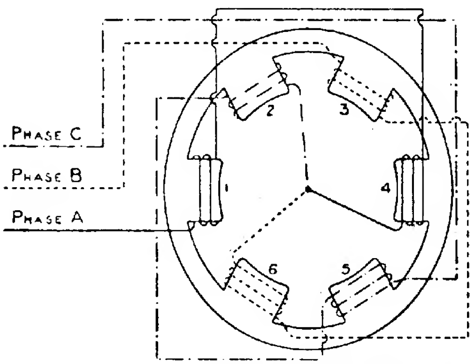

Fig. 193.—Incorrect Winding of a Two-Pole Three-Phase Motor Field.

in their windings, as shown in Fig. 194. In Fig. 193, on the other hand, the left-handedly wound poles are Nos. 1, 2, and 3, in phases *A*, *C*, and *B* respectively, and these are 60° instead of 120° apart. Thus their "pole-distance," so to speak, is not equal to the phase-difference between the currents in the coils. And it is because of this inequality that the rotation of the field in this case would not be uniform.

Thus it may be accepted as an axiom that the rotation of the field of a motor stator with one pair of poles per phase will only be uniform when the angular *pole-spacing*

(or distance between similarly-wound poles) is equal to the phase-difference of the currents. If there are two, three, or more pairs of poles per phase, the angular pole-spacing of similarly-wound poles must be one-half, one-third, etc., the phase-difference of the currents. By similarly-wound poles are meant those which are connected either right-handedly or left-handedly. (See footnote on p. 286.)

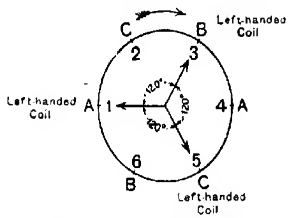

Fig. 191.—Clock-Face Diagram.

It is necessary to mention once more that the poles of an induction motor do not really project from the rest of the stator core, as seen in Figs. 186, 190, etc. They are shown projected merely to simplify the figure. (See § 96.)

95. **REVERSAL OF POLYPHASE INDUCTION MOTORS.** —To provide for the reversal of the direction of rotation of a two-phase induction motor, it is only necessary to insert a reversing switch in the circuit of one phase, so that the connections of the pole windings therein may be reversed. Thus, referring to Figs 186 and 188, with the windings as there shown, the polarities—at four equal intervals (ordinates *c*, *e*, *g*, and *a*, Fig. 188) during one cycle—before and after the reversal of the phase *B* winding, would be:—

	Phase → . .	A.	B.	A.	B.
	Ordinate → . .	(*c*.)	(*e*.)	(*g*.)	(*a*.)
Before Reversal.	Poles . .	1—3	2—4	3—1	4—2
	Polarity . .	N. S.	N. S.	N. S.	N. S.
After Reversal.	Poles . .	1—3	4—2	3—1	2—4
	Polarity . .	N. S.	N. S.	N. S.	N. S.

Before reversal, the north polarity travels round to pole

1, 2, 3, and 4 in turn; that is to say, the rotation of the field is clockwise. After reversal, which makes poles 2 and 4 change places in Fig. 188, the north polarity travels round *via* poles 1, 4, 3, and 2, *i.e.*, in a counter-clockwise direction. And since the direction of the rotation of the rotor corresponds with that of the field, the reversal of the connections of the *B* circuit will thus effect the reversal of the motor. The reversal of the *A* instead of the *B* circuit would do the same thing.

With a three-phase motor, two of the phase connections must be interchanged, though which two they are does not matter. Thus if—in Figs. 190 and 191—we change poles 2 and 5 over from phase *C* to phase *B*, and poles 3 and 6 from phase *B* to phase *C*, we shall get a reversal of rotation. Before changing the connections, the polarities would be as indicated below, where the results diagrammed in Fig. 192 are arranged in tabular form.

ONE CYCLE.

FRACTION OF CYCLE		1st ⅙.	2nd ⅙.	3rd ⅙.	4th ⅙.	5th ⅙.	6th ⅙.
	Ordinate	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*
Before change of connections.	Poles	5 2	6 3	1 4	2 5	3 6	4 1
		6 3	1 4	2 5	3 6	4 1	5 2
	Polarity	N. S.	N. S.	N. S.	N. S.	N. S.	N. S.

The alteration of connections mentioned above would interchange poles 3 and 5 and poles 6 and 2 in Figs. 191 and 192, with the following result:—

FRACTION OF CYCLE		1st ⅙.	2nd ⅙.	3rd ⅙.	4th ⅙.	5th ⅙.	6th ⅙.
	Ordinate	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*
After interchange of B and C connections.	Poles	3 6	2 5	1 4	6 3	5 2	4 1
		2 5	1 4	6 3	5 2	4 1	3 6
	Polarity	N. S.	N. S.	N. S.	N. S.	N. S.	N. S.

On referring to Fig. 190, where the pole numbers remain unchanged though the connections are altered, it will be evident that, in the first case, the resultant field of the north

and south polarities travels round in a clockwise direction. But after the change of connections, the field rotation will be counter-clockwise.

A diagram of a switch and its connections for effecting the reversal of a three-phase motor is given in Fig. 194A. It consists of two copper blades *kk*, pivoted at *a* and *b*, and connected rigidly together, but insulated from each other, by the handle *h*. The fixed contact-jaws *c* and *f* are permanently interconnected by a wire *l*, and the jaws *d* and *e* by a wire *m*. Two of the supply leads (*B* and *C*) are

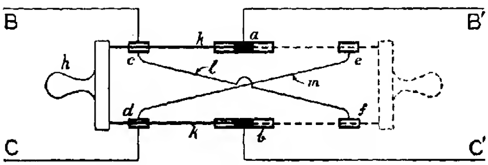

Fig. 194A.—Connections of a Reversing Switch.

connected to *c* and *d*, and two of the motor leads (*B′* and *C′*) to *a* and *b*. The other supply lead (or leads) are taken directly to the motor.

When the switch is placed to the left (as shown by the full lines in Fig. 194A), *B* is connected to *B′* through *cka*, and *C* to *C′* through *dkb*. When the switch is put over to the right (shown dotted), *B* is connected to *C′* through *clfkb*, and *C* to *B′* through *dmeka*. The motor leads *B′* and *C′* will then have been interchanged, and the rotation will consequently be reversed.

96. **STATOR CORES AND WINDINGS OF POLYPHASE INDUCTION MOTORS.**—As stated in § 90, the stators of motors are similar in construction to those of the corresponding type of alternator. Thus, they all have slotted cores without the projecting poles shown in Figs. 186 and 190; the latter arrangement being adopted to simplify those

diagrams. The field-face of a motor stator consequently resembles that of the alternator in Fig. 125, except that it is generally much smaller.

Further, the illustrations of alternator windings in Figs. 121 and 124 apply equally well to two- and three-phase motor stators.

In an alternator, the function of the stator windings is to have e.m.fs. induced in them by the mechanically-rotated field-system. In a motor, current from an external source is sent through the windings, with the result that regions of N. and S. polarity sweep round the face of the stator core, as was proved in the three previous sections.

We have now to see how this rotating field imparts motion to the rotor.

97. **ACTION OF POLYPHASE INDUCTION MOTORS.**—Since the rotating field set up by the stator cuts the rotor, electromotive-forces—and hence currents—are generated in the rotor windings, in exactly the same way and with the same effect as if these windings formed the armature of a dynamo: the e.m.f. being reversed in each individual coil or conductor as many times as the field adjacent to it changes polarity. In fact, the behaviour of a polyphase-motor rotor may easily be studied by reference to a 2-pole field-magnet excited by continuous current (or permanently magnetised); this magnet being supposed to rotate outside an armature or rotor consisting of a number of bars of copper embedded in a cylindrical core of laminated iron, and interconnected at each end. Let us now investigate the action of the rotor upon these lines.

The polyphase current delivered to a stator winding sets up a rotating field of constant strength. In Fig. 195 this field is represented by the poles *N*, *S*, which are supposed to revolve about the rotor in the direction indicated by the arrows. The rotor *R* consists of a laminated core mounted on a shaft *S*, and a number of copper rods

or bars, 1, 2, 3, etc., laid in slots or "tunnels" around its periphery; these bars being all connected together at each end by heavy copper or brass rings.

This construction is known as the *squirrel cage*, and is better illustrated in Figs. 213 to 215. In a rotor of this description it is not always necessary to insulate the copper rods, as their conductance is so much higher than that of

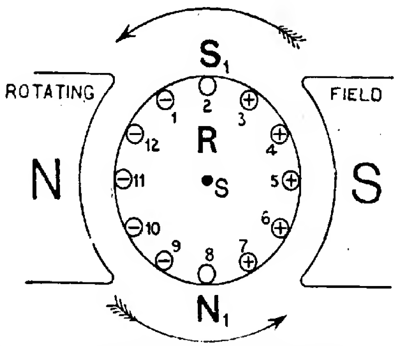

Fig. 195.—The Interaction between a Rotating Field and a Rotor, assuming that the Rotor Bars have no Inductance.

the surrounding iron that there is little tendency for the induced currents to leave them. The rotor bars in Figs. 214 and 215 are not insulated.

Returning once more to Fig. 195, as the field *NS* revolves, e.m.fs. will be induced in the bars, and currents will flow therein under the influence of the e.m.fs. *Assume for the present that the bars have no inductance, i.e.*, that the currents will be in phase with the e.m.fs. producing them; then, at the instant represented in the figure, the bars 3, 4, 5, 6, and 7 will have currents induced in one direction; while those diametrically opposite (Nos. 9, 10, 11, 12, and 1)

will have currents induced in them in the reverse direction. The direction of the currents is indicated by the + and − signs, which signify that they are flowing out and in respectively.*

Fig. 196 is a top view of the rotor in Fig. 195, showing the rotor bars and the copper end-ring R, R'. The numbering of the bars corresponds with that in the former figure.

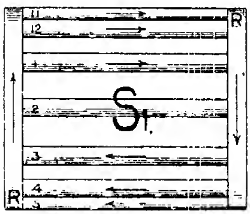

Fig. 196.—Top View of Rotor in Fig. 195, showing some of the Active Bars and the End-Rings.

The arrows in Fig. 196 indicate the directions of the e.m.fs. at the particular moment considered in Fig. 195; those in bars 11, 12, and 1 being from left to right, and those in 3, 4, and 5 being from right to left. Hence currents are produced which flow in a clockwise direction *via* bars 11, 12, and 1, end-ring R', bars 3, 4, 5, and end-ring R. The region between bars 1 and 3 consequently becomes of S. polarity. There is no current in bar 2, firstly, because no e.m.f. is being generated in it. And secondly, because though some of the current due to the e.m.fs. in 11, 12, and 1, tends to flow back from right to left in it, there is an equal tendency for part of the current due to the e.m.fs. in 3, 4, and 5 to flow through it from left to right; these two tendencies being equal and opposite.

The polarity of the rotor as a whole is shown in Fig. 195, it being south (S_1) at the top, and north (N_1) at the bottom. Repulsion is then set up between NN_1 and SS_1, and attraction between SN_1 and NS_1; and the rotor will rotate in the

* For Left-Hand Rule for finding the direction of induced currents see the Author's *Electric Lighting and Power Distribution*, Vol. I.

same direction as the inducing field *NS*, and will attempt to keep pace therewith, though it will never quite succeed in doing so.

Now it must be remembered that in the case of a bi-polar polyphase motor, whose action we are attempting to illustrate, the field *NS* revolves at a high speed; the number of the revolutions per minute being 60 times the frequency of the supply.

On account of this high speed the currents set up in the rotor bars at starting will be relatively large, unless the latter with their end-connections have a high resistance, or unless the rotating field *NS* be very weak.

98. **ACTION OF POLYPHASE INDUCTION MOTORS** (*continued*).—In the previous section it was assumed that the rotor bars had no inductance. In reality, however, they have a very appreciable amount, chiefly because of their embedment in the slots or tunnels of the iron core. And as the reactance due to this inductance (§ 28A) adversely affects the starting of an induction motor, it is necessary to see why this is and how matters can be remedied.

In § 29 it was explained that, in an inductive circuit, the current lags behind the applied voltage by an angle ϕ. Now the tangent* of this angle is given by $\frac{2\pi fL}{R}$, R and L being the resistance and inductance respectively. Anything which increases the tangent will also increase the angle, hence the greater the frequency and the inductance, and the smaller the resistance, the greater will be the angle of lag between the current and the voltage.

* The *tangent of an angle* a (Fig. 19) is given by the ratio of the perpendicular DE to the adjacent side BE, that is, by $\frac{DE}{BE}$. Hence in Fig. 73:—

$$\tan \phi = \frac{AC}{OA} = \frac{OB}{OA} = \frac{2\pi fL}{R}.$$

Tangent values can be obtained directly from the Table on p. 271.

Now, at starting, the frequency of the e.m.fs. induced in the rotor bars in Fig. 195 will be a maximum, and will be that of the rotating field; consequently the lag of the currents in the bars will also be a maximum. So that instead of the currents being as in Fig. 195, where no lag is considered, they would be somewhat as in Fig. 197. Thus, although the poles have got past bars 2 and 3, and 8 and 9,

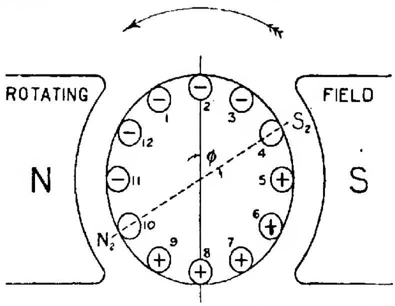

Fig. 197.—Effect of Inductance of Rotor Bars upon Fig. 195.

the currents will still be flowing therein; and the reversal of current will only just be taking place in bars 4 and 10. In consequence of this, the field due to the rotor currents will now be in the direction N_2S_2, *i.e.*, ϕ degrees behind N_1S_1 in Fig. 195.

The effect of the shifting back of the rotor field is to cause the latter to oppose more or less the main field NS, and therefore to weaken the actual flux cutting the rotor bars, and so decrease the magnetic drag on the rotor. The greater the angle ϕ, the greater will be this demagnetising action of the rotor currents; and the smaller will be the magnetic drag or the torque on the rotor, since the drag or the torque depends upon the product of the current in the conductors and the flux cutting them. In fact, the torque

due to bars 3 and 9 would tend to turn the rotor in a clockwise direction; and if the angle ϕ was 90°, *i.e.*, if the rotor winding was purely inductive, the torques in the two directions would be equal and opposite, and the rotor would not revolve at all.

The foregoing explanation has been given at some length, since it is necessary for making clear the reasons for the different types of rotor to be explained presently.

The simplest type of rotor construction is the one already referred to (§ 97), namely, the *squirrel-cage* or *short-circuited rotor.* With this construction it is evident that the resistance R is exceeding low, consequently $\frac{2\pi fL}{R}$ is high, and the angle of lag (ϕ) large; so that a motor with this kind of rotor has a very poor starting torque, and is only able to start under a light load.

The obvious ways to improve the starting torque are to decrease f and L and to increase R. This is one reason why low frequencies are desirable for power circuits (§ 7). As f cannot be varied for motors running off any given supply, and as an appreciable decrease in L would adversely affect the performance of the motor; R remains the only quantity that can be varied to obtain a good starting torque. Various ways in which this is done will now be explained.

99. **ACTION AND CONTROL OF POLYPHASE INDUCTION MOTORS.**—One method of obtaining a fairly good starting torque, while still retaining the squirrel-cage construction, is to make the rotor bars of greater length, and of some material (such as brass) having a higher resistance than copper; the end-connections (or parts interconnecting the ends of the bars) being also given plenty of resistance. But since this resistance is always in circuit, the efficiency of the motor is lowered; consequently the above method is only adopted in special cases.

The most usual method of obtaining a good starting torque is to employ a *wound rotor*, *i.e.*, a rotor wound for three-phase in accordance with Fig. 124; and to connect the three leads from the winding to slip-rings on the shaft. Brushes bearing on these slip-rings are connected to a non-inductive 3-arm rheostat, as shown in Fig. 198. This *starting rheostat* consists of three groups of resistances, one for each phase; and three contact arms, a, a, a; the latter being connected rigidly together, both mechanically and electrically, so as to form a star point.

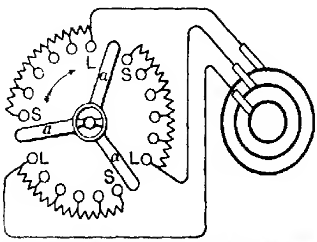

Fig. 198.—Starting Resistance for a Three-Phase Slip-Ring Motor.

To start the motor, the arms of the rheostat switch are put in the "off" position, as depicted in Fig. 198, and current is applied to the stator by means of, say, an ordinary triple-pole switch. The rheostat handle is then turned in the direction of the curved arrow (Fig. 198). When its arms reach the studs *S*, *S*, *S*, the rotor starts, all the resistance being then in circuit. As the arms are moved further round, the resistance is gradually cut out, until the last stops *L*, *L*, *L*, are reached. At this point the brushes and slip-rings are virtually short-circuited, and the motor runs at full speed.

In the case of the smaller machines, it is usual to leave the brushes down all the time the motor is running. In larger motors, however, arrangements are often made for short-circuiting the rotor rings and raising the brushes when the motor has been run up to speed; the starting

resistance being then entirely disconnected from the rotor

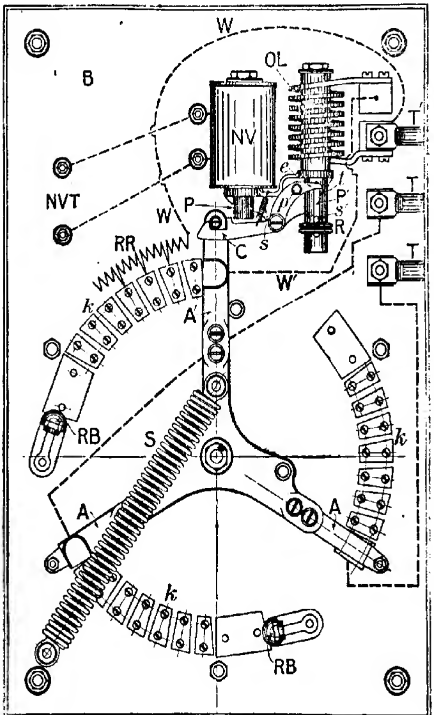

Fig. 199.—Starter for Slip-Ring Induction Motor. (*Ellison.*)

winding. This is the case with the motors illustrated in Figs. 216 and 221.

Fig. 199 gives a front view of a *motor starter*, the parts

of which are mounted on an enamelled-slate base *B*. The switch-arms *A'*, *A*, *A*, are seen in the full-on position, where they are held by the catch-piece *C*, which engages with a stud on arm *A'*. The arms are operated by a handle which is not shown in this illustration, as it is mounted on a cover which fits over the apparatus. This handle (for operating with both hands) can be seen in the external view in Fig. 199A; the purpose of the small lever in the top right-hand corner being to open the circuit by releasing the catch-piece—as explained presently.

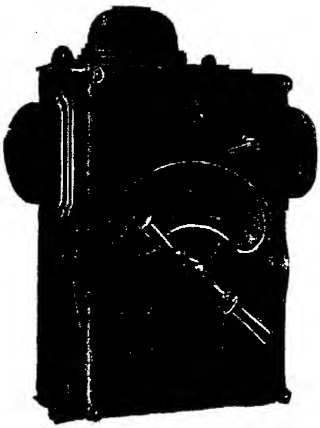

Fig. 199A.—External View of Starter for Slip-Ring Induction Motor. (*Ellison.*)

Returning to Fig. 199, the helical steel spring *S* always tends to pull the switch-arms round to the "off" position when *C* releases them; and the shock of the off movement is taken by the rubber buffers *RB*.

The resistance wires* are mounted on porcelain insulators inside a sheet-iron box behind the slate base *B*; *k*, *k*, *k* being the contacts to which they are connected.

The starter is fitted with *no-voltage* and *overload*

* These are made partly of "Hecknum" and partly of "Alpacca," which are special grades of german-silver wire, claimed to have resistivities of 18·9 and 11 microhms per inch cube respectively. (*See* the Author's *Electric Circuit Theory and Calculations.*)

releases. The former is for opening the rotor circuit should the supply fail; and the latter for doing the same thing should too much load be put upon the motor, and the current therein consequently become excessive. The overload release is thus otherwise termed the *excess-current release.*

The no-voltage release consists of a fine-wire solenoid *NV*, which is connected through the terminals *NVT* across two of the stator phases (see *NVT* in Fig. 200), and which holds up its plunger core *P* as long as the supply is maintained. This core is just above the catch-piece *C*, which, with the help of the little spring *s*, holds the rheostat or switch arms in their full-on position. Should the supply fail, the plunger *P* drops onto the catch-piece *C* and forces it down, so allowing the rheostat arms to be pulled to their off position by the spring *S*.

The overload release consists of a thick-wire solenoid *OL* with comparatively few turns, this being connected in one of the rotor phases through the top main terminal *T′* (see *OL* in Fig. 200). The effect of the normal maximum current in this solenoid is insufficient to suck up its plunger core *P′*, which slides in a brass tube *t*, a stud *s′* on its lower end normally resting on the adjustable tapped rings *R*. Although not shown so in the figure, *t* is screwed to receive *R*, which may thus be raised or lowered to suit different degrees of overload.

If the current in *OL* becomes excessive, the plunger *P′* is sucked up, and its stud *s′* knocks up an extension *e* of the catch-lever, the other end *C* of which is thereupon forced down, so that the rheostat arms are released as already explained. The release can also be effected by hand, when it is desired to stop the motor, by raising the little lever on the outside of the cover (Fig. 199A), this engaging with the pin *p* on *e*.

The three main sweating-lug terminals are seen at *T″*, *T′*, *T*. These terminals, as well as *NVT*, are also marked in

Fig. 200; so that the connections of the starter to its motor can be fully understood.

As regards the connections on the starter itself, it should be noted that the overload coil *OL* is not put into circuit until the arm *A'* reaches its last or full-on stop. Thus, when this arm is passing over the contacts of the resistances *RR*, the circuit to terminal *T'* is made through the connecting wire *W*. But when *A'* reaches the full-on position shown in the figure, the connection to *T'* is *via* *W'*

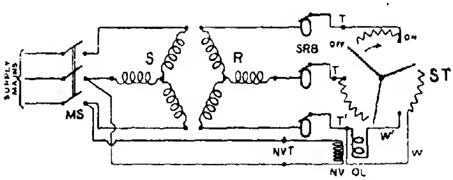

Fig 200.—Diagram of Connections of a Slip-Ring Motor with its Main Switch and Starter.

and *OL*. *W* and *W'* are also shown in Fig. 200. The reason for keeping the overload coil *OL* out of circuit until the starting rheostat is full on, is that the heavy rush of current in the rotor on starting would suck the plunger *P'* right up, and the latter would tend to keep up, and so prevent the locking-on of *A'* by *C*.

The projecting portions on the top and sides of the complete apparatus (Fig. 199A) are cable-entry boxes.

A complete diagram of the connections for a motor fitted with the above-described type of starter is given in Fig. 200. Here *S* and *R* are respectively the stator and rotor windings, *SRB* the slip-rings and brushes, *MS* the main triple-pole switch in the stator circuit, and *ST* the starting rheostat with the no-volt coil *NV* and the overload coil *OL*.

The windings of the stator and rotor in Fig. 200 are both shown star-connected; but either or both may be mesh-connected. The chief disadvantage of the mesh arrangement is that a greater number of turns is required, and the cost of winding is consequently higher.

Even when the stator is wound two-phase, it is customary to wind the rotor for three-phase. The chief reason for this is that one design of rotor and starting gear does for the two types of motor. Fig. 218, for example, shows a two-phase motor with a three-phase rotor.

Machines fitted with wound rotors (page 304) are generally referred to as *slip-ring motors;* and actual examples are illustrated in § 106.

100. **SLIP AND SPEED.**—A little consideration should make it clear that as the rotor of an induction motor increases in speed, the rate at which the rotating field of the stator cuts the rotor conductors must decrease. Thus the greater the speed of the rotor, the less the induced e.m.fs. therein, and the lower their frequency. If the rotor could attain the same speed as the stator field, the rotor bars would not be cutting any lines of force, and there would be no e.m.fs. or currents induced in them; consequently, the torque on the rotor would be zero. Actually, this condition of things is impossible, since a torque, however small, is always necessary to keep the rotor revolving. In other words, there is always a difference in speeds between the rotating field and the rotor, and this difference becomes greater the greater the torque required.

The difference between the rotor and field speeds, when expressed as a percentage of the synchronous speed (*i.e.*, the speed of the rotating field), is known as the *percentage slip* of the motor. Thus:—

$$\text{Percentage Slip} = \frac{\text{synchronous speed} - \text{speed of rotor}}{\text{synchronous speed}} \times 100 \quad \textbf{(49)}$$

As mentioned in § 91, the synchronous speed is given by the formula $R = \frac{60f}{p}$, where f is the frequency of the supply, and p the number of pairs of poles on the stator.

EXAMPLE.—*An 8-pole motor is supplied at a frequency of* 50, *and the speed of the rotor under a certain load is observed to be* 712 *r.p.m. Determine the slip of the motor.*

$$\text{Synchronous speed} = \frac{60 \times 50}{4} = 750 \text{ r.p.m.}$$

$$\text{Slip} = \frac{750 - 712}{750} \times 100 = 5{\cdot}07 \text{ per cent.}$$

As the load on a motor increases, the torque which the rotor is required to exert also increases; and in order to provide this increasing torque, the slip must be larger to generate sufficient current in the rotor. Hence, the greater the load, the greater is the slip; and the value of the slip at full load varies from about 2 per cent. for large motors to about 8 per cent. for small ones.

101. **METHODS OF STARTING SQUIRREL-CAGE INDUCTION MOTORS.**—It was explained on p. 301 that at starting, very large currents are induced in a low-resistance squirrel-cage rotor if its stator be switched direct onto the supply. The effect of this is to cause the stator to take a very large current from the mains. In fact, if a medium-sized squirrel-cage motor were switched directly onto the line (by means of a triple-pole switch), the starting current would be several times the full-load current; and this current would lag very much behind the voltage because of the inductance of the motor. Thus the starting of such a machine would have a very disturbing effect upon the voltage of adjacent portions of the supply system.

To prevent this disturbance, only very small motors are

allowed to be switched directly onto the mains, and some other means of starting must be adopted with medium-size and large machines.

Fig. 200A shows how a small three-phase squirrel-cage motor, M, can be started with a triple-pole switch, T, in the stator circuit. If the motor stator be star-wound, and if it be provided with six terminals, three for the supply and three for the star, a coupled "Twinob" tumbler switch can be connected, as in Fig. 200B, to take the place of the more expensive triple-pole switch. This switch can deal with currents up to 10 amperes at voltages not exceeding 250, and externally resembles Fig. 200C. Its porcelain base is only 3½ inches in diameter, and it can easily be mounted on the motor itself. One advantage of having the starting-switch at the star end of the windings (Fig. 200B), instead of at the supply end (Fig. 200A), is that there is no chance of short-circuiting. An interior view of the switch, with its knobs uncoupled as for ordinary use, is given in Fig. 200D.

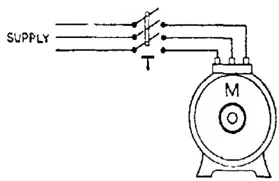

Fig. 200A.—Starting of a small Three-Phase Squirrel-Cage Motor by means of a Triple-Pole Switch.

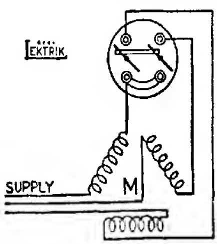

Fig. 200B.—Starting of a very small Three-Phase Squirrel-Cage Motor by means of a coupled "Twinob" Tumbler Switch. (*Lundberg.*)

One method of limiting the starting current of squirrel-cage motors would be to connect a variable choking coil (§ 71) in each phase-circuit of the stator. Another and

less economical method would be to insert a rheostat in each phase of the stator circuit. But the two most usual methods of starting are by means of—(*a*) a change-over *star-mesh* connection for the stator, or (*b*) an *auto-starter*. The former method is much the simpler.

The star-mesh method of starting is obviously only applicable to three-phase motors, whereas the auto-starter

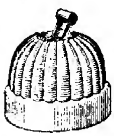

Fig. 200c.—Coupled "Twinob" Switch. (*Lundberg.*)

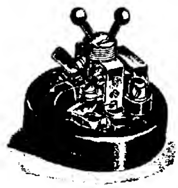

Fig. 200D.—Interior View of "Twinob" Switch with Knobs uncoupled for ordinary use. (*Lundberg.*)

method may be used with either three- or two-phase machines. A series-parallel method of starting a two-phase motor is described in § 103.

102. **STAR-DELTA STARTING OF THREE-PHASE INDUCTION MOTORS.**—For the *star-mesh* or *star-delta* method of starting, the two ends of each phase are brought out from the stator winding and connected to the starting switch. The latter has three "positions":—off, starting, and running. In the starting position, it connects the stator phases in star, so that the voltage per phase is only $\frac{1}{\sqrt{3}}$ ($= \cdot 58$) of the line voltage (Formula 34). When the switch is put over to its running position, the stator is mesh-connected; each phase then getting the full line voltage.

The action of a star-delta switch is thus to start the motor with less than two-thirds of the supply voltage on

each phase, and then to apply the full voltage when it has got up speed.

Two views of one form of star-delta switch are given in Figs. 201 and 202. Fig. 201 shows the lid of the upper portion opened, and the terminals and fuses exposed to view. The switch itself is beneath, and is enclosed in an oil-tank there.*

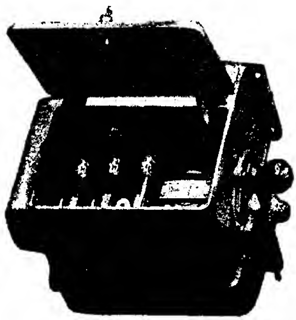

Fig. 201. — Star-Delta Switch, with Lid opened, showing Fuses. (*British Thomson-Houston Co.*)

Fig. 202 is a view of the bottom of Fig. 201, the oil-tank being removed so as to expose the switch. The fixed spring-clip contacts of the switch are provided with *sparking fingers*, at which the actual break takes place; the main contacts being thus saved from *sparkwear*.

The electrical connections for the switch are given in

* ***Oil-immersed Switches.*** The motor-starting switch in Figs. 201 and 202 belongs to the class of ***oil-immersed*** or ***oil-break switches***; which, by the way, are chiefly used in alternating-current work.

The immersion of the switch contacts in oil has the following advantages: (*a*) It allows of closer grouping of the switch contacts, and consequently more compact construction, as the oil quenches the sparking at the contacts, and prevents arcing over from one to another. (*b*) Because of the action just mentioned, with a given switch, oil-immersion permits of the breaking of circuits at much higher voltages than would be possible in air. (*c*) The ***oil switch*** (as it is sometimes briefly termed) always breaks the circuit when the alternating current is passing through its zero value; consequently there is practically no sparking at the switch contacts.

Oil-immersed switches are not so satisfactory on continuous-current circuits, because they have to break an unvarying current. The sparking which occurs carbonises the oil and lowers its insulating value.

Fig. 203. *LLL* are the three-phase supply mains. These are connected to the switch-blade holders, *MMM*, mounted on (but insulated from) the shaft *SS*, which is shown by dotted lines. *DDDD* are the copper switch-blades.

AA', *BB'*, and *CC'* represent the three-phase windings on the stator of the motor. The ends *A*, *B*, and *C* are joined-up to the fixed spring-clip contacts *NNN* and *RRR*. The other ends, *A'*, *B'*, and *C'*, are connected to M_1, *P*, *P*, and (through fuses *FFF*) to other spring-clip contacts *QQQ*.

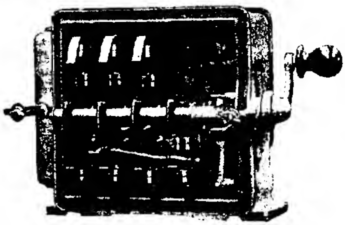

Fig. 202.—Star-Delta Switch, with Oil-Tank removed, showing the Switch itself. (*British Thomson-Houston Co.*)

Fig. 202 shows the switch in its off position. In order to start the motor, the handle *H* (Fig. 203) is turned so that the blades *DDDD* are in the position there shown. Each line conductor *L* is then connected to the start of the corresponding phase through *M*, *D*, and *N*; whilst the ends *A'*, *B'*, *C'* of the phases are interconnected at M_1, *D*, *P*, *P*; that is, the windings of the motor are then connected in star.

After the motor has run up to speed, the handle *H* is turned through half a revolution in the other direction, thereby bringing the blades *DDDD* to the positions (*MRQ*) shown dotted in Fig. 203. The end *A'* of the first phase will then be connected through 1, fuse *F*, and contacts *QR* to the beginning *B* of the second phase; *B'* will be similarly connected through 2 to *C*, and *C'* through 3 to *A*. The stator windings will then be connected in mesh, and each phase will have the full line-voltage across it.

An interlock is provided on the switch to prevent it being thrown into the "running" position before it has

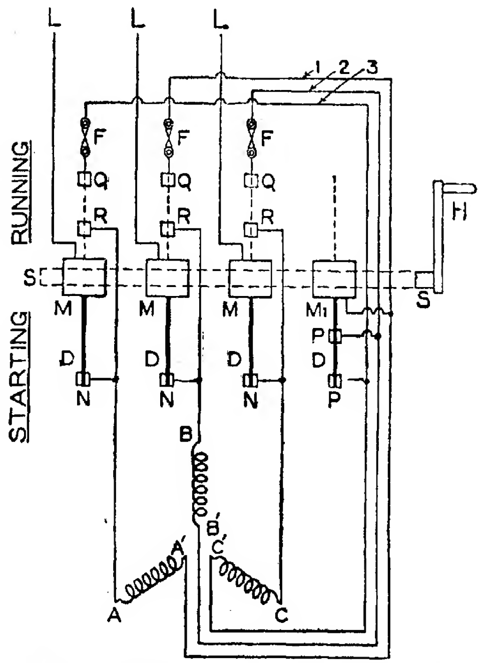

Fig. 203.—Diagram of Connections for the Star-Delta Switch in Figs. 201 and 202.

been put in the "starting" position; and there are certain other details which cannot be described here.

It should be noticed that the fuses *FFF* are only in

circuit in the "running" position of the switch. A good deal of unnecessary trouble is thus avoided, since if they were put in the starting circuit, frequent renewals would be necessary, as the starting current would often exceed the maximum current allowable in the "running" circuit.

A disadvantage of the star-delta method of starting is that it is not suitable for a motor which is required to exert an appreciable starting torque. *The starting torque of an induction motor varies as the square of the voltage per phase.* This statement may be made clear with the help of Fig. 195. The torque is proportional to the product of the flux and the rotor current. Now if the voltage applied to produce the excitation of the poles *NS* be, say, halved, then (if saturation be neglected) the flux due to *NS* will be halved, and the e.m.fs. (and their currents) induced in the rotor bars will also be halved; consequently the torque will only be one-fourth its original value. In the star-mesh method of starting, the voltage per phase at starting is $\frac{1}{\sqrt{3}}$ (or ·58) of the line voltage (p. 312); hence the starting torque is only a third ($\cdot 58 \times \cdot 58 = \cdot 34$) of the value obtainable by switching the motor directly onto the mains. This shows that the star-delta method is not suitable when an appreciable starting torque is necessary.

103. **SERIES-PARALLEL STARTING OF TWO-PHASE INDUCTION MOTORS.**—A two-phase squirrel-cage motor which is too large to be switched directly onto the mains could be started with a variable choking coil in each phase; or by means of two variable non-inductive resistances (rheostats), one in each phase. Neither of these ways, however, are adopted in practice.

A method sometimes employed, which necessitates dividing each of the two phase-windings into two parts,

is to *start* with these parts in series, and to *run* with them in parallel, as diagrammed in Fig. 204. The windings are such that when in parallel they are suitable for the full line voltage; so that when they are connected in series, the starting current cannot be very large.

A starting switch for changing the connections as shown in Fig. 204 could be constructed on very much the same lines as that in Figs. 201 and 202.

For very small low-voltage motors the control switches

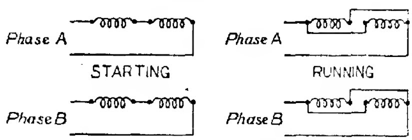

Fig. 204.—Series-Parallel Starting of Two-Phase Squirrel-Cage Motor.

can be of very simple form as follows:—In Fig. 205, *Sws* are two series-parallel tumbler switches, whose knobs are coupled together by a handle-bar, as seen in Fig. 206, so that the switches are operated simultaneously.

Each switch has four terminals, and the connections of the supply lines and motor windings thereto are as shown. In the starting or series position of the coupled switch, the pairs of terminals *s*, *s*, are interconnected, with the result that the halves of each phase-winding are put in series. In the running or parallel position of the switch, the pairs of terminals *p*, *p*, are interconnected, and the sections of winding are then in parallel. The coupled switch can be provided either with or without an "off" position. In the former case there would be no interconnections at all between the switch terminals when the switch was off; and in some uses, an extra switch for cutting-off the current could then be dispensed with.

The usual method of starting a two-phase squirrel-cage

induction motor is by means of an *auto-starter*, as described in the latter part of the next section.

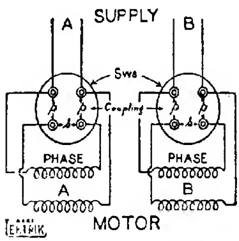

Fig. 205. — Diagram of Coupled Switch for Series-Parallel Starting of a very small Two-Phase Induction Motor. (*Lundberg.*)

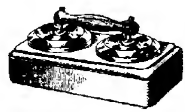

Fig. 206.—Coupled Tumbler Switches.

104. **AUTO-STARTER STARTING OF THREE-PHASE AND TWO-PHASE INDUCTION MOTORS.**—An auto-starter consists of a species of change-over switch working in conjunction with two auto-transformers (§ 87).

Starting through auto-transformers possesses the following advantages :—

(*a*) By suitably winding the auto-transformers, the starting voltages for any given size of motor can be adjusted to various conditions.

(*b*) When starting, only a small current is drawn from the line, this being transformed into a larger current at a lower pressure through the stator windings.

The principle of the auto-transformer method is shown in Figs. 207 and 208, where A, B, and C are the supply leads, and A', B', and C', the terminals of the motor stator M. T_1 and T_2 are the two auto-transformers, which at starting are connected between the supply leads as seen in Fig. 207. In order that the amount of reduction of voltage may be changeable, the "auto." windings are arranged with a number

of loops or tappings; so that their points of connection c, c, to the motor may, when being installed, be suited to the starting torque required. If the connections are made midway as in the figure, the starting voltages at the terminals of the motor will obviously be half those on the mains; and in this case, the stator current, as already explained on p. 256, will be practically twice that taken from the mains.

After the motor has run up to its full speed, the

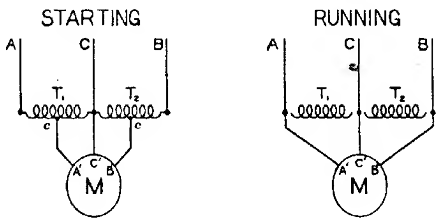

Fig. 207 and 208.—Simple Diagram of the Starting and Running Connections of an Auto-Starter for a Three-Phase Induction Motor.

connections are altered (by means of the switch) to those in Fig. 208; that is, the auto-transformers are cut out, and the stator windings are put directly across the mains.

An actual auto-starter, with its cover removed, is illustrated in Fig. 209. A diagrammatic view of the same is given in Fig. 210, where A, A', B, B', C, and C'', represent the contact jaws seen on the right-hand side in Fig. 209; and 1, 2, 3, etc., are the jaws on the left-hand side of the same figure. L, M, and N are U-shaped contact blades clamped to (but insulated from) a steel shaft S S, which is turned by the handle H. N consists of four interconnected blades. T_1 and T_2 are the two auto-transformers, which are connected to the contact jaws as shown; the jaws A and 1, B and 3, C and 5, C' and 6, being permanently interconnected. The leads from the supply mains are

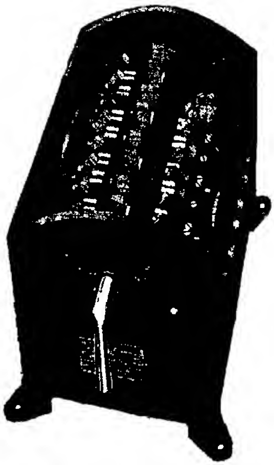

Fig. 209.—An Auto Starter. (*British Westinghouse Co.*)

taken to A, B, and C; and those to the motor go from A', B', and C'.

When starting the motor, the handle H is turned so that the blades L, M, and N are in the position seen in the diagram. The supply circuit between A and C will then be:—A, 1, 2, T_1, 7, 5, C; and that between B and C:—B, 3, 4 T_2, 8, 5, C. The motor circuits will be A' c, T_1, 7, 6, C' and B', c, T_2, 8, 6, C'. If these apparently complicated connections be traced out, they will

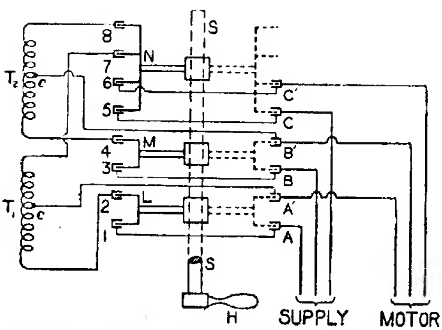

Fig. 210.—Diagram of Connections of Auto-Starter in Fig. 209.

be found to correspond exactly with those in Fig. 207.

When the motor has started up, the handle H is rotated through 180°, so that the blades L, M, and N are thrown

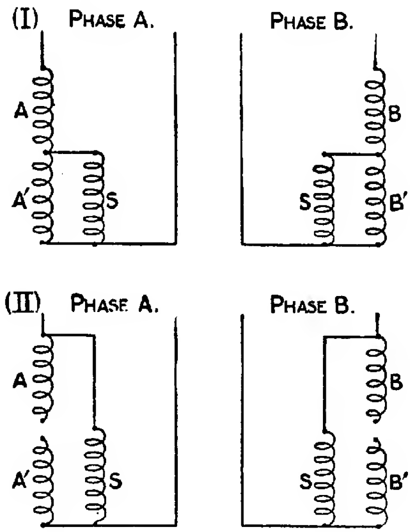

Fig. 211.—Simple Diagram of Auto-Starter Connections for a Two-Phase Motor.

over to the dotted position in Fig. 210. The effect of this is to connect together AA', BB', and CC' respectively, and thereby place the motor windings straight across the mains; the transformers T_1 and T_2 being entirely cut out. The connections will then correspond with Fig. 208.

If it be desired to vary the starting torque of the motor,

this can be accomplished by altering the position of the connections c, c, on T_1 and T_2; spare tappings being generally provided for this purpose.

Fig. 211 shows how the same principle can be applied to the starting of a two-phase motor. Here AA' and BB' are the two auto-transformers, and SS the motor stator windings.

When in the starting position, the connections are as shown in the top half (I) of the figure. The voltage on each of the stator phases SS is then obviously only half that on the line, whilst the current passing through the stator windings will be nearly double that drawn from the line, as explained on p. 256. When the switch is put over to its running position (II), SS are connected straight across the supply mains, and the auto-transformers are disconnected.

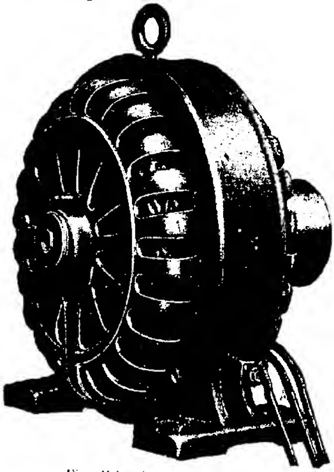

Fig. 212. Squirrel-Cage Motor.
(*Metro-Vickers Ltd.*)

A starting switch for effecting the above connection changes could be constructed on the lines of that in Fig 209.

105. SQUIRREL-CAGE MOTORS. — Fig. 212 shows a complete squirrel-cage motor, and Fig. 213 is the rotor of the same. In Fig. 212, the stator terminals are at the ends of the loose leads; and the pulley is partly seen on the right.

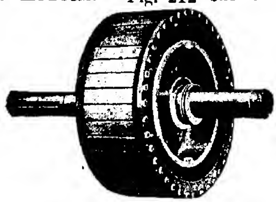

Fig. 213.—Rotor of Motor in Fig. 212. (*Metro-Vickers Ltd.*)

The rotor is shown without its pulley in Fig. 213; and the conductors thereon consist of thick insulated copper rods (one per slot) riveted into the end rings.

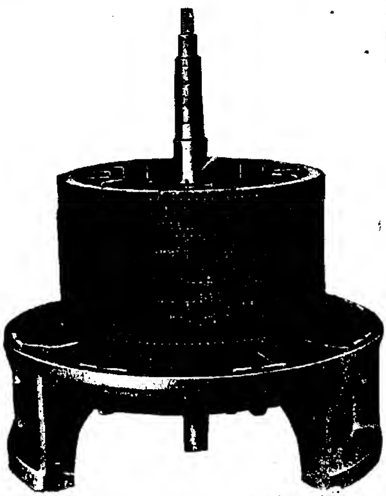

Fig. 214. Rotor and Base of a Vertical Squirrel-Cage Motor. (*Metro-Vickers Ltd.*)

The rotor and base of a *vertical motor* of similar type is seen in Fig. 214. In this case, the conductors, which are rectangular in shape, are uninsulated, and are fixed to the end rings by riveted bolts. In both cases the riveting ensures

secure jointing. The soldered joints employed by some makers have been known to melt, and the solder to fly off, under the heavy currents circulating in the rotor (p. 301). It should be noted (in Figs. 213 to 215) that the rotor bars are mainly kept in position by the projections of the teeth overhanging the slots in which they are embedded; so that little or no mechanical strain is put upon the riveting.

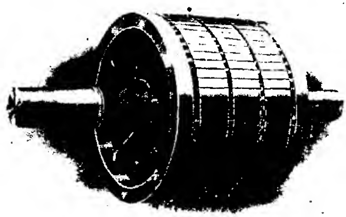

Fig. 215.—Rotor of a Squirrel-Cage Motor. (*Bruce Peebles & Co.*)

In Fig. 215 is shown an extremely robust construction for a squirrel-cage rotor. The conductors consist of a number of bare copper bars forced into the slots. These bars are afterwards connected together at each extremity of the rotor by rings of special alloy which are cast solid onto the ends of the bars. A splendid permanent and yet inexpensive connection is thus secured.

106. **SLIP-RING MOTORS AND THEIR WINDINGS.**—Fig. 216 shows a complete slip-ring induction motor, and Fig. 217 gives separate views of its stator and rotor, with certain portions missing. The latter figure shows the stator and rotor windings very clearly, both of these being arranged as in Fig. 124, except that the motor in question has been wound for eight poles.

The slip-rings are seen on the left in Fig. 216, and the handle projecting on the extreme left is for operating a device for short-circuiting the rings and lifting the brushes

clear of them. The figure shows the brushes raised as when the motor is running, the slip-rings being then short-

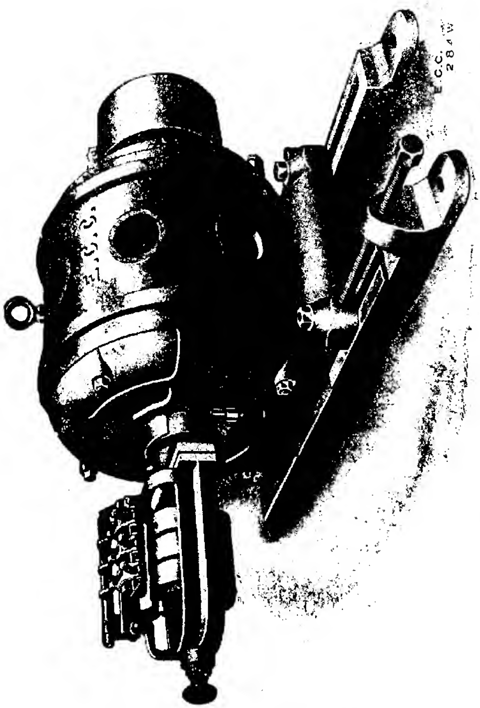

Fig. 216.—Slip-Ring Motor. (*Electric Construction Co.*)

circuited. To put the gear in readiness for starting, the handle is given a quarter-turn and then pulled out slightly.

The first operation puts the brushes onto the rings, and the second unshort-circuits the latter. A further description of this particular gear cannot be given, but another kind is more fully described on p. 331.

Machines are often supplied without the slip-ring short-

Fig. 217.—Stator and Rotor of Motor in Fig. 216. (*Electric Construction Co.*)

circuiting gear above mentioned. In such cases, however, the brushes are always rubbing on the slip-rings, and the rotor currents have to flow through the brushes. There is thus an appreciable loss of power due to brush friction and resistance, and a corresponding decrease in the efficiency of the machine.

The stator terminals cannot be seen in Fig. 216, as they are on the far side of the motor; but their position may be gathered from Fig. 217, where the ends of the windings are shown in readiness for the connection of the terminals.

The half of the shaft on the slip-ring side is hollow, as

seen in Fig. 217 ; and the connections from the rotor windings to the slip-rings are threaded through. As the slip-rings are on the outer side of the bearing, they cannot be fitted until after the machine has been "assembled." When this is done, the three loose connections (Fig. 217) are joined-up to their respective rings.

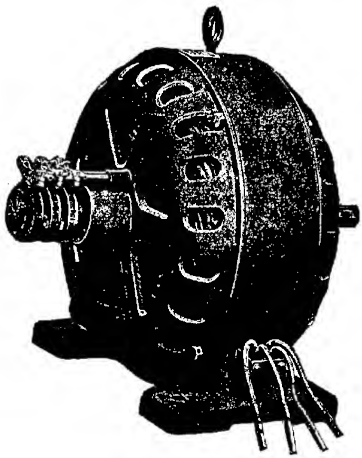

Fig. 218.—Slip-Ring Motor. (*Metro-Vickers Ltd.*)

Fig. 218 shows a Westinghouse slip-ring induction motor, the rotor of the same being illustrated in Fig. 219. The connections between the slip-rings and the rotor winding are made through the shaft as in Fig. 217. The ends of the stator windings (as may be seen) terminate in connectors for the attachment of the supply leads. The reason for there being four connections is that the machine is intended for working off a two-phase circuit (Fig. 95).

It should be noted that both the stator and the rotor windings differ in appearance from those in Fig. 217 ; in fact, they look much more like the windings of a continuous-current armature. The advantage of this type of winding is that all the coils can be separately wound on "formers"

to the proper shape. They are afterwards mounted on the cores by putting the wires, one or two at a time, through the slot openings; the latter being generally very small on alternating-current machines. The forming and subsequent assembling of the windings on the cores is consequently a comparatively easy and expeditious matter. The chief disadvantage of this kind of winding is that the slot-openings must be comparatively wide to enable the wires to be put through, and this lowers the power factor of the motor.

Fig. 219.—Rotor of Motor in Fig. 218. (*Metro-Vickers Ltd.*)

The windings in Fig. 217 have to be threaded through the core slots turn-by-turn by hand, and skilled workmanship is required; but it is obvious that the windings are held more securely in position, and that the arrangement is free from the drawbacks mentioned above.

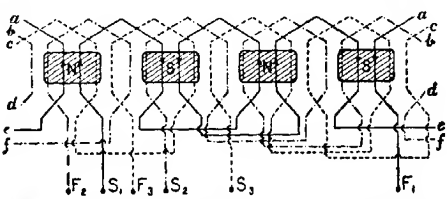

Fig. 220.—Diagram of Three-Phase Windings in Figs. 212 and 219.

Fig. 220 shows the manner in which the coils of 3-phase windings of the type shown in Figs. 212, 218 and 219 are interconnected; each coil being represented by one turn,

though in reality, there are several. The position of the rotating magnetic field at a particular instant corresponds to the shaded areas. If the connections of each phase are

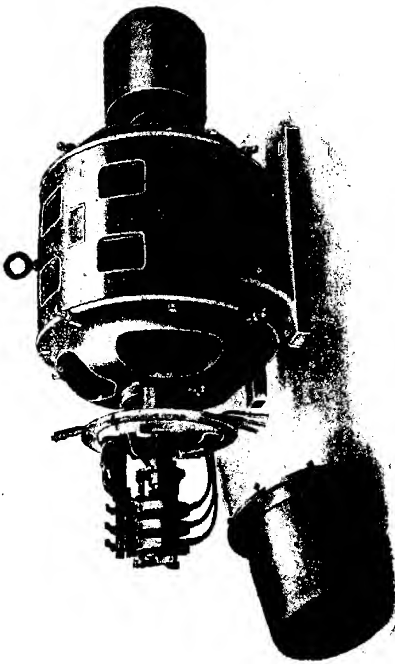

Fig. 221.—Slip-Ring Motor. (*Bruce Peebles & Co.*)

traced round, and compared with those in Fig. 122, it will be seen that the two windings are identical in effect.

This type of winding (Fig. 220) is known as *lap winding,* since it is really similar, so far as any one phase

is concerned, to the lap winding of some continuous-current armatures.*

Another make of slip-ring motor is shown in Fig. 221, this—like the machine in Fig. 216—being fitted with slip-

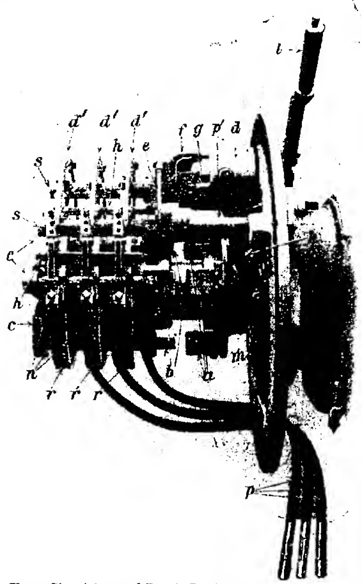

Fig. 222.—Short-Circuiting and Brush-Lifting Gear. (*Bruce Peebles & Co.*)

ring short-circuiting and brush-lifting gear. In the present example, a cover fits over the gear.

The gear is shown separately in Fig. 222, where it looks more complicated; the reason being that there are three

* *See* the Author's *Electric Lighting and Power Distribution*, Vol. I. Seventh Edition.

brushes to each ring instead of one. This, however, makes no difference to the principle on which the mechanism works.

The three leads from the rotor winding are brought out through the hollow shaft, and are connected at *c*, *c* to their respective slip-rings by means of rods passing through—but insulated from—the other two slip-rings. These rods are fitted at their other ends with switch-blades, *b*, opposite each of which is a jaw, *a*, rigidly fixed to the short-circuiting ring, *m*, which slides on the shaft.

At starting, the gear is in the position shown in Fig. 222. The brushes, *n*, (one set only is seen on this side of the figure) are then bearing on the slip-rings, and the contact jaws, *a*, are a good distance from the blades, *b*. The motor is run up to speed by a starter (similar to that in Figs. 199 and 199A) connected to the brushes *n*, through the three leads *p*. The lever handle *l*, which can be locked in either the starting or the running positions, is then moved over to the running position. This operation first of all rotates a small drum *d*, in which there is a skewed slot (seen in Fig. 221); and this slot moves a pin on one end of a lever *k*, pivoted to the right of *p′*. The other end of *k* slides the short-circuiting device *m* along the shaft, until the jaws *a* are well over the blades *b*. The three slip-rings are thereby electrically connected together.

A slight further movement of the lever handle *l*, to its full running position, causes a pin *g*, projecting horizontally from the drum *d*, to move further round and press against the vertical piece *f*, which is capable of rotating on the top brush spindle *s*. When down, the brushes are pressed against the slip-rings by means of springs, but the above-mentioned slight rotation of *f*, acting through two rods *e* (one only is seen in figure), and three rods *h*, parallel to the motor shaft (only two seen), is sufficient to raise the three sets of brushes off the rings.

The arched strips d', d', d', are for connecting the three brushes of each slip-ring together.

In all short-circuiting and brush-lifting gears, it is important that the short-circuiting should be effected

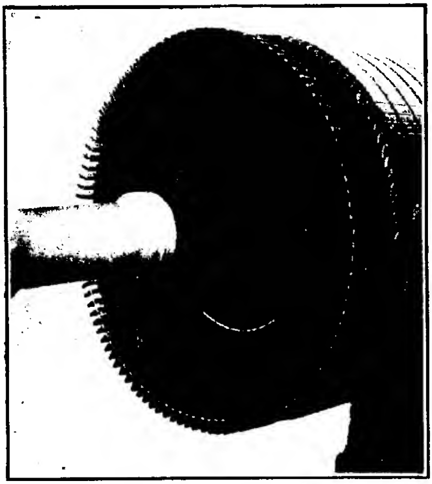

Fig. 223.—Bar-Wound Rotor. (*Bruce Peebles & Co.*)

before the brushes are lifted; otherwise the rotor circuits would be opened, and the motor might stop.

Fig. 223 shows an end view of a slip-ring rotor for a large induction motor. The winding consists of copper strips arranged exactly like the winding of a large continuous-current armature, and it is generally referred to as a *bar winding*. The principle of the method of connection employed on this rotor is given in Fig. 224, from which it will be seen that the winding is very similar to an ordinary

*wave winding.** On tracing out the circuits of the various phases relative to the poles (shown shaded), it will be evident that the arrangement gives the same effect, so far as e.m.f. is concerned, as the windings in Figs. 122 and 220.

It may be added that one or the other of the methods of winding shown in Figs. 220 and 224 is generally adopted for the stators of heavy-current alternators, since it would be almost impossible in such cases to bend thick strips into the shape of the windings shown in Figs. 125 and 130.

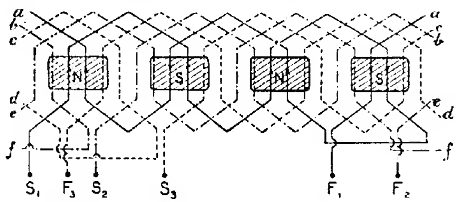

Fig. 224.—Diagram of Connections of Rotor in Fig. 223.

107. **PRINCIPLE OF SINGLE-PHASE INDUCTION MOTORS.**—If we refer back to Figs. 188 and 189, and consider the variation in the magnetic field due to one phase alone, it will be clear that such a field only pulsates or oscillates; in other words, the field only varies from a maximum in one direction to a maximum in exactly the opposite direction; so that it certainly *does not rotate.* Thus in Fig. 189, I and II, the field due to poles 4 and 2 in phase B (Fig. 188) is from left to right, in III it is zero, in IV it is from right to left, and so on. Consequently, no matter how a two-pole stator be wound for single-phase current, it can only have two polar regions, which are alternately and respectively N. and S.

Because of the above, single-phase induction motors are not self-starting.

* *See* the Author's *Electric Lighting and Power Distribution*, Vol. I.

If, however, we were to take a two-phase motor, run it up to speed, and then cut off the supply to one phase; the motor would continue to run, and would be capable of taking a comparatively large load. The reason is given later (p. 337).

It follows from the foregoing, that in whichever direction a single-phase motor is started, it will continue running in that direction.

The most common method of starting a single-phase

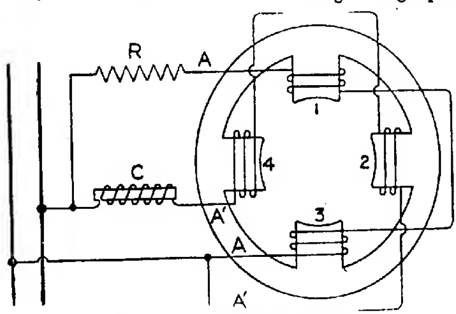

Fig. 225.—Split-Phase Rotating Field.

induction motor is to convert it for the time being into a two-phase motor—by *splitting the phase.*

One way of accomplishing this is to wind the stator with two distinct windings, like a two-phase motor. Such an arrangement is shown diagrammatically in Fig. 225, the first winding AA being on poles 1 and 3, and the second winding $A'A'$ on poles 2 and 4. In series with AA is put a non-inductive resistance R, whilst a choking coil C is inserted in series with $A'A'$. When these circuits are connected in parallel across the mains, the result is that the current in coils AA is practically in phase with the voltage, whereas the current in $A'A'$ lags considerably

behind the voltage. The supply current, in fact, is divided into two currents having a large phase-difference, and approaching very closely to a two-phase supply; in other words, the phase is "split." In consequence, the two windings AA and $A'A'$ will produce a rotating magnetic field in the manner described in § 93, and a squirrel-cage rotor will be set in motion thereby.

As already mentioned, and as will be explained later, it is unnecessary to continue splitting the phase after the motor has attained its full speed. It would therefore be very uneconomical to leave the resistance R and the choker C in circuit, because the constant presence of R would consume a great deal of energy, whilst the choking coil C would reduce the voltage actually applied to the winding $A'A'$. Hence, as soon as the motor has got up speed, R, C, and the *starting winding* $A'A'$ are all cut out, leaving only the *running winding* AA in circuit.

Let us now see how the rotor, when once set well in motion, continues to rotate.

Assume that the 4-pole stator in Fig. 225 has only one set of coils, say AA, in circuit. Also, suppose that a squirrel-cage rotor R is placed inside, as depicted in Fig. 226, and that it is driven at a good speed by another motor. When the windings AA are supplied with alternating current, a flux is produced in the direction of the axis of the poles 1 and 3; and this flux varies in magnitude and direction with the current. Assume that, at a particular instant, the poles 1 and 3 are respectively N and S, and that the rotor is revolving in a clockwise direction. Conductors 1, 2, 7, and 8 will then be cutting the flux from a N. pole, and 3, 4, 5, and 6 will be cutting that from a S. pole: consequently, e.m.fs. will be induced in them, sending currents in the directions indicated in Fig. 226, + and − denoting currents flowing out and in respectively. From this diagram it will be evident that

the induced currents will produce a magnetic field n_1s_1 in the rotor, and that this field will pass through the poles or portions of the stator core marked 2 and 4, magnetising them S_1 and N_1 respectively.

Let us now consider the phase of this new flux N_1S_1 in relation to that of NS. The e.m.fs. induced in the rotor conductors are strictly in phase with the inducing field NS; and they are directly proportional to the rate at which the lines of force are cut. But these e.m.fs. act upon a very inductive circuit, for the conductors 1, 2, etc. on R, although in parallel, act like the winding of a choking coil, the magnetic circuit of which is completed through poles 2 and 4. This will be understood better from Fig. 227, where E, E represent the end connections of the squirrel-cage rotor R in Fig. 226.

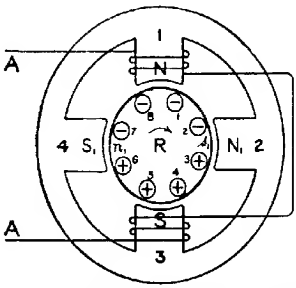

Fig. 226.—Effect of Rotating the Rotor of a Single-Phase Motor.

It has been explained that when an e.m.f. is impressed on the ends of a very inductive circuit, the current produced therein, and the flux due to this current, lag considerably behind the voltage. This is the case in Fig. 227, although the e.m.f. therein is induced, not applied. Consequently the flux n_1 s_1 in Fig. 226 will lag nearly 90° behind the e.m.f. induced in the rotor conductors, and therefore also the same amount behind the inducing flux NS.

Suppose curve L in Fig. 228 represents the flux entering the rotor from poles 1 and 3 (Fig. 226), then M represents the e.m.f. induced in the rotor conductors, and N (lagging

nearly 90° behind M) represents the flux produced in the poles 2 and 4.

The above shows that when the rotor of a single-phase motor is in motion, the stator is magnetised in a very similar manner to that due to two-phase current. Thus, compare curves L and N in Fig. 228 (the ordinates of which are proportional to the flux) with Fig. 187. The chief difference between them is that the maximum heights of the two flux curves in Fig. 228 are different, whereas in Fig. 187 they are equal. At starting, the flux N in Fig. 228 (corresponding with S_1N_1 in Fig. 226) is zero, since the rotor is at a standstill; but when the machine has been given a fair start, the fluxes L (due to NS in Fig. 226) and N in Fig. 228 will combine to form a rotating field, like that described in § 93. The motor will consequently run up to speed, and will then take a considerable load.

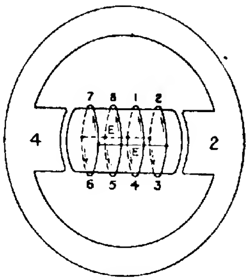

Fig. 227.—Choking-Coil Action of Rotor and Poles 2 and 4 in Fig. 226.

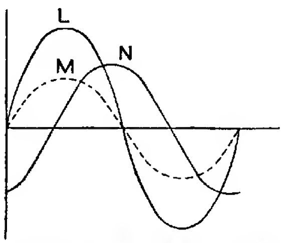

Fig. 228.—Curves of E.M.F. and Fluxes in Fig. 226.

From the foregoing explanation it should be evident that a single-phase motor will rotate in the direction in which it is started, since the direction of the flux N_1S_1 in Fig. 226, depends upon the direction of rotation

of the rotor. Thus, if the rotation be reversed, the curve A (due to N_1S_1) in Fig. 228 is reversed, and therefore the direction of the rotating field due to L and N will also be reversed, as explained in § 95. It follows, then, that the diagram-

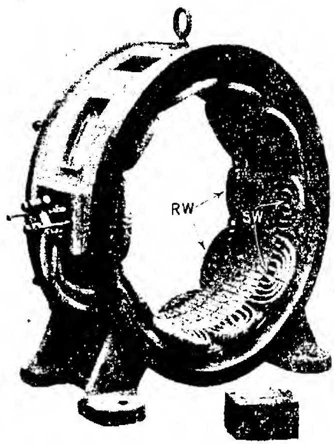

Fig. 229.—Stator of a Single-Phase Motor. (*Harding Churton & Co.*)

matic motor in Fig. 225 could be made reversible by connecting the winding $A'A'$ to a reversing switch. (Fig. 194A.)

In the above treatment, the stator has been shown in the figures as having salient poles; but in actual machines of course the stator core is identical in construction with those shown in Figs. 125, 217, and 229. This, however, does not affect the explanations given.

Although a single-phase induction motor can cope with a considerable load when once started, care must be taken

not to overload it. If the load were increased beyond a certain value, the rotor would stop dead and would need restarting. The rotor of a polyphase induction motor, on the other hand, would quickly regain its normal speed directly the overload was removed; since the rotation of its stator field is quite independent of the speed of its rotor.

108. **SINGLE-PHASE INDUCTION MOTORS AND THEIR CONTROL.**—The stator of one make of single-phase motor is shown in Fig. 229. The cover of the terminal

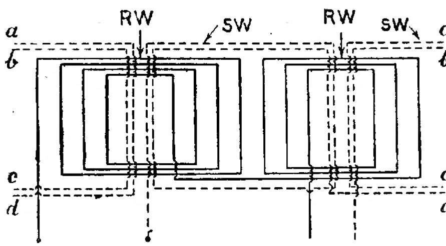

Fig. 229A.—Diagram of a Portion of the Winding of the Stator in Fig. 229.

block is removed, and the large and small pairs of terminals seen thereon form the ends of the running and starting windings respectively; the latter, as explained later, being longer and of smaller size.

This particular stator is wound for 10 poles, and has 90 slots, 70 of which are filled by the running winding *RW*, and the remaining 20 by the starting winding *SW*. A diagram of the connections for one-fifth (*i.e.*, 2 poles) of the windings is given in Fig. 229A: the remaining portion would be simply a repetition of this figure. To show the coils clearer, *SW* has been indicated with two conductors per slot, and *RW* with one conductor per

slot. The actual windings, of course, have a number of conductors in each slot.

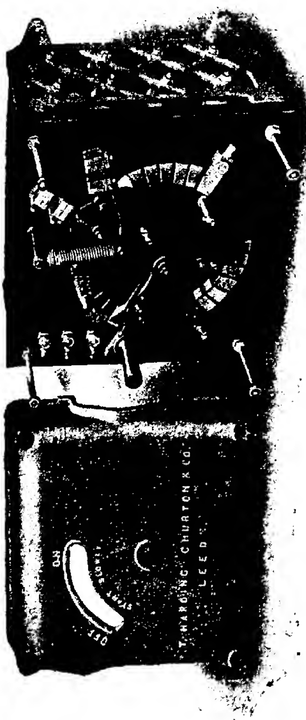

Fig. 230.—Starter for Single-Phase Slip-Ring Motor. (*Harding Churton & Co.*)

It should be noted that Fig. 229A is similar to Fig. 121, except that in the former one winding occupies more slots than the other, whereas in the latter, each winding takes an equal number. The reason for this difference is that, in the single-phase motor (Fig. 229), the starting winding is in circuit only for a few seconds at each start, so it can consist of comparatively thin wire, which will pack into fewer slots, and so leave more slots available for the main or running winding.

These machines are fitted with either squirrel-cage rotors

or wound rotors with slip-rings, exactly like those used with polyphase motors.

A machine with a squirrel-cage rotor must be started up on very light load, or on a loose pulley, since with a starting current about 25 per cent. greater than the full-load current, the starting torque is only about $\frac{1}{4}$ to $\frac{1}{3}$ of the full-load torque. If the machine be fitted with a wound rotor with slip-rings and external starting resistance, the

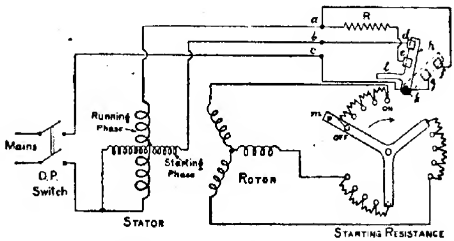

Fig. 231.—Diagram of Connections of the Starter in Fig. 230.

starting current is decreased, while the starting torque is improved, as explained at the end of § 97 and in §§ 98 and 99.

Fig. 230 illustrates a *starter* for a single-phase slip-ring motor. This consists of a triple-arm rheostat switch and a special switch, mounted on an enamelled slate face-plate. The latter is fixed to the front of a cast-iron frame containing the resistance wires, and is provided with a cast-iron cover with a slot through which the operating handle protrudes. A diagram of the electrical connections is given in Fig. 231. This starter differs from that in Fig. 199 in that there are no automatic overload and no-voltage releases. It is provided with an arrangement (seen on the right-hand side in Figs. 230 and 231) for cutting-out the non-inductive resistance R (in series with the running

phase) and also the starting phase, when the motor has got up speed. These operations are performed by a copper switch-blade *h* pivoted about *k*, and fitting into copper jaws *d* and *e* on the one side and *f* and *g* on the other.

It should be noticed that in this arrangement the choking coil shown in series with the starting phase in Fig. 225 is omitted. The reason for this is that the starting phase of the machine in Fig. 229 is wound with a much greater number of turns than the running phase, and therefore has a much higher impedance than the latter, so that it is unnecessary to add to it externally. Thus the reactance of such a winding is very great compared with its resistance, and the current in the starting phase will lag considerably behind the voltage; whereas in the running phase, the current and the voltage will be practically in step on account of the large non-inductive resistance *R*. Hence the effect is exactly the same as that obtained in Fig. 225.

Before closing the circuit at the main D.P. switch (Fig. 231), the starter handle *m* is put in the starting or "off" position, and the blade *h* is then held in contact with jaws *d* and *e* by the interconnected mechanical arrangement seen in Fig. 230. Both these figures show the starting position. When the double-pole switch between the motor and the mains is closed, the current flows through *c*, Fig. 231 (one of the three terminals on the left in Fig. 230), then through pivot *k*, blade *h*, jaw *e*, non-inductive resistance *R*, and terminal *a*, to the running coils. The current to the starting coils flows *via* *c*, *k*, *h*, *d*, and *b*.

The motor having started, the starter handle is turned slowly in a clockwise direction; and when nearly all the rotor starting resistance has been cut out, and the machine has reached practically full speed, the projecting part *m* of the handle comes against an extension *l* of the blade *h*, and rotates the latter about its pivot *k* until its makes

contact with jaws *f* and *g* (as shown dotted). The result is that the running coils will be connected directly across the mains through *c*, *g*, *h*, *f*, and *a*; whilst the resistance *R* and the starting phase will both be put out of action. It should be noted that under running conditions the pivot *k* does not carry current, there being a direct connection between *c* and *g* *via* *k*. Any danger of heating due to imperfect contact at *k* is thus avoided.

The motor is stopped by opening the main D.P. switch; and before it can be started again, the starter handle must be brought back to the "off" position. As it is moved to this position, it automatically puts the blade *h* back into contact with *d* and *e*, thereby rendering it impossible to try and start the motor without the resistance *R* and the starting phase being in circuit.

There are numerous uses for very small single-phase squirrel-cage motors of less than one horse-power, and there is no necessity to use expensive and elaborate starting switches therewith. Messrs Lundberg & Sons have devised special tumbler switches for controlling such motors, as seen in the following diagrams.

Figs. 232 and 233 illustrate a switch, and Fig. 234 the connections, for a single-phase control. The switch is a modified "Twinob," and will carry up to 10 amperes. When the switch is off, the depression of the little box piece *B* forming part of one lever, ensures the closing of both the starting and running circuits. Directly the finger is removed, the box lever flies off, so opening the starting circuit, and leaving the running lever on as in Fig. 232. To stop the motor, the depressed round-knob lever is put off, *i.e.*, up; and the switch levers are then in the position seen in Fig. 233.

The control just described can also be effected by the

type of switch shown in Fig. 235. This is a modification of the "All-or-Part-and-Off" switch, is of 10 amperes capacity, and is shown in its off position in the figure. To start the motor the switch knob is put up, and is held there (against the force of the push-off spring *S*) until the motor has got up speed. The knob is then moved through the central (off) to the down or running position.

When there is no external resistance or choking coil to

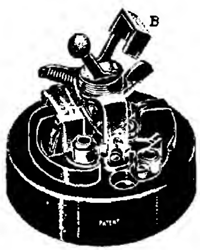

Fig. 232.

Fig. 233.

"Twinob" Control-Switch for very small Single-Phase Motor. (*Lundberg.*)

be connected, the "Twinob" is generally preferred to the modified "All-or-Part-and-Off" switch; unless a break is wanted on both poles, in which case the coupled arrangement depicted in Fig. 236 is adopted. The switch for this control externally resembles Fig. 206.

Another coupled arrangement, with a break on one pole only, provides for currents up to 20 amperes if the voltage be low.

When an external resistance or choker, or both, are to be connected, the "Twinob" switch cannot be used. Fig. 237 shows the connections of the modified "All-or-Part-and-Off" switch (Fig. 235) when a resistance is employed;

and Fig. 238 the connections of the same for starting through a choker as well as a resistance. The latter figure should be compared with Fig. 225.

Starting circuits with resistance or choker, or both, can also be dealt with by coupled switches (Fig. 206) giving a break on both poles.

Single or coupled switches of the Fig. 235 type can be fitted with a fool-proof automatic lock-and-release device to prevent them being put into the running position first.

It may be of interest to note that a "Twinob" switch, coupled as in Fig. 200c, can be connected in an ordinary circuit to act as:—(*a*) a large-capacity single-way switch, (*b*) a quadruple-break single-way switch, or (*c*) a pilot switch. With a very slight modification it may also be connected as (*d*) a double-pole switch.

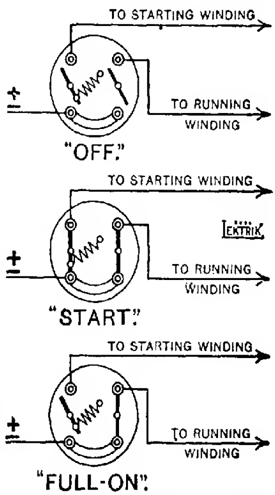

Fig. 234.—Connections of "Twinob" Control-Switch for very small Single-Phase Motor. (*Lundberg.*)

Messrs Lundberg's special switches have effected quite a revolution in lamp-circuit controls, and have greatly added to the conveniences of electric lighting in consequence.* They are also largely used on heating apparatus.

* *See* the Author's *Electric Wiring Diagrams*, or his *Lektrik Lighting Connections*,—the latter obtainable from Messrs Lundberg & Sons.

108A. **SPEED-VARIATION OF INDUCTION MOTORS.** —The two types of alternating-current motor already considered are essentially constant-speed machines. Thus the speed of a synchronous motor is absolutely constant at all loads, and if it be over-loaded sufficiently to prevent its rotor keeping pace with the stator field, it will stop at once (§ 91). The variation in the speed of an induction motor between no-load and full-load is so very small that it may also be said to run at a constant speed. The effect of overloading a motor of this kind beyond a certain limit would be to cause it to stop; the stator meanwhile taking such an excess of current that the circuit fuses would probably be blown. (See page 374.)

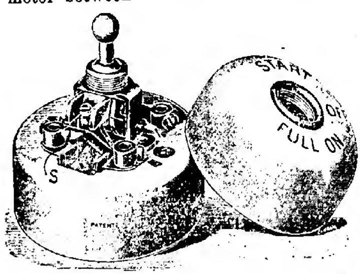

Fig. 235.—Control Switch for very small Single-Phase Motor. (*Lundberg.*)

Different speeds can be obtained from induction motors by so winding the stator that the number of poles can be varied. This method, however, involves such undesirable complications in the stator winding, that it is very seldom adopted.

The most common way of arranging for the variation of the speed of an induction motor is to fit it with a slip-ring rotor, and to provide for the insertion of non-inductive resistances in series with the rotor winding, in exactly the same way as is done when starting-up a motor of this type. But this method has the great drawback that the speed varies with every fluctuation of the load, unless the rotor

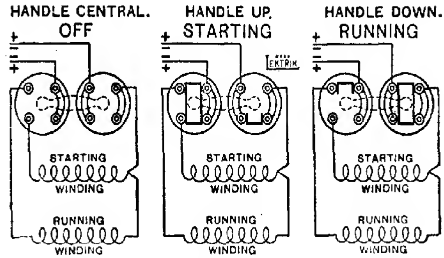

Fig. 236.—Single-Phase Motor Connections for Coupled Switches as Fig. 235, giving a Break on Both Poles. (*Lundberg.*)

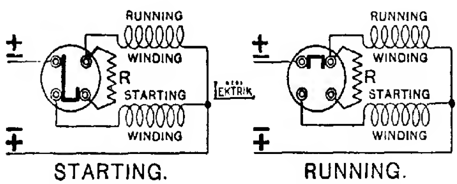

Fig. 237.—Connections of Switch for Controlling very small Single-Phase Motor through a Starting Resistance. (*Lundberg.*)

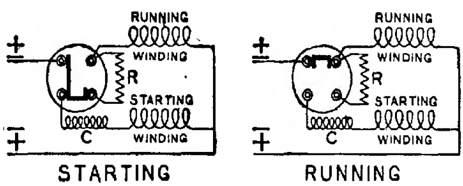

Fig. 238.—Connections of Switch for Controlling very small Single-Phase Motor through a Starting Choker and Resistance. (*Lundberg.*)

resistances are constantly readjusted to each fluctuation. Thus, take the case of a 10-b.h.p., 50-cycle, 6-pole, slip-ring motor, having a full-load slip (with brushes short-circuited) of 5 per cent. :—

From the Formula on p. 310 :—

$$\text{Synchronous speed} = \frac{60 \times 50}{3} = 1000 \text{ r.p.m.}$$

And from Formula 49, p. 309 (and p. 310),

$$\text{(Percentage Slip.)} \ \frac{5}{100} = \frac{1000 - x \text{ (speed of rotor)}}{1000}$$

$$i.e. \text{:—(Slip)} \ \frac{5 \times 1000}{100} = 1000 - x = 50$$

$$\therefore \text{ speed of rotor } (x) \text{ at full load} = 1000 - 50 = 950 \text{ r.p.m.}$$

That is, the speed varies between practically 1000 r.p.m. at no-load and 950 r.p.m. at full load.

Next, suppose resistances are inserted in the rotor circuits to reduce the speed to, say, 600 r.p.m. with a load of 6 b.h.p. ; it being noted that with a constant torque, the horse power is practically proportional to the speed. If this load is taken off, the speed will rise to practically 1000 r.p.m., though the rotor resistances have not been altered. That is, the speed will now vary between practically 1000 r.p.m. at no-load and 600 at 6 b.h.p. This wide fluctuation of speed would in many cases be highly undesirable.

Further, when the speed of a motor is varied by resistances in the rotor circuit, the stator will continue to take the same power from the mains in order to exert a given torque, whatever the speed may be. Thus, in the above example, when the output of the motor has been reduced to 6 b.h.p. at 600 r.p.m., the stator will still continue to absorb from the mains, power equivalent to 10 b.h.p., the additional 4 b.h.p. being wasted in heating the rotor resistances.

The reader may be somewhat surprised at such

behaviour on the part of the induction motor, and will perhaps be asking himself, (*a*) Why should inserting resistances in the rotor circuits produce a variation in the speed of this type of motor? and, (*b*) Why should that variation be dependent upon the load?

It was explained on p. 302 that the torque exerted by an induction motor depended upon the product of the flux and the rotor currents, and also upon the angle of lag of the currents behind the e.m.fs. induced in the rotor. With the supply voltage constant, the flux produced will be practically the same at all loads; also, when the motor is running near synchronism, or when it has high non-inductive resistances in the rotor circuits, the angle of lag in the rotor will be practically negligible (p. 303). Hence we can say that the torque is proportional to the rotor currents; that is, the greater the load on the motor, the greater must be the rotor currents.

Now, when the slip-rings are short-circuited, the rotor resistance is low; consequently, the e.m.f. required to set up a given current in the rotor is also low. This means that the slip—upon which the magnitude of the induced e.m.fs. depends (p. 309)—will be small.

Next, suppose that non-inductive resistances are inserted in the rotor circuits, as shown in Fig. 198. Then, in order to set up a given current in the rotor winding, the e.m.fs. induced must be greater, consequently the slip will have to increase correspondingly, and the rotor will therefore be running at a lower speed. This explains why the the introduction of resistances into the rotor circuits produces a variation of speed.

As regards question (*b*) above, the answer to this will be more or less evident from what has just been said, namely, that the rotor current increases with the load on the motor, so that the e.m.fs. induced in the rotor windings must also increase correspondingly. Now, on no-load, the

rotor e.m.fs. only require to be small, even with large resistances in the rotor circuits, consequently the slip will be very small. With high rotor resistances on full load, on the other hand, there must be a large slip in order to produce the required currents. This explains why the motor in the example on p. 348 varies in speed from 600 r.p.m. at 6 b.h.p. up to about 1000 r.p.m. when running light.

Further, the presence of the resistances in the rotor circuits means that a large amount of power is being wasted in them, so that with a constant power being taken by the motor from the mains, the useful output decreases in proportion to the speed, the remainder of the power being converted into heat in the rotor resistances.

It is therefore evident that regulation of speed by rotor resistances is exceedingly uneconomical, and this method is only adopted when efficiency and constancy of speed are of secondary importance.

When a variable-speed single-phase motor is required, one of the commutator types should be used; and when variable speed is required with three-phase motors, it is best to secure it by mechanical means. These matters are explained in the following sections.

109. **COMMUTATOR MOTORS.** — To secure variable speeds without the above drawbacks, machines have been evolved (both for single-phase and polyphase working) in which the rotors are fitted with commutators. Up to the present, however, *polyphase commutator motors* have only been adopted to a very limited extent; chiefly because their high initial cost and their comparatively complicated construction outweigh the advantage of variable speed. In other words, the simplicity and the extremely satisfactory performance of three-phase constant-speed induction motors, whether of the squirrel-cage or slip-ring type, is so much superior to that of three-phase commutator motors, that when variation of speed is required, it is often preferable

to instal one of the former types, and to vary the speed by mechanical means, *i.e.*, by one of the various sorts of speed-changing gear.

Single-phase commutator motors, on the other hand, are vastly superior to single-phase induction motors. The latter are not very satisfactory even at their best, since they are not self-starting unless fitted with an auxiliary or starting-phase; and even then their starting torque is poor. Further, their efficiencies, power factors, and overload capacities are lower than those of corresponding polyphase motors.

For such work as electric traction on railways, single-phase is preferable to three-phase for distributing the power, since only one insulated conductor is necessary (§ 117). It is also essential in such work to have a variable-speed motor, and one that is able to exert a powerful starting-torque. These desiderata are only obtainable with the commutator motor, which is thus a machine of great and increasing importance.

Practically all the different types of single-phase commutator machines can be placed in one or the other of the following groups:—(*a*) *series motors* and (*b*) *repulsion motors.*

110. **THE SINGLE-PHASE SERIES MOTOR.**—This machine is somewhat similar in construction and action to its continuous-current relative.

In the continuous-current series motor the current passes through the armature and field in series, and causes the armature to rotate in a certain direction. Now if this current were suddenly reversed, the direction of rotation would still be the same, because the current in both the armature and the field would have been reversed. It follows, therefore, that if such a motor were fed with a slowly alternating current, it would not alter materially in its action. The only differences would be:—(*a*) a lower efficiency, due to the current round the field-magnet inducing eddy currents

in the solid magnet core (§ 74); and (*b*) the induction by the alternating magnetic flux of "extra" or secondary currents in certain portions of the armature windings, namely, in the coils which were being short-circuited by the brushes. The magnitude of both of these effects would depend upon the frequency of the current.

Extra or secondary currents in the armature are only troublesome because of their tendency to cause sparking at the commutator; and this disadvantage may be overcome by careful design, and by the introduction of special windings on the stator. Eddy currents in the field-magnets are prevented by laminating the magnetic circuit. Thus, the chief difference between continuous-current and single-phase series motors is that the latter must have laminated field-magnet cores.

It is clear that a series motor built for single-phase current will work equally well when fed with continuous current. In proof of this, on some railways in the United States, part of the track is supplied with continuous current, and part with single-phase current; and the train motors have to work with one or the other, according to the portion of the line the train happens to be traversing.

Like their continuous-current cousins, single-phase series-wound motors exert their maximum torque at starting, and are thus adapted for the same important kinds of work. As compared with ordinary induction motors, their chief disadvantage lies in the fact that they have commutators, there being consequent sparking troubles. The latter are more pronounced than in continuous-current machines; since in addition to the ordinary causes, there is a transformer action between the field and some of the armature coils, as already mentioned. This action may be explained with the help of Fig. 239, which shows a Gramme-ring armature in a 2-pole field *A* and *B*. In order not

to confuse the figure, the connection between the brushes *BB* and the field-coils are broken between *l*, *l*, and *m*, *m*.

When the machine is supplied with alternating current, an alternating flux is produced between the poles *AB*, this flux passing through the armature as indicated by the dotted lines. At the moment shown in the diagram, coils 1 and 5 on the armature are short-circuited by the brushes *BB*, hence they will each act like a closed secondary circuit of a transformer(p.223); the alternating flux inducing e.m.fs. and heavy circulating currents in them. Immediately afterwards, the commutator sections of coils 1 and 5 pass from under the brushes *BB*; so that the circulating currents are abruptly broken, and there will consequently be a great tendency to spark at the brushes.

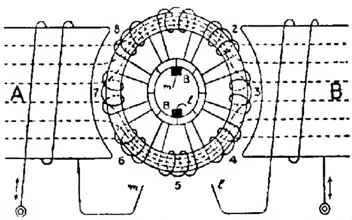

Fig. 239.—Transformer Action in a Single-Phase Commutator Motor.

It is therefore essential to reduce this transformer action to a minimum; this is done:—

(*a*) By having a large number of commutator segments, so that there are only a few turns per coil.

(*b*) By using very narrow brushes, so that only one or two coils are short-circuited simultaneously; and

(*c*) By using high-resistance carbon brushes, and inserting resistances *r*, *r*, etc., between the windings and the commutator segments, as in Fig. 240. These resistances decrease the circulating currents in the short-circuited coils, and therefore decrease the sparking and sparkwear.

The increase in the resistance of the armature will be comparatively small, since at any given instant only those resistances are in circuit whose corresponding commutator sections are in contact with the brushes.

Apart from the above-mentioned points of design, the sparkwear at the commutator may be reduced by employing as low a supply frequency as possible; since the transformer e.m.f. is clearly proportional to the frequency. (Formula 43 p. 227). Thus it is that a frequency as low as 15 is generally adopted in single-phase railway work.

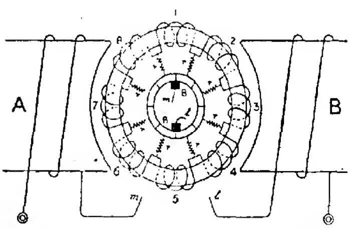

Fig. 240.—Arrangement of Resistances between the Windings and the Commutator Segments in a Single-Phase Motor.

An example of a series commutator motor, and a goods locomotive into which it is being lowered, are illustrated in Fig. 241. This locomotive is one of various types used on the Prussian State Railways. The trolley wire is at present being supplied at 10,000 volts,* 15 cycles; this voltage being stepped-down to about 400 volts by means of a transformer carried on the locomotive. The actual pressure supplied to the motor can, however, be varied to a certain extent, partly by means of tappings on the secondary of the transformer, and partly by means of an *induction regulator*. The latter is a transformer with a rotatable secondary core and winding, the pressure induced in the secondary depending upon the position of the movable portion with reference to the fixed primary core and winding.

* It was proposed to raise this pressure later to 15,000 volts.

The locomotive is fitted with two *bow collectors* for taking the current from the overhead wire, but only one of these is shown in Fig. 241. The motor is connected to the four driving axles by a combination of oblique and horizontal connecting rods, as seen in the figure.

This locomotive is capable of drawing 700 tons (including

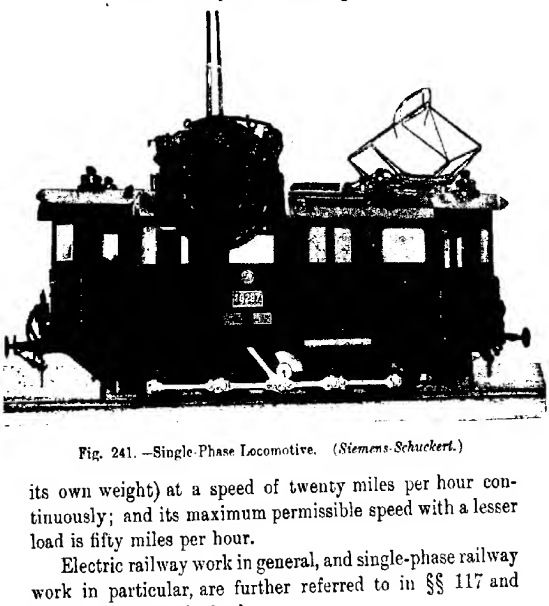

Fig. 241.—Single-Phase Locomotive. (*Siemens-Schuckert.*)

its own weight) at a speed of twenty miles per hour continuously; and its maximum permissible speed with a lesser load is fifty miles per hour.

Electric railway work in general, and single-phase railway work in particular, are further referred to in §§ 117 and 118 at the end of the book.

111. **THE SINGLE-PHASE REPULSION MOTOR.**—This is the other type of single-phase commutator motor mentioned on page 351. Its characteristics are very much like those of the series motor just described; but it has the

advantage that no connections between the mains and the commutator are necessary, so that it is possible to use higher pressures on the stator windings. Its action may be explained as follows.

If we arrange a laminated magnetic circuit such as *M* in Fig. 242, and send a single-phase current through the

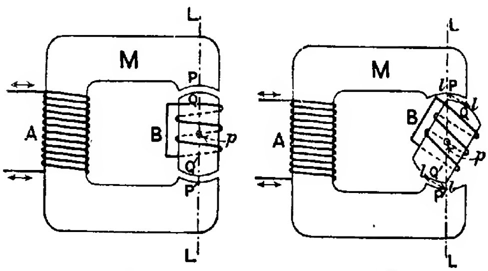

Fig. 242. Fig. 243.
Principle of the Single-Phase Repulsion Motor.

coil *A*, a similar but opposite current will be induced in the coil *B*, which is wound on a rotatable iron core pivoted at *p*. The current in *B*, being almost directly opposite in phase to the current in *A*, will try to produce north and south polarities at *Q* and *Q'* respectively when *P* and *P'* are magnetised north and south respectively by the current in *A*. This state of things will obviously cause repulsion to take place between *P* and *Q*, and between *P'* and *Q'*, along the line *L L* joining *P* and *P'*; but since these two forces will be equal and opposite, no motion will take place. If, however, the secondary winding *B* and its core be slightly turned, as shown in Fig. 243, there will still be a certain amount of repulsion between *P* and *Q*, and between *P'* and *Q'*. But instead of being set up along the line *L L*, the repulsion will now take place along the lines

$l\,l$, $l\,l$, so that the coil B with its core will tend to move round in a clockwise direction, until its axis lies at right angles with $L\,L$. There would then be no tendency for further motion, because the induction of current in the coil B would have stopped.

It might at first be supposed that the motion was in some measure due to the shape of the core, but this is not the case, since, even if the core be spherical, the same tendency to rotate will be evident, provided that the axis of the coil be inclined to the line $L\,L$ to start with, as in Fig. 243.

The above illustrates the principle of the single-phase repulsion motor, except that the method of securing continuous rotation has yet to be explained.

An actual machine of this kind has an armature and commutator very similar to those of a continuous-current motor; but the brushes are unconnected with any external circuit, their function being simply to short-circuit each coil (or group of coils) in turn as it (or they) reaches the best position.

One such arrangement is diagrammed in Fig. 244 (A), where it will be noticed that each brush B, B, short-circuits the three coils marked 2, 3, and 4, which may be assumed to have currents induced in them in the same way as the coil B in Figs. 242 and 243.

Since, as has been shown, the short-circuited coils tend to set themselves so that they enclose no magnetic flux, the armature in Fig. 244 (A) will move in the direction indicated by the arrow, when alternating current is sent through the field-coils at F, F. Directly any coil (say No. 1) arrives under the centre of the adjacent pole, the sections of the commutator to which it is connected will have just left the brush, so that it will then be open-circuited; and another coil (say No. 4) will enter into action to take its place. This action will take place on both sides

of the armature (in a 2-pole machine), and continuous rotation will be maintained in this way. This particular arrangement of the brushes, however, is seldom used now, as the width necessary to span so many sections involves considerable brush friction, and consequent heating of the commutator.

A more usual arrangement of the brushes is that

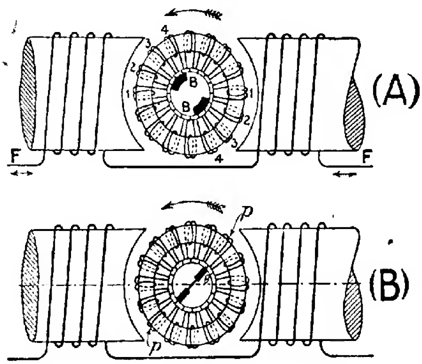

Fig. 244.—Principle of the Single-Phase Repulsion Motor.

shown in Fig. 244 (B), where, as before, the armature is of the Gramme-ring type; but the two brushes are only of ordinary width, and are connected together. In this case, the whole armature consists virtually of two windings in parallel, consequent poles being set up continually at the points p, p. Thus the rotation is still counter-clockwise, as shown by the arrow, although the position of the brushes upon the commutator is 90° removed from that in Fig. 244 (A).

Modern repulsion motors are fitted with drum armatures, similar to those on continuous-current machines.

It should be mentioned that similar precautions to obtain good commutation have to be taken with repulsion motors as with series motors; the transformer action in the short-circuited coils having to be minimised in the ways described on p. 352.

In Fig. 245 are shown the parts of a 2-b.h.p., 4-pole, single-phase *repulsion-induction motor.* The stator *St* is wound with an ordinary single-phase winding similar to that shown in Fig. 119; and the four leads brought out from it enable the halves of the winding to be connected either in parallel or in series, to suit the voltage off which the machine has to work. Thus, in the particular machine shown, the halves would be in parallel for 100 volts, and in series for 200 volts. This is obviously a very convenient arrangement.

The stator *St* is mounted on the bedplate *BP*, and its position adjusted by the screw or bolt *S*. The winding of the rotor *R* is exactly similar to that of a ordinary continuous-current armature. Its pulley is seen at *P'*.

The commutator, which is at the end marked *C*, is of the radial or disc type, its face being at right angles to the shaft, instead of parallel therewith, as in the ordinary or drum form of commutator. Bearing on this commutator are four carbon brushes *A*, which are attached to a brush-gear fitted on the inner side of the end cover *E*. This gear is capable of being rotated through a small angle by a lever *L*, which is operated by a handle on the outside of *E*.

The brushes are short-circuited together, so that when the stator winding is switched onto the mains the machine starts up as a repulsion motor, like Fig. 244 (B). As the speed approaches its correct value, the weights *WW* arranged in the form of heavy plates pivoted about points *PP*, tend to fly outwards under centrifugal force, just like the weights of an engine governor. The outward movement of these weights causes rods inside the rotor core to push forward

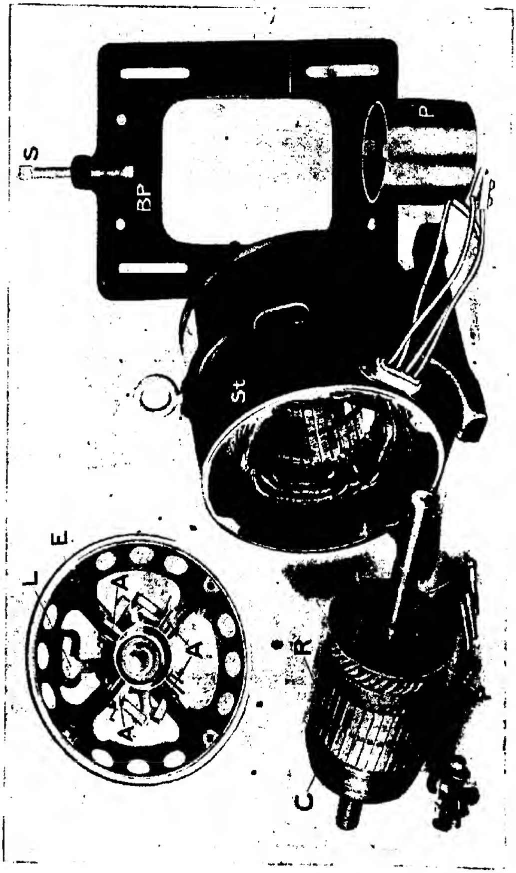

Fig. 245.—2-B.H.P. Single-Phase Repulsion-Induction Motor (Adnil Electric Co.)

a ring which short-circuits all the commutator segments. This movement also causes the brushes A to be pressed away from the commutator surface, thereby eliminating brush friction and wear at the points of contact. The rotor is thus converted into an ordinary short-circuited rotor, and the machine continues to run as a single-phase induction motor. When the machine is stopped by switching-off the current, a spring on the rotor shaft causes the weights WW to resume their original position, in readiness for the next start.

The change from a repulsion motor to an induction motor and *vice versa* takes place quite automatically at a certain predetermined speed, which can be varied by altering the position of WW relative to the pivots PP.

The chief advantage of this type of motor over the ordinary single-phase induction motor (§§ 107, 108) is that it is capable of starting under full load, and that no auxiliary phase or phase-splitting arrangements are necessary. Its starting torque can be varied by slightly rotating the lever L, the effect being to increase or decrease the angle θ in Fig. 244 (B).

112. **SINGLE-PHASE MOTOR CALCULATIONS.*** (*See also* § 45.)

(Ex. 1). **To find**, The Current taken by a Single-Phase Motor: **Given**, Supply Voltage, and the Output, Efficiency, and Power Factor of the Motor.

Find the current for a single-phase motor fed at 100 *volts, and having an output of* 5 *b.h.p., if its percentage efficiency is* 78, *and its power factor* 80 *per cent.*

* The matter in this and the following two sections has been taken, with some slight rearrangement, from the Author's *Electric Circuit Theory and Calculations.*

It should be noted that power factor is sometimes expressed as a percentage, although it is more usual to denote it as a fraction. Efficiency, on the other hand, is usually given as a percentage; but it can equally well be expressed as a fraction.

Solution.—By transposing Formula 32, p. 146, and splitting the kVA into current and voltage in so doing we get:—

$$\text{Current} = \frac{\overbrace{\text{watts output}}^{(\text{b.h.p.} \times 746)}}{\text{volts} \times \text{eff.} \times \text{p.f.}} \qquad \textbf{(50)}$$

$$= \frac{5 \times 746 \times 100 \times 100}{100 \times 78 \times 80}$$

$$= \frac{50 \times 746}{78 \times 8} = 59{\cdot}8 \text{ or say 60 amperes.}$$

(Ex. 2.) **To find**, The Power Factor of a Motor: **Given**, The Actual Watts absorbed, and the Apparents Watts as indicated by the product of the readings of an Ammeter and a Voltmeter.

A certain single-phase motor on a 240-*volt circuit takes* 25 *amperes, and a wattmeter placed in the circuit shows a reading of* 5040 *true watts. Find the power factor of the motor.*

The working involves the use of Formula 29 (p. 142), the power being expressed below in watts and volt-amperes instead of in kW and kVA, as is sometimes done.

$$\text{Power factor} = \frac{\text{actual watts}}{\text{apparent watts}} = \frac{\text{true input in watts}}{\text{volt-amperes}}$$

Thus:—

$$\text{p.f.} = \frac{5040}{240 \times 25}$$

$$= \frac{84}{4 \times 25} = {\cdot}84$$

(Ex. 3.) **To find**, The B.H.P. of a Single-Phase Motor: **Given**, The Voltage and Current it takes, its Efficiency, and its Power Factor.

Find the b.h.p. of a single-phase motor when taking

40 amperes at 200 volts, if its efficiency is 81 per cent., and its power factor 86 per cent.

Transposing Formula 50, on p. 362, we get:—

$$\text{B.H.P.} = \frac{\text{volts} \times \text{amperes} \times \text{p.f.} \times \text{eff.}}{746} \qquad \textbf{(50A)}$$

$$= \frac{200 \times 40 \times 86 \times 81}{746 \times 100 \times 100} = \frac{8 \times 86 \times 81}{7460}$$

$$= 7{\cdot}47.$$

113. **TWO-PHASE MOTOR CALCULATIONS.**—The simplest manner to work out calculations on two-phase motors is to consider their stator windings as two single-phase circuits. For instance, a 10 b.h.p. two-phase motor can be regarded as a machine giving out 5 b.h.p. per phase. The remainder of the calculation will then be identical with that for a single-phase motor. (*See also* § 45.)

(Ex. 4.) **To find**, The Current taken by a Two-Phase Motor with Two Separate Circuits: **Given**, The Supply Voltage, and the B.H.P., Efficiency, and Power Factor of the Motor.

A certain 200-volt two-phase motor, having an efficiency of 85 per cent. and a power factor of ·88, develops 12·5 b.h.p. Find the current taken per phase—i.e., *in each of the two pairs of conductors.*

1st Step.—B.H.P. per phase $= \frac{12{\cdot}5}{2} = 6{\cdot}25.$

2nd Step.—Find current per phase.

Here we use Formula 50, p. 362:—

$$I = \frac{\text{b.h.p.} \times 746}{\text{volts} \times \text{eff.} \times \text{p.f.}}$$

$$= \frac{6{\cdot}25 \times 746}{200 \times {\cdot}85 \times {\cdot}88} = 31{\cdot}1 \text{ amps.}$$

Thus each of the four wires has to carry, say, 31 amperes.

(Ex. 5) **To find,** The Current per Phase taken by a Two-Phase Three-Wire Motor, and the Current in the Middle Wire; also the Voltage across the Outers: **Given,** The Phase Voltage, B.H.P., Efficiency, and Power Factor of the Motor.

A two-phase 220-volt motor supplied from three-wire distribution mains, and having an efficiency of 90 per cent. and a power factor of ·86, develops 20 b.h.p. Find the current per phase taken by the motor, the current in the middle wire, and the voltage across the two outer wires.

1st Step.—B.H.P. per phase $= \frac{20}{2} = 10.$

2nd Step.—Find current per phase.

By Formula 50, p. 362 :—

$$I = \frac{\text{b.h.p.} \times 746}{\text{volts} \times \text{eff.} \times \text{p.f.}}$$

$$= \frac{10 \times 746}{220 \times \cdot 9 \times \cdot 86} = 43 \cdot 8 \text{ amps.}$$

3rd Step.—Find the current in the middle wire.

By Formula 32A, p. 150,

$$\text{current in middle wire} = 1 \cdot 414 \times \text{phase current}$$
$$= 1 \cdot 414 \times 43 \cdot 8$$
$$= 62 \text{ amps.}$$

4th Step.—Find voltage between the two outer conductors.

By Formula 32B, p .150.

$$\text{Voltage across the outers} = \text{phase voltage} \times \sqrt{2}.$$
$$= 220 \times 1 \cdot 414$$
$$= 311 \text{ volts.}$$

(Ex. 6) **To find,** The B.H.P. of a Two-Phase Motor: **Given,** its Apparent Input, Efficiency, and Power Factor.

A two-phase 200-*volt motor, having an efficiency of* 87 *per cent. and a power factor of* 90 *per cent., takes a current of* 47·6 *amps. per phase. What is its b.h.p.?*

The B.H.P. per phase can be calculated from Formula 50A, p. 363, thus:—

$$\text{B H.P per phase} = \frac{\text{volts} \times \text{amps.} \times \text{eff.} \times \text{p.f}}{746}$$

$$= \frac{200 \times 47{\cdot}6 \times {\cdot}87 \times {\cdot}9}{746} = 10.$$

$\therefore$ Total B.H.P. of motor $= 2 \times$ B.H.P. per phase.

$$= 2 \times 10 = 20.$$

114. **THREE-PHASE MOTOR CALCULATIONS.** *(See also* § 45.)

From Formula 38, p. 162.

Power in watts given to a three-phase motor

$$= I_L \times E_L \times 1{\cdot}73 \times \text{p.f.}$$

$$\therefore \text{ Output in kilowatts} = \frac{I_L \times E_L \times 1{\cdot}73 \times \text{p.f.} \times \text{eff.}}{1000} \quad \textbf{(51)}$$

$$\text{and Output in b.h.p.} = \frac{I_L \times E_L \times 1{\cdot}73 \times \text{p.f.} \times \text{eff.}}{746} \quad \textbf{(52)}$$

(Ex. 7.) **To find,** The Current taken by a Three-Phase Motor: **Given,** the Line Voltage, and the B.H.P., Efficiency, and Power Factor of the Motor.

Calculate the current taken in each line by a certain three-phase motor, which is fed at 500 *line volts and develops* 200 *b.h.p. At this load the motor in question has an efficiency of* 92 *per cent., and its power factor is* ·91.

Solution.—The formula to be used here is a transposition of Formula 52.

$$\text{Line current} = \frac{\text{B.H.P.} \times 746}{\text{line volts} \times 1{\cdot}73 \times \text{eff.} \times \text{p.f.}} \qquad \textbf{(53)}$$

$$= \frac{200 \times 746}{500 \times {\cdot}92 \times {\cdot}91 \times 1{\cdot}73}$$

$$= \frac{1{,}492}{4{\cdot}6 \times {\cdot}91 \times 1{\cdot}73}$$

$$= 206 \text{ amperes.}$$

As explained on p. 162, in reference to Formula 38, from which No. 52 is derived, it makes no difference in this case whether the motor be mesh- or star-connected.

(Ex. 8.) **To find**, The kVA Input, and the Full-Load Line and Phase Currents of a Three-Phase Motor: **Given**, the Supply or Line Voltage, B.H.P., Efficiency, and Power Factor of the Motor.

Find the apparent input, in kVA, of a 16-*h.p., three-phase motor supplied at* 190 *line volts, if its efficiency is* 86·6 *per cent., and its power factor* 90 *per cent. at that load. Also find the full-load line current, and the phase current, assuming the machine to be mesh-connected.*

1st Step.—Find the apparent input in kVA

Here we use Formula 32, p. 146.

$$\text{kVA} = \frac{\text{b.h.p.} \times 746}{1{,}000 \times \text{eff.} \times \text{p.f.}}$$

Substituting the known values we get :—

$$\text{kVA} = \frac{16 \times 746}{1000 \times {\cdot}866 \times {\cdot}9} = \frac{11{,}936}{779{\cdot}4} = 15{\cdot}3.$$

2nd Step.—Find the full-load line current.

First, bring the kVA to volt-amperes by multiplying by 1000. Thus :—

$$VA = 15{\cdot}3 \times 1000 = 15{,}300.$$

Then, find line current I_L by dividing the VA or apparent watts by the line voltage multiplied by $\sqrt{3}$. (See Formula 37, p. 162.)

Thus :—

$$I_L = \frac{\text{volt-amperes}}{E_L \sqrt{3}} \qquad \textbf{(54)}$$

$$i.e.,\ I_L = \frac{15{,}300}{190 \times 1{\cdot}73} = 46{\cdot}5 \text{ amperes.}$$

3rd Step.—Find the full-load phase current.

From Formula 36, p. 159, it follows that :—

$$\text{Phase current} = \frac{\text{line current}}{\sqrt{3}}$$

$$= \frac{46{\cdot}5}{1{\cdot}73} = 26{\cdot}9 \text{ amps.}$$

(Ex. 9.) **To find**, The Power Factor of a Three-Phase Motor: **Given**, the Supply Voltage, the Current in the Line Conductors, and the B.H.P. and Efficiency of the Motor.

A 440-*volt three-phase motor takes a line current of* 60 *amperes, and develops* 50 *b.h.p.; its efficiency being* ·92. *Find its power factor.*

A transposition of Formula 52, p. 365, is used here.

Thus :—

$$\text{Power factor} = \frac{\text{b.h.p.} \times 746}{E_L \times I_L \times \text{eff.} \times 1{\cdot}73} \qquad \textbf{(55)}$$

$$= \frac{50 \times 746}{440 \times 60 \times {\cdot}92 \times 1{\cdot}73}$$

$$= {\cdot}887.$$

(Ex. 10.) **To find**, The B.H.P. of a Three-phase Motor: **Given**, Its Voltage, Current, Efficiency, and P.F.

A three-phase motor having an efficiency of 89 *per cent.*, *and a power factor of* ·92, *takes a current of* 50 *amperes at* 500 *volts. Find its b.h.p.*

Solution.—The formula required is No. 52, p. 365, thus :—

$$\text{B.H.P.} = \frac{E_L \times I_L \times \text{eff.} \times \text{p.f.} \times 1{\cdot}73}{746}$$

$$= \frac{500 \times 50 \times {\cdot}89 \times {\cdot}92 \times 1{\cdot}73}{746}$$

$$= \frac{25 \times 89 \times 9{\cdot}2 \times 1{\cdot}73}{746}$$

$$= 47{\cdot}4.$$

115. **CIRCLE DIAGRAM FOR POLYPHASE INDUCTION MOTORS.**—The *circle diagram of an induction motor* is a diagram which enables its behaviour under various conditions to be predetermined. Thus, when the circle diagram for any given motor has been drawn, its line current, input, internal losses, power factor, and efficiency, for any given output; and also its overload capacity, can be at once determined.

The main reason for introducing the circle diagram here is that it helps in the understanding of the characteristics or behaviour of the induction motor.

The complete theory of such a diagram is too advanced for treatment in this book; and further, the diagram is complicated when fully drawn. Thus in what follows, the diagram is given the simplified form usually employed; and the reader must take its correctness for granted.

We will now proceed to construct the diagram, the quantities being assumed to relate to some particular three-phase motor.

In Fig. 246 :—

Draw OX for base line.

Draw OV (= vector of voltage applied to stator) at right angles to OX.

Draw OA (= line current taken on no load*) at angle VOA got from cos VOA obtained as follows :—

$$\cos VOA = \text{power factor on no load.}$$

$$= \frac{\text{watts absorbed on no load}}{\sqrt{3} \times \text{line volts} \times \text{line amps. on no load}} \qquad \textbf{(56)}$$

This is akin to Formula 55, on p. 367, except that there is no mention of efficiency. The latter quantity does not enter here, or in Formula 58, because only the input power is being considered.

The quantities from which cos VOA is calculated are obtained with test readings on a wattmeter, voltmeter, and ammeter.

Draw OC (=line current taken on "short-circuit," *i.e.*, when rotor is "locked" or held stationary,†) at angle VOC got from cos VOC as obtained on page 371.

OC is the current that would be due to the full pressure; but as this current would be very large, and would overheat the windings, it is usual to calculate it from a smaller current obtained when a reduced voltage is applied. This reduced voltage is about 20 to 25 per cent. of the supply voltage.

* A motor is said to be "running light" or to be on "no load" when its rotor is unconnected with any external load.

† The term "short-circuit" is used here in a special and rather obscure way. In ordinary use, "short-circuit current" would mean that due to the cutting-out of all the resistance, etc., in its circuit; and it would be exceedingly large compared with the normal current. In the above special use, the short-circuit current of the stator, viz., that flowing when the rotor is prevented from moving, is only about 4 to 6 times the full-load current in a motor of medium size.

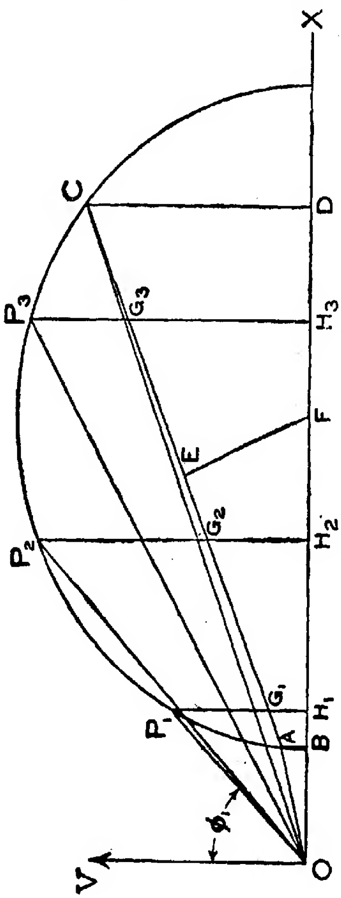

Fig. 246.—Circle Diagram for a Polyphase Induction Motor.

Thus :—

$$\left.\begin{array}{l}\text{“Short-circuit}\\ \text{current” } OC \text{ at}\\ \text{line or supply}\\ \text{voltage.}\end{array}\right\} = \left\{\begin{array}{l}\text{“Short-circuit}\\ \text{current” ob-}\\ \text{served on test}\\ \text{with reduced}\\ \text{voltage.}\end{array}\right\} \times \frac{\text{line volts.}}{\text{voltage employed on test.}} \qquad (57)$$

And :—

$$\cos\ VOC = \frac{\text{watts absorbed on short-circuit test.}}{\sqrt{3} \times \text{test volts on stator} \times \text{test amps. to stator.}} \qquad (58)$$

When performing the above test on slip-ring motors, the rotor slip-rings must be short-circuited just as they are when the machine is running on load.

From the above quantities, the circle diagram in Fig. 246 is completed by joining A and C, bisecting AC at E, and dropping a perpendicular EF thereto to meet the horizontal line OX at F. Then with centre F, and radius equal to FA or FC, a semi-circle AP_1P_2C is described.

It could then be proved (though not here) that *if the line current taken by the motor when supplying any given load be represented to scale by a line OP (not marked), the end P_1, P_2, or P_3, etc., of that line will always lie on the arc of the semicircle just described.*

The angle POV (*i.e.*, P_1OV, P_2OV, or P_3OV, etc.) will give the angle of lag (ϕ) of that current behind the applied voltage; and cos POV will be the corresponding power factor. Thus, for a point P_1, OP_1 represents the stator current, and cos VOP_1 the corresponding power factor.

In other words, suppose we know the line current, and have found its vector length to the proper scale, then if its vector be drawn from O at such an angle that its other end rests on the arc of the semicircle, say at P_3, P_3OV will be the angle of lag of the current.

Now draw P_1H_1 parallel to OV. Then, angles P_1OV

and OP_1H_1 are equal, and cos OP_1H_1 is given by

$$\frac{P_1H_1}{OP_1} \text{ (see p. 100).}$$

$$\therefore\ P_1H_1 = OP_1 \cos OP_1H_1 = OP_1 \cos \phi.$$

As OP_1 represents the stator current at some particular load, it follows from Formula 38 (p. 162), that the power absorbed by the motor (if three-phase) may be stated as follows:—

$$\text{Power (in watts)} = \sqrt{3} \times \text{line voltage} \times OP_1 \cos \phi.$$
$$= \sqrt{3} \times \text{line voltage} \times P_1H_1.$$

The above shows that the power absorbed by the motor (*i.e.*, its input) is proportional to P_1H_1 in Fig. 246. In other words, the perpendicular from any point P on the arc of the semicircle to the base line OX is directly proportional to the power taken by the motor from the mains, when the distance of that point from O represents the corresponding stator current.

It could also be shown (though not here) that the length of the portion of the perpendicular to OX between the arc and the line AC is proportional to the output of the motor for any given case. Thus, for point P_1, P_1G_1 represents the output of the motor to the same scale as P_1H_1 represents the input; and G_1H_1 will consequently represent the total losses in the machine. Hence (from Formula 31, p. 145) the percentage efficiency at that particular load is given by $\frac{P_1G_1}{P_1H_1} \times 100$.

With any other stator current, say OP_2, cos VOP_2 is the power factor; P_2H_2 and P_2G_2 are respectively proportional to the input and the output of the motor, and G_2H_2 is proportional to the total losses.

These chief features of the circle diagram are shown in the simplified Figure 246A.

Another characteristic to be observed from the circle diagram is the variation of the power factor with load. Now at no load (*i.e.*, no external load), the current taken is represented by OA (Fig. 246), and the angle of lag of the current behind the voltage then approaches 90°. But as the current increases up to, say, OP_1, the phase difference decreases, and the power factor consequently improves, since the cosine increases as the angle decreases (see p. 271).

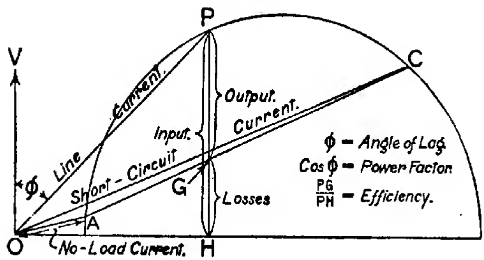

Fig. 246A.—Chief Features of the Circle Diagram.

But beyond a certain value of the stator current, the phase angle increases instead of continuing to decrease; for instance, VOP_2 is greater than VOP_1, and VOP_3 is greater still.

This shows that as the load on an induction motor is increased from its minimum, the power factor first increases from a very small value up to a maximum, and afterwards decreases. It is therefore always desirable so to design an induction motor that its power factor is as nearly as possible a maximum at the load it is intended to carry normally.

It has been stated that the length of P_2G_2 in Fig. 246 is proportional to the output of the machine. Now it will be observed that if the load be increased until the stator

current equals say OP_3, the corresponding (output) length of P_3G_3 is less than that of P_2G_2. This shows that the mechanical power which the motor is capable of giving out, actually decreases after a certain increase of load, even though its current continues to increase. Thus, if the point where the output began to decrease were passed, the motor would "pull up" and come to a standstill; and the current, unless prevented by fuses or circuit-breakers, would increase to OC, which is the "short-circuit" current (p. 369), and the machine would be dangerously overheated. At this point, the power CD taken by the motor is all absorbed within the machine, there being no output.

This illustrates one great difference between a continuous-current and an induction motor. With the first, however much the load is increased, the armature or rotor will always tend to keep in motion, for its torque increases as the current increases. With induction motors, on the other hand, the rotor will immediately stop dead when the load exceeds a certain amount, which is generally about from 2 to 2½ times full load. This characteristic renders induction motors less suitable than continuous-current or a.c. commutator motors for loads which fluctuate in value between very wide limits.

The *overload capacity* of an induction motor is generally expressed as the ratio of the maximum output of which the motor is capable to the rated output of that machine; and is generally about from 2 to 2½ times the rated or *full-load capacity*, according to the design of the machine.

115A. **CIRCLE DIAGRAM FOR POLYPHASE INDUCTION MOTORS** (*contd.*). It should be evident from the foregoing that once the circle diagram for any given induction motor is drawn, the most important characteristics of that machine can easily be determined.

The following example will assist in elucidating the calculations involved.

EXAMPLE.—*The understated readings were observed on the no-load test of a three-phase 4-pole induction motor*:—

Terminal volts = 440, *line current* = 5 *amperes, power absorbed* = 800 *watts. On the "short-circuit" test, current* = 15·5 *amps., terminal volts* = 105, *and power absorbed* = 900 *watts. The rated output of the motor was* 10 *b.h.p. Draw the circle diagram for this motor, and from it determine* (a) *the power factor at full load,* (b) *the full-load efficiency, and* (c) *the overload capacity.*

(I.) **To Construct the first part of the Circle Diagram.**

From Formula 56, p. 369, we have:—

$$\text{Power factor on no load} = \frac{800}{1{\cdot}73 \times 440 \times 5} = {\cdot}21$$

∴ Angle of lag (of which ·21 is the cosine) of current behind volts = 78°. (See p. 271.)

From Formula 57, p. 371,

$$\text{Short-circuit current at 440 volts} = 15{\cdot}5 \times \frac{440}{105}$$
$$= 65 \text{ amperes.}$$

From Formula 58, p. 371,

$$\text{Power factor on short-circuit} = \frac{900}{1{\cdot}73 \times 105 \times 15{\cdot}5}$$
$$= {\cdot}32.$$

∴ Angle of lag of current behind volts = 71°.

Now draw OV in Fig. 247 vertically from the left-hand end O, of a base line OX. OV represents the voltage vector. Next draw OA at an angle of 78° with OV, and of such a length that it represents the no-load current to a scale of, say, 1 cm. = 5 amperes. Also draw OC at an angle of 71° with OV to represent to the same scale the short-circuit current of 65 amperes. Then construct the semi-circle $AP_1\ P_2\ C$ in the manner described on p 371.

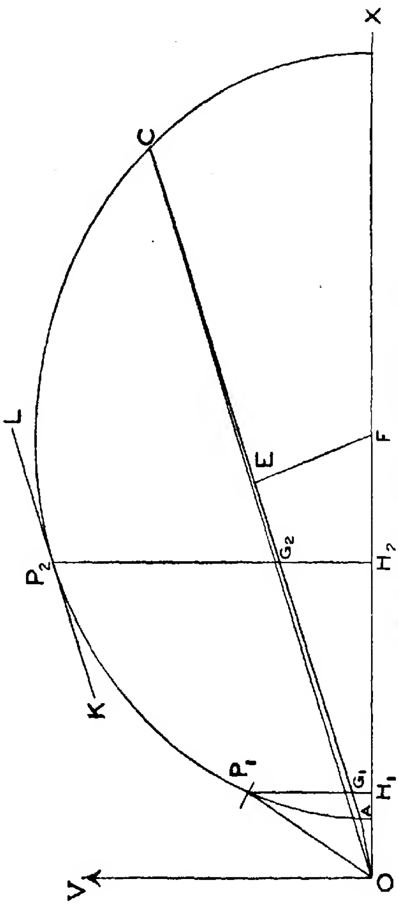

Fig. 247.—Circle Diagram for Example on p. 375.

(II.) **To find the Full-Load Power Factor.**

Assume the full-load efficiency to be 84 per cent., and the full-load p.f. to be ·85.*

Then from Formula 53, p. 366 :—

$$\text{Full-load current} = \frac{\text{h.h.p.} \times 746}{1{\cdot}73 \times \text{line volts} \times \text{eff.} \times \text{p.f.}}$$

$$= \frac{10 \times 746}{1{\cdot}73 \times 440 \times {\cdot}84 \times {\cdot}85}$$

$$= 13{\cdot}7 \text{ amperes.}$$

With centre O, and radius representing to scale (1 cm. =5 amps.) 13·7 amperes, describe an arc cutting the semicircle at P_1. Join OP_1, and measure the angle VOP_1.

* Those who are always working amongst these figures know approximately what they ought to be. For the assistance of the beginner, however, the following Table has been compiled, giving average values of the full-load efficiencies and power factors for different outputs. These figures must be taken only as guides in order to start the above calculations. If the calculated values agree pretty closely with those assumed from the Table, they are correct; if they differ, the whole calculation must be repeated with the calculated values for the efficiency and the power factor, as explained overleaf.

Table of Efficiencies and Power Factors for Three-Phase Induction Motors.

B.H.P.	Percentage Efficiency.	Power Factor.			
		Two-pole.	Four-pole.	Six-pole.	Eight-pole.
5	81	·87	·82	·77	·70
10	84	·89	·85	·81.	·77
20	86	·91	·88	·85	·82
50	88	·92	·90	·88	·86
100 and upwards	90	·93	·92	·91	·89

In Fig. 247, $VOP_1 = 31°$

$\therefore$ Full-load p.f. $= \cos VOP_1 = \cos 31°$

$= \cdot 857$

(III.) **To find the Full-Load Efficiency.**

Draw P_1H_1 parallel to OV and cutting AC at G_1. Then, as already stated (p. 372):—

$$\text{Percentage efficiency} = \frac{P_1G_1}{P_1H_1} \times 100.$$

Now, in calculating this efficiency value, it is immaterial in what units or scale the lengths P_1G_1 and P_1H_1 are measured, so long as both are expressed in the same units. For convenience, let us measure both directly in centimetres.

In Fig. 247, $P_1G_1 = 1{\cdot}98$ cms., and $P_1H_1 = 2{\cdot}35$ cms.

$$\therefore \text{Full-load efficiency} = \frac{1{\cdot}98}{2{\cdot}35} \times 100 = 84{\cdot}2 \text{ per cent.}$$

It will be observed that the values of both the p.f. and the efficiency, as calculated from the circle diagram, happen to agree practically with the assumed values. If this had not been the case, as often occurs in practice, the values of the p.f. and the efficiency so determined must be substituted instead of the assumed values in the expression for the full-load current, and the above calculations for the full-load power factor and efficiency repeated. It will be found that this second set will probably agree closely with those assumed for determining them; if not, the calculations must be repeated with the last set of values.

(IV.) **To find the Maximum Output and the Overload Capacity.**

It follows from what was stated on p. 374, that the first thing is to determine the maximum distance between the arc AP_1C of the semicircle and the line AC; and the simplest way of doing this is to draw KL parallel to AC by means of set squares, so that KL just touches the arc. Suppose the

point of contact be P_2, then draw P_2H_2 parallel to OV and cutting AC at G_2. Now, by geometry, P_2G_2 is the maximum distance—along a perpendicular to OX—between the arc AP_1P_2C and the line AC; *i.e.*, P_2G_2 represents the maximum output of the motor.

As measured from Fig. 247, $P_2G_2=4{\cdot}35$ cms.

$$\therefore \text{ Overload capacity of motor} = \frac{\text{maximum output}}{\text{full-load output}}$$

$$= \frac{P_2G_2}{P_1G_1} = \frac{4{\cdot}35}{1{\cdot}98} = 2{\cdot}2$$

i.e., the maximum b.h.p. which the motor under consideration would be capable of giving would be $2{\cdot}2 \times 10$, or 22 b.h.p.

116. **THE APPLICATIONS OF ALTERNATING-CURRENT MOTORS.**—If, say, five hundred different machines for the most varied purposes and of all sizes were considered, the difficulty would be to pick out those which could *not* be driven by some type or other of a.c. motor. Thus a whole volume could be filled with illustrations of machines (coupled to a.c. motors) for use in every conceivable kind of work or trade.

The very few examples dealt with here, in addition to one or two already illustrated and mentioned below, have been selected to show some of the severest conditions under which alternating-current motors are called upon to work.

Fig. 185 shows a synchronous motor driving a continuous-current generator or dynamo. The motor takes the place of an engine, and performs the work much more economically and efficiently in the situations in which such motor-generator sets are used.

The electric locomotive in Fig. 241 is a striking example

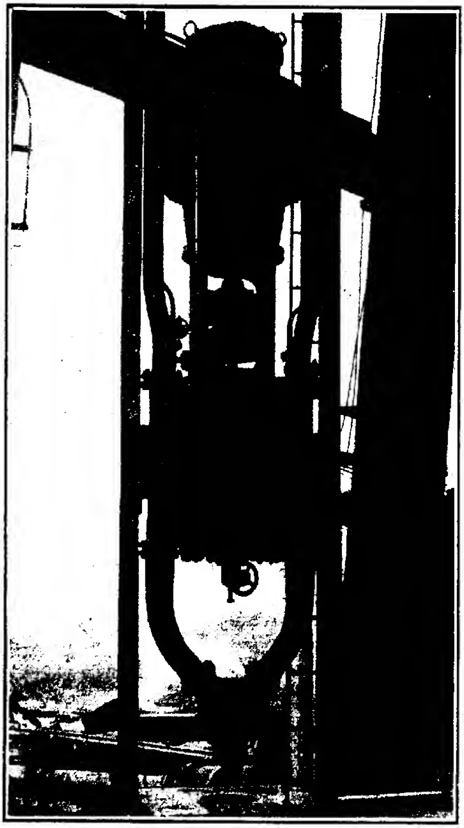

Fig. 248—A Dick-Kerr Induction Motor coupled to a Sulzer Pump.

of heavy traction work, and so also in the complete electric train in Fig. 258.

Fig. 248 shows how admirably electric motors are adapted for use during such operations as shaft-sinking, etc., where space is very confined. This particular figure illustrates a Dick-Kerr three-phase slip-ring induction motor (at the top) directly coupled to a Sulzer pump; and

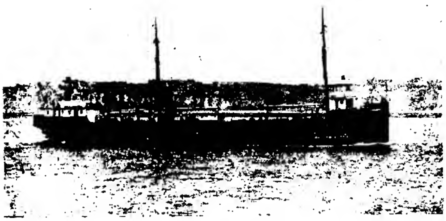

Fig. 249.—The Oil-Electric Ship *Tynemount.*

it will be noticed that the whole equipment is exceedingly compact. It is chiefly because of its compactness that the electric motor has almost entirely displaced every other type of engine in connection with operations of this kind.

The following example relates to one of the most recent developments in the application of alternating-current motors. In Fig. 249 * is illustrated the electrically-driven ship *Tynemount*, built by Messrs Swan, Hunter, & Wigham Richardson, Ltd., of Wallsend-on-Tyne, to the order of

* The blocks for Figs. 249 and 250 were kindly supplied by the Proprietors of *Electrical Engineering*; in which Journal a full description of the *Tynemouth* appeared on Oct. 23rd, 1913.

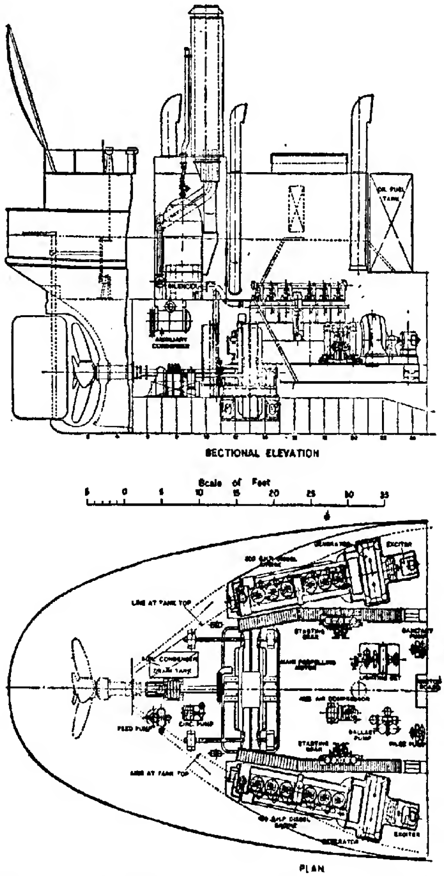

Fig. 250.—General Arrangement of Propelling Machinery in *Tynemount*

the Electric Marine Propulsion Co., for cargo service on the great lakes and canals of N. America. The system of driving is due to Mr H. A. Mavor, M.I.E.E.

The equipment consists of two six-cylinder four-stroke vertical Diesel engines, each capable of developing 300 b.h.p. at 400 r.p.m. To each engine is coupled a three-phase 235-k.v.a. alternator and exciter. One of the alternators,

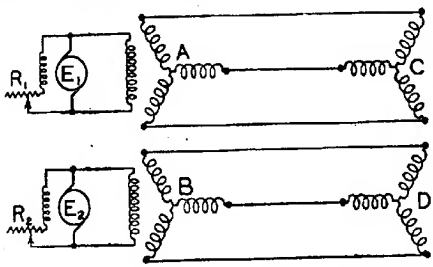

Fig. 251.—*Tynemount* Alternator and Motor Connections for Maximum Speed.

which we shall refer to as A, is fitted with six poles, and the other B, with eight poles; hence, by Formula 1, p. 34, the frequency from A is $\frac{400\times3}{60}=20$ cycles per second, and that from B is 26·6 cycles per second.

The current from these two generating sets is led to a 500-b.h.p. induction motor, the rotor of this being mounted direct on the propeller shaft. The rotor is of the squirrel-cage type, but the stator is provided with two different and entirely separate windings, which we shall refer to as C and D. C has 30 poles and D has 40.

When alternator A is connected to windings C on the motor, as in Fig. 251, the synchronous speed of the motor would, by Formula 1, p. 34, be $\frac{60\times20}{15}=80$ r.p.m. Similarly,

when B is connected to D, the synchronous speed of the motor would also be 80 r.p.m. Thus when both sets are connected at the same time, the motor absorbs the total power of the two engines, and drives the propeller at an actual speed of about 78 r.p.m.; this being the highest speed of the propeller.

To obtain a lower speed, the alternator A is connected to windings D of the motor (Fig 252), so that the synchron-

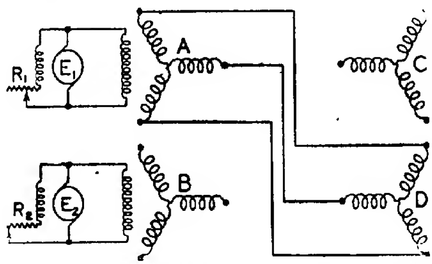

Fig. 252.—*Tynemount* Alternator and Motor Connections for Medium Speed.

ous speed of the motor is then $\frac{60 \times 20}{20} = 60$ r.p.m. Now, the power required by the propeller decreases faster than the speed, so that with a speed about three-fourths of full speed, the power absorbed is only about half of that absorbed at full speed. The power of the one engine driving alternator A is therefore ample, and the engine driving B can be shut down.

This is one of the chief advantages of this *oil-electric system* over the ordinary practice in which all the engines are coupled to a propeller (or propellers), and have to be kept going even when the load is only a fraction of its full value. The efficiency under such conditions is obviously low.

A further advantage of this system is that the alternators when working together are connected to entirely separate circuits, and are therefore never run in parallel. This avoids the need for synchronising apparatus (§ 67), and simplifies the handling of the plant very materially.

Another advantage is that there is no mechanical connection between the propeller-and-rotor shaft and the fixed portion of the machinery; so that the latter is not affected by the sudden and frequent variations of load thrown upon the propeller in rough water.

The control gear consists of two parts—the first, a main switch for effecting the change of connections shown in Figs. 251 and 252, and for reversing the motor (§ 95); the second, a switch for varying the resistances R_1 and R_2 in the shunt circuits of the exciters E_1 and E_2. These two switches are so interlocked that the first switch can only be operated when the whole of the resistances R_1 and R_2 have been introduced into the shunt circuits. The excitation is then practically zero, and the main switch consequently does not have to break any large currents between the alternators and the motor; the latter being started and stopped by increasing and decreasing the excitation of the alternators by means of the resistances.

As will be seen in Figs. 249 and 250, all the propelling machinery is located in the stern of the boat; and a sectional elevation and plan showing its general arrangement are given in the latter figure.

Electric-ship work has developed very rapidly, as may be judged from the fact that the *President Coolidge* and *President Hoover* are electrically-propelled.

In Fig. 253 is shown the "crab" C (traversing and hoisting portion) and a portion of the "bridge" BB of a 40-ton electric overhead travelling crane, which as a whole can

" travel " from one end to the other of the building or yard in which it is installed. The crab runs on the rails r, r, and is

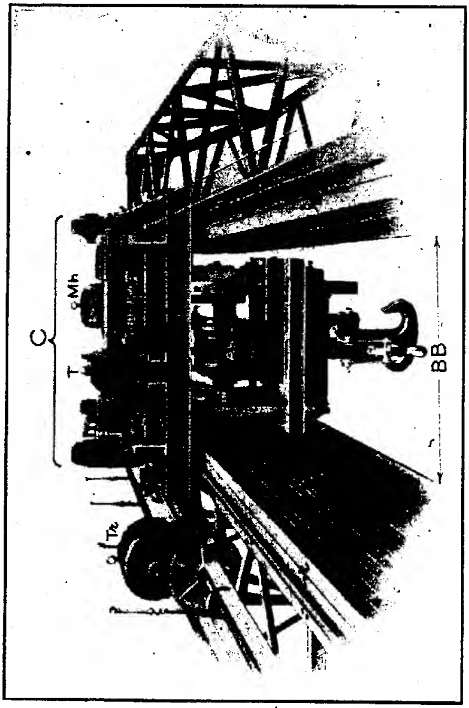

Fig. 253.—Overhead Electric Travelling Crane. (*Babcock & Wilcox.*)
(*British Thomson-Houston Motors.*)

fitted with three three-phase slip-ring motors. Of these, T is for traversing (*i.e.*, moving the crab to any portion of the bridge), and *Mh* and *Ah* for operating the main (40-ton) and auxiliary (5-ton) hoists respectively. A fourth motor *Tr*

on one side of the bridge, is for moving the whole to any desired point between the ends of the place in which it is installed. Current is supplied to the motors, and connections are made between the rotors and their starting resistances, by means of the wires seen suspended alongside the girders of the bridge.

Fig. 254 gives a side view of a similar crane, or rather of

Fig. 254.—One of the many Uses for an Overhead Travelling Crane. (*Babcock & Wilcox.*) (*Metro-Vickers Ltd.*)

two cranes—one behind the other, these being engaged in lifting and carrying a locomotive engine. The crab of the front crane and the hanging cage in which the control gear is fixed, and in which the driver sits, can be seen to the left of the figure.

Fig. 255 shows a new pattern of wharf crane. Here, again, there are four distinct movements. The crane tower or pedestal as a whole can travel along the wharf; the

crane proper can turn on a vertical axis on the top of its pedestal; and the main and luffing jibs can be moved through wide angles.

One of the most severe duties that can be imposed upon the alternating-current motor is that of driving winding gears for collieries, etc.

Fig. 256 gives an example of a three-phase motor instal-

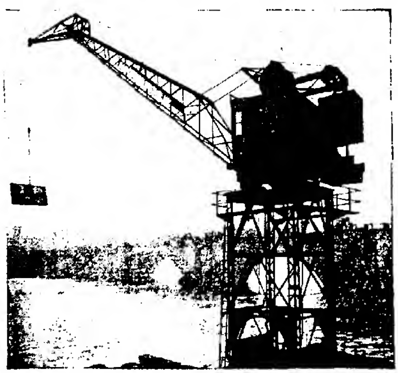

Fig. 255.—A Travelling Jib Crane. (*Babcock & Wilcox.*)
(*Gen. Elec. Co.'s Motors.*)

lation for such a purpose. This set has been constructed and erected at the Burley Pit of the Midland Coal, Coke, and Iron Company by the General Electric Company of Witton.

The plant is designed to complete a wind from the full depth of the shaft (458 yards) in 80 seconds, with a gross load of 30 cwts. or 12 men. The motor M is of the slip-ring type, and is rated at 145 b.h.p. The normal motor

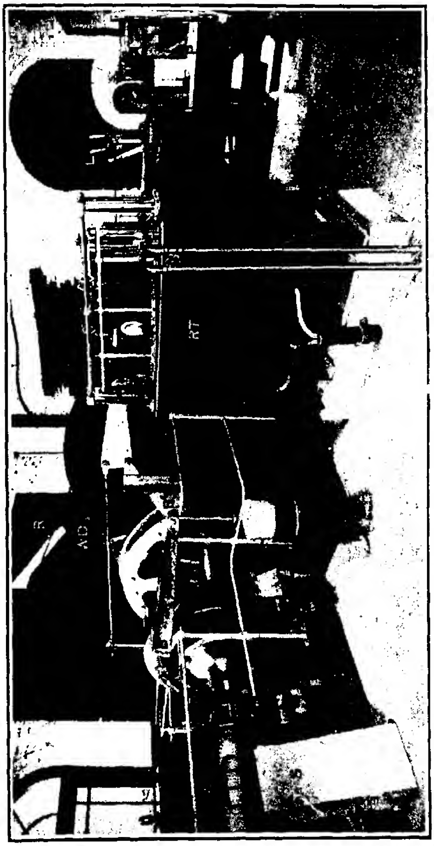

Fig. 256.—Electric Winder. (*Gen. Elec. Co.*)

speed is 240 r.p.m., this being reduced by gearing running in an oil bath to give a speed of 39 r.p.m. to the winding drum *WD*.

The leads from the slip-ring brushes (the slip-rings are out of the illustration on the left) are taken to the resistance tank *RT*, and may be seen passing into the bottom thereof. This tank consists of three chambers insulated from one another, each of which is nearly filled with a resistance electrolyte, such as water with soda in solution; and from the top of the tank are suspended three blades which are joined together both mechanically and electrically. This arrangement is shown diagrammatically in Fig. 257. The motor is started by switching the current onto the stator, and allowing the resistance tank blades to descend into the electrolyte. The greater the area of the blades immersed in the electrolyte, the smaller the resistance in the rotor circuit; and when the blades reach the contacts *K, K, K*, all the resistance is cut out of the slip-ring circuit. The advantage of this type of starter is that a very smooth acceleration of the winding drum *WD* is obtained. If the metal-resistance type of starter were employed, there would be an appreciable increase in the stator current as each step was cut out, so that the cage would be subjected to sudden jerks.

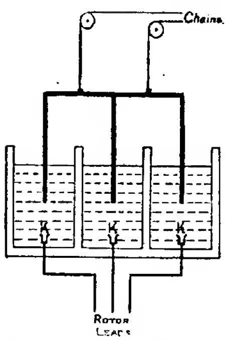

Fig. 257.—Simple Diagram of Liquid Starter for Slip-Ring Motor.

An adequate description of the plant in Fig. 256 is beyond our scope, but the following additional parts may

be noted. *P* is the raised platform on which the driver stands; the three levers at *L*, with one or two others, giving him full control over the plant through the main switches, rotor resistance, brake gear, etc.

R is the steel hoisting and lowering rope, which passes through a slot in the wall over a pulley outside, and then down the mine shaft. The motor is geared to the winding drum through oil-immersed reduction gear at *RG*; and *B*,*B'* are portions of brake gear acting on a pulley on the motor shaft and on the winding drum respectively.

The main cables from the stator pass out, down, and below the floor, behind the shield *S*.

The applications of a.c. motors are so very varied that there has been some temptation to extend this portion; but one must draw the line somewhere. The following matter on electric-railway work, however, forms a sort of extension. As stated at the beginning of this section, motor applications would fill a whole volume; and this would be so even if they were treated in the same brief way as the foregoing. A thorough treatment, including the various interesting and elaborate systems of control, some of which are much more intricate than the simple methods described in this book, would fill several volumes. There are one or two large books in existence which treat of electric cranes solely!

117. **ELECTRIC - RAILWAY WORK.**—*Electric-railway work* offers perhaps the widest scope of all the varied applications of electricity to the young electrical engineer who desires to specialize (*see* page 4); but in order to succeed in it he must also be a very good mechanical engineer. And some acquaintance with civil engineering would be essential.

Electric railways may be worked with either continuous, single-phase, or three-phase current; but for reasons that follow, it would seem that the single-phase system will eventually be the most extensively employed.

In comparing the above three systems of electric railway, we may start with the three-phase. The chief advantage of this system is that the line conductors can be worked at a very high voltage; and it was explained on p. 15 that this is most desirable when large powers have to be transmitted over long distances. Another favourable feature is the absence of commutator troubles. The system, however, possesses a serious drawback, namely, that two or three insulated line wires have to be provided; the rails being employed as the third-phase conductor when two insulated lines are used. The overhead construction is thus exceedingly complicated at junctions and crossings. A further disadvantage of the three-phase system is that induction motors are not suitable for economical speed-variation (§ 108A). So far, three-phase railway working has been almost entirely limited to the Italian State Railways, and it does not seem likely that it will extend very much.

The field for the continuous-current system seems to be practically limited to tramways (which are a species of railway), and to suburban and other railways a few miles in length. In this field, continuous current is likely to hold its own. The reasons are simple. As regards tramways, the introduction of pressures above the usual 500 or 600 volts in the streets would be dangerous; and at these pressures, continuous-current working, with either an overhead or underground line conductor, is superior to single-phase. Railways, however, generally take much more power than tramways, and require a massive live conductor if worked at low voltage. Nevertheless, there is not very much difficulty in disposing of this large conductor on the track; so that such lines, if comparatively

short, are likewise best worked at from 500 to 600 volts with continuous current. It would obviously be too expensive a matter to have the heavy conductor overhead.

It is true that abroad there are several continuous-current lines working at 1000 volts, and a few at 2000 volts, presumably with overhead conductors. On account of commutator troubles with the motors, the latter voltage appears to be as high as it is possible to get on the continuous-current system, and it is far below the limits possible with the single-phase system.

When we come to main-line railways with their long mileage, the problem is quite different. For economical working, the line conductors must be fed at a high voltage (p. 15), that is, 5000 to 15,000 volts or so. It will be obvious that line conductors on the track are entirely unsuitable for this pressure, so that overhead construction must be resorted to. The choice of system consequently lies between the three-phase and the single-phase. The drawbacks of the former are absent in the latter, and so we arrive at the conclusion that the single-phase is the most suitable system for long railways.

As regards electric railways generally, it is interesting to reflect that the Giant's Causeway and Portrush Tramway in the North of Ireland, and Volk's Electric Railway at Brighton, were the first permanent electric lines to be erected in the United Kingdom—if not in the world. They were both opened in 1883, the first in March, the second in August; and they are both working now. Their historical interest is self-evident.

Excluding the thousands of miles of electric tramways, there are some hundreds of miles of underground, overhead, and surface railways now in operation on the continuous-current system in this country; the latest addition (not yet quite completed) being nearly 150 miles of suburban lines on the Southern Railway.

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118. **SINGLE-PHASE RAILWAY WORK.**—The single-phase railway system originated in the United States, and the British Westinghouse Electric and Manufacturing Company first demonstrated its possibilities in the United Kingdom by erecting a short experimental line at their works at Manchester about ten years ago.

The first public line to be operated on this method in this country was the 9½ miles of route between Lancaster, Morecambe and Heysham Harbour, on the Midland Railway; the opening taking place in 1908. The overhead line voltage is 6,600.

The only other British line on which the single-phase* system was adopted was the original London, Brighton and South Coast Railway. The first section was opened in 1909, and another in 1911, the two comprising about 20 route miles and 65 track miles. Work was in progress on an additional 200 track miles, and would have been very near completion but for the Great War. It is quite within the bounds of probability that the whole of this Company's railway will be electrified in due course. The overhead line voltage was 6,700; and it is interesting to note that, all the electrical power required was taken from an outside electricity-supply company.

Fig. 258 shows a train on the above line. This had no locomotive, the collecting and control gear being fitted at one end of a passenger coach. Instead of the trolley pole and wheel used on tramcars, a *bow collector* is employed. The overhead trolley conductor is hung from two parallel suspension wires above it by pairs of short wires disposed V-wise; the suspension wires running over massive porcelain insulators mounted on steel girder arches every 50 yards or so. The suspension wires naturally sag towards the middle

* The whole of this system, now forming part of the Southern Railway, has been changed over to continuous-current, working at 600 volts.

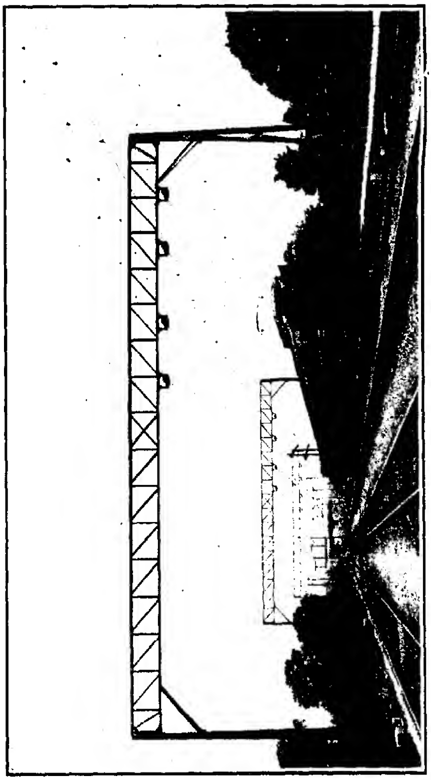

Fig. 258.—Single-Phase Electric Train on the Southern Railway, Brighton Section.

of each span, but by making the V-wires shorter here than near the arches, and graduating their length in between, the conductor wire is kept level. This method of supporting the overhead conductor is known as the *double catenary suspension.*

The square wooden troughs seen on one side of the track contain the feeder cables, these passing through small sub-stations located every few miles along the line.

In America and on the Continent there are already over 70 lines, with a total route mileage exceeding 2,500, working on the single-phase system; so that, quite apart from the example of the Brighton Line before us at home, it is evident that the system has certain advantages.

Some description of the types of motors used, and a short reference to and an illustration of a single-phase railway locomotive, were given in § 110. It is impossible to deal with the subject further herein, as the details of equipment and control gear are very intricate.

Though, as stated at the commencement of the previous section, three-phase current is not very suitable for direct application to train driving, it can be used in the feeding portions of railway networks. Thus on some continuous-current railways, the electrical energy is generated as high-tension three-phase at the generating station, and is converted to low-tension continuous current by means of rotary converters at various sub-stations.

CHAPTER VI.—QUESTIONS.

In answering these Questions give Sketches wherever possible.

NOTE.—Questions marked * range slightly beyond the subject-matter of this Book. Those marked † can only be partly answered therefrom.

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†1. Explain how a rotating field is produced by means of a three-phase current and give reasons why, in an induction motor, it is desirable to have a sine wave supply voltage and also windings designed to give a sine distribution of magnetic induction over the polar arc. (*Grade II.*, *A.C.*, 1914.)

2. State and explain the principle on which a three-phase non-synchronous motor works. (*Ele.*, 1909.)

3. Describe the principle of action of a non-synchronous three-phase motor. Illustrate by a circle diagram. (*Grade II.*, *A.C*, 1913.)

†4. On which points in the design of a three-phase induction motor does the power factor depend? Why is a resistance inserted in the rotor circuit of an induction motor with slip-rings at starting? How does an induction motor behave if, without any alteration in its connections with the supply circuit, it is driven by mechanical power so that its speed is raised above that of synchronism? (*Ord.*, *A.C.*, 1910.)

5. Explain the terms power factor and slip. (*A.M.I.E.E. Exam.*, 1914.)

6. Describe the relative advantages and disadvantages of squirrel-cage and slip-ring induction motors. (*A.M.I.E.E. Exam.*, 1914.)

7. Explain the action of an induction motor. State the relative advantages and disadvantages of squirrel cage and wound rotors, and describe, with diagrams, the methods used in starting motors of each type. (*Grade II.*, *A.C.*, 1912.)

8. Describe the differences between a squirrel cage and wound rotor, stating the advantages of each. Give diagrams of connections for motor and starting gear in each case. (*Wiremen's Final*, 1914.)

9. Give a diagram of connections for a three-phase auto-starter, and a diagram of connections for a rotor starter for a 3-phase motor. (*A.M.I.E.E. Exam.*, 1914.)

10. Describe suitable methods of starting three-phase induction motors in the following cases:—(*a*) small squirrel induction motors in factories, (*b*) large induction motor driving continuous current generator, (*c*) large induction motor operating haulage gears. (*Final*, 1*st Paper*, 1914.)

11. Explain why in a three-phase induction motor (*a*) the torque is approximately proportional to the slip if the applied potential difference is constant, (*b*) the torque exerted for a given slip is proportional to the square of the applied potential difference. (*Ord.*, *A.C.*, 1911.)

12. Explain why (*a*) in a three-phase induction motor the power wasted in the rotor circuit is approximately proportional to the square of the slip, (*b*) the starting torque of a motor with a wound rotor is increased by inserting resistance in the rotor circuits. (*Grade II.*, *A.C.*, 1912.)

†13. Explain why, in an induction motor (1) the torque exerted by the motor is proportional to the slip, (2) when the voltage at the motor terminals is varied the torque exerted by the motor for a given slip is proportional to the square of the applied voltage, (3) the air gap of the motor must be short if it is to have a good power factor. (*Grade II.*, *A.C.*, 1913.)

†14. What modifications are necessary in the design of a continuous-current series motor, in order to make it suitable for running with alternating current (single-phase)? How does the frequency affect the power-factor and the limiting output of a motor of that type? (*Honours*, 1*st Paper*, 1910.)

†15. Explain, with the help of a diagrammatic representation of the windings and circuits, the construction and mode of operation of a modern form of single-phase repulsion motor suitable for railway work. Give the speed-torque curve of

such a motor, and describe the means employed for starting and speed regulation. (*Honours, 2nd Paper*, 1910.)

16. Explain, with sketches, the construction and principle of action of some form of single-phase commutator motor. Why is commutation more difficult in a single-phase series motor than in one working on continuous current? (*Honours, 1st Paper*, 1911.)

17. A two-phase motor fed by three conductors takes 50 amperes in each outer conductor. The voltage of each phase, *i.e.*, between the common middle wire and the two outers, is 200 volts. Calculate the horse power developed, the current in the middle wire, and the voltage across the outers. Efficiency, ·86; power factor, ·86. (*Wiremen's Final*, 1912.)

Ans. 19·8 b.h.p., 70·7 amps., 283 volts.

18. Calculate the current in the conductors supplying a 3-phase motor developing 10 b.h.p. at 400 volts. Efficiency of motor 85 per cent., power factor 85 per cent. (*Wiremen's Final*, 1911.)

Ans. 14·9 amps.

19. An installation of four 10-h.p., two 25-h.p., and one 35-h.p. motors, and 100 40-c.p. metal filament lamps is to be supplied from a public supply 3-phase 4-wire main. The pressure across phases is 380 volts, and the lighting is to be, as near as possible, balanced between phases and the neutral. For what pressure must the lamps be supplied; and what must be the section of each core of the service main, neglecting drop in volts? (*A.M.I.E.E. Exam.*, 1914.)

Ans. 220 volts. Each conductor must carry 195·9 or (say) 200 amps.

The efficiencies, p.fs., and number of poles of these motors must be assumed. This answer has been worked out for 4-pole machines, and with the values tabulated on p. 377. The lamps have been assumed to absorb 1 watt per c.p.

20. The resources of a works are not sufficient to test a large induction motor under full load. What tests can you make on that motor to predict with a fair degree of approximation its power factor at different loads? (*Final, 2nd Paper*, 1914.)

21. Construct, and explain fully, the circle diagram of the induction motor, showing in particular the manner in which the performance on load of an induction motor can be completely determined with the help of this diagram and certain no-load measurements. (*Honours, 2nd Paper*, 1911.)

*22. What is the "circle diagram" of a three-phase motor and why is it given that name? Show a typical diagram for a three-phase motor, and explain how the current, power factor, slip, torque, and power are found from the diagram. How would the diagram change if the motor had more magnetic leakage? (*Grade II., A.C.*, 1914.)

*23. Show a typical circle diagram for a three-phase motor, including the graphic representation of current, power factor, slip, torque and power. How would the diagram change if the motor had more magnetic leakage? (*Ord., A.C.*, 1911.)

†24. Describe the working parts of a three-phase induction motor. Two such motors, identical in every other respect, have different width of air-gap. How will they differ from one another in their performance, under identical conditions, in respect of current, power-factor, slip, and torque, at normal-load, at no-load, at over-load, and at starting? (*Ord., A.C.*, 1909.)

25. A certain three-phase slip-ring induction-motor, rated at 45 B.H.P., 400 volts, 50 cycles per second, and 1,500 r.p.m. no-load speed, is tested running light, with the following results:—

No-load current at 400 volts, 50 cycles = 17 amperes.
Energy (*sic*) absorbed at do. do. = 1,600 watts.

The motor was then tested at stand-still with short-circuited rotor, when it was found that the stator current at 100 volts was 80 amperes, the power-factor of this current being 0·25; the resistance per phase of the stator winding (measured warm) was 0·15 ohm.

With the help of a vector diagram, pre-determine close approximate values for:—

(*a*) The full-load current.

(*b*) The full-load power-factor.

(*c*) The overload capacity of this motor, when running under normal conditions. (*Honours, 2nd Paper*, 1909.)

Ans. (*a*) 60 amps. (*b*) ·9. (*c*) 2·4 times full load (= 108 b.h.p.).

26. A three-phase star-connected induction motor gives the following data on test at a frequency of 50:—

Running light—6000 volts, 39 amperes per phase. Power taken 21 kW.
Rotor locked—1200 „ 128 „ „ „ Power taken 48 kW.

The resistance between any two of the primary terminals of the motor is 0·567 ohms. Assuming that the power taken by the motor when locked is proportional to the square of the voltage, find the maximum power which the motor will give, and determine the slip. Take the supply pressure to be 6000 volts at 50 frequency. (*Final, 1st Paper*, 1914.)

Ans. 3500 b.h.p., 10·8 per cent.

AUXILIARY NOTE.—The point at which the output of the motor is maximum is determined as described on p. 378, and the corresponding values of the current, p.f., and the efficiency, are measured on the diagram. The b.h.p. is then calculated by means of Formula 52, p. 365.

As regards the slip at the maximum load, the simplest method of determining this is as follows:—

$$\text{Slip (at any load)} = \frac{\text{Copper losses in the rotor circuits at that load.}}{\text{Corresponding input to the rotor (by induction from the stator).}} \quad \textbf{(59)}$$

(The derivation of this formula cannot be given here.)

$$i.e., \ \text{Slip} = \frac{\text{Input to rotor—Output of rotor (to shaft).}}{\text{Input to stator—Losses in stator.}}$$

The input to the stator can be calculated by Formula 38, p. 162, the current and the p.f. being determined from the circle diagram, as already described. The output of the rotor in watts or kilowatts is obtained from the b.h.p., which has just been determined.

The stator losses consist of (*a*) iron loss in the core, and (*b*) copper loss

in the windings. The iron loss can be assumed to be the same as the watts absorbed by the motor on no-load, since the copper, friction, and windage losses actually included in that reading are generally so small in comparison with the other losses that they can be neglected throughout this calculation. Hence in the above example, the iron loss will be 21 kW.

The stator copper loss can be calculated from the current (already obtained from the circle diagram) and the resistance of each phase. In the present case, the resistance of each phase is $\frac{\cdot 667}{2} = \cdot 333$ ohm, and the current per phase (from the diagram) is about 417 amperes; so that the total copper losses in the three phases will be $3 \times (417)^2 \times \cdot 333 = 174$ kW.

Hence, total stator losses $= 21 + 174 = 195$ kW.

Further, the diagram gives the maximum output of the rotor as 2610 kW (=3500 b.h.p.), and the corresponding input to the stator is 3120 kW.

$$\therefore \text{ Slip} = \frac{3120 - 195 - 2610}{3120 - 195} = 10 \cdot 8 \text{ per cent.}$$

27. A three-phase induction motor takes, when running light on a pressure of 120 volts, a current of 7·5 amperes per phase at a power factor of 0·29. When the rotor is clamped the motor takes 24 amperes per phase with a pressure of 24·5 volts at a power factor of 0·45. Find the maximum power at which the motor will operate and the load at which it will "pull out." (*Final*, 1*st Paper*, 1913.)

Ans. 10 b.h.p. The maximum load at which the motor will operate will obviously be just a little less than that at which it will "pull out."

*28. Discuss the propositions which have been made for operating main line railways of full gauge with electricity supplied at pressures exceeding 1,000 volts, and point out the considerations which are involved in the adoption of the particular types of motor, modes of motor control and means of collecting the current. (*Honours*, 2*nd Paper*, 1908.)

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The figures refer to the pages.

(NOTE.—Lists of Sections, Illustrations, and Formulæ are given at the front of the Book; and a List of Worked Numerical Examples will be found on p. 415.)

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WORKED NUMERICAL EXAMPLES.

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(NOTE.—Lists of Sections, Illustrations, and Formulæ are given at the front of the Book.)

APPENDIX A.

CONCERNING LAG, ETC.

To show that the Current in a Circuit with Inductance only, and therefore with Reactance only, would lag 90° behind the Applied Voltage.

Consideration of Circuits with both Inductance and Resistance.

ON page 62 it was stated that if a circuit could consist of inductance only, an alternating current therein would lag 90° behind the applied voltage. This statement was based upon the results obtained by mechanical means (Figs. 45, 46, and 48), and consequently may not be convincing to some students.

In this Appendix, the truth of the statement is proved by considering what actually occurs in such a circuit when an alternating current is passed through it. A circuit with inductance only is, of course, quite an imaginary thing; for however thick we make the conductor, the latter cannot be entirely devoid of resistance; and there is bound to be some capacity also in the circuit. But it is useful to consider this hypothetical purely inductive circuit in order to demonstrate that we should then get the maximum lag of 90°. It is shown on p. 99 that as the resistance in the circuit increases, the lag diminishes. Capacity also helps to reduce the lag; but as the capacity of ordinary circuits is very small indeed, it will simplify matters if we ignore it.

Suppose we wind a large number of turns of very thick wire or cable on a laminated iron core, as in Fig. 259. This arrangement would certainly have very high inductance and a negligible resistance. Coil C then represents our imaginary circuit with inductance only, A and B being its two ends. Call the current *positive* when it is flowing from A through C to B, as indicated by the arrow marked + ; and *negative* when it is flowing from B through C to A, as shown by the arrow marked –. The terms positive

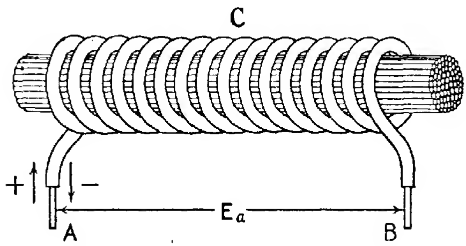

Fig. 259.—Imaginary Circuit with Inductance only.

and negative are only used to distinguish one direction of the current from the other. The names right-hand and left-hand currents might almost equally well be used, but they would not be quite so definite. Let E_a stand for the alternating voltage to be applied to the ends A and B to produce a current I through the coil. It will now be shown that in such a circuit, I lags 90° behind E_a.

Suppose the curve I in Fig. 260 represents the variation of current flowing through the coil C. During the time oa, this current is increasing from zero to a maximum in the positive direction (A to B in Fig 259). Now in § 9, it was explained that

when the current in a circuit is varied (*i.e.*, started varied in strength, or stopped), an e.m.f. is induced in such a direction as to try and prevent the variation; and also that the magnitude of this induced e.m.f. depends upon the inductance of the circuit and upon the rate at which the current varies. It consequently follows that during the time *oa* (Fig. 260), an e.m.f. is induced in coil *C* acting from *B* towards *A*, *i.e.*, in the negative direction. This is indicated at the beginning of the dotted curve E_i. The rate of change in the value of the current *I* is greatest when the current is passing through zero, and is zero when the current is at its maximum. This will be quite evident if we consider the interval of time *t* from a zero (Fig. 260), during which the change is x; and another and equal interval t_1, just before a maximum, during which the

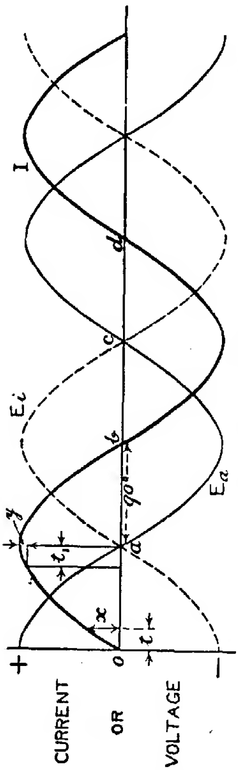

Fig. 260.—Curves of Current and of Applied and Induced E.M.Fs. in an Imaginary Circuit with Inductance (and Reactance) only.

change is only y. Hence the induced e.m.f. E_i is at its maximum in a negative direction at o (Fig. 260), and decreases to zero at a.

Between a and b, the current I is still flowing in the positive direction, but it is now decreasing in value, so that the direction of the induced e.m.f. is now positive also, as it has to oppose the decrease of the current; and this induced e.m.f. attains its maximum value at b, when the current is zero.

During the interval bc, the current is increasing in the negative direction, *i.e.*, from B to A in Fig. 259; so that the induced e.m.f. is still positive in order to oppose this growth of the current. Finally, between c and d, the current is decreasing to zero, and consequently the induced e.m.f. is now in the same direction as the current, and reaches its maximum negative value at d.

This cycle of changes is repeated as long as the alternating voltage is applied to the coil C.

From an inspection of Fig. 260, it will be obvious that the induced e.m.f. of reactance (§ 28A) has its maximum positive and negative values 90° (or a quarter of a cycle) after the corresponding values of the current; in other words, the induced e.m.f. lags 90° behind the current.

The foregoing observations may be tabulated as follows :—

Quarter Cycle.	*Current* (I).	*Time.*	*Induced E.M.F.* (E_i).
(i.)	Increasing + at diminishing rate.	o to a	Decreasing − at increasing rate.
(ii.)	Decreasing + at increasing rate.	a to b	Increasing + at diminishing rate.
(iii.)	Increasing − at diminishing rate.	b to c	Decreasing + at increasing rate.
(iv.)	Decreasing − at increasing rate.	c to d	Increasing − at diminishing rate.

The above Table bears out at (i.) and (iii.) the fact that in electromagnetic induction an increasing current is opposed by an induced e.m.f. in the *opposite* direction; and at (ii.) and (iv.) the kindred fact that a decreasing

current is opposed by an induced e.m.f. in the *same* direction (see p. 36). It will also be noted that a diminishing rate of change of current is always opposed by an increasing rate of change of induced e.m.f., and *vice versa.* Further, any variation in I is followed by an exactly similar variation in E_i during the succeeding quarter of a cycle: thus, changes in I that occur during, say, (i.) are followed by the same changes in E_i during (ii.), etc. This simply confirms the statement already made, namely, that the induced e.m.f. lags 90° behind the current.

Let us now consider the applied voltage E_a (Fig. 260) which gives rise to the current and the induced e.m.f. Now, in a circuit which is assumed to possess inductance only, this applied voltage must at every instant be equal and opposite to the induced voltage (p. 86). This is bound to be so; for if E_a exceeded E_i, the surplus applied voltage would immediately send a larger current through coil C (Fig. 259), and so bring up the value of E_i (which depends upon the current) to that of E_a. Hence the applied voltage can be represented by a curve E_a (Fig. 260) which is exactly the reverse of curve E_i.

On comparing curves I and E_a, it will be evident that the maximum positive and the maximum negative values of E_a occur 90° before the corresponding values of I, i.e., *the current in a circuit with inductance only, lags* 90° *behind the applied voltage*; which is what we set out to prove in this Appendix.

The facts that the current lags 90° behind the applied voltage while the equal and opposite induced voltage lags 90° behind the current are easy to reconcile. Starting with E_i, say at a (Fig. 260), 90° behind I at o; we next find E_a at c, 180° behind E_i at a, and 270° behind I at o, or 90° in advance of it at d. That is to say, I is 90° behind E_a.

If, by reason of the fact that the induced voltage is at any instant equal and opposite to the applied voltage, the reader is unable to understand why there should be any current at all, he should refer to § 38.

Though this explanation has been given for the imaginary case of a circuit possessing inductance only, it is of great assistance in the consideration of one having both inductance and resistance, such as that dealt with in § 29 and below.

Application of Curves to the Consideration of a Circuit possessing both Resistance and Inductance.

Let R and L be respectively the resistance in ohms and the inductance in henries of the kind of circuit to be investigated. For present purposes, it is con-

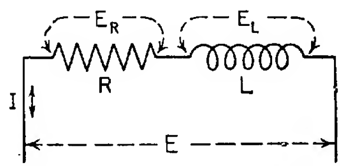

Fig. 261.—Circuit with Inductance and Resistance in Series.

venient to consider one having a resistance R (possessing negligible inductance) connected in series with an inductance L (possessing negligible resistance) as indicated in Fig. 261.

Let E_R be the voltage that would have to be applied across R to produce a current I through it alone, and let E_L be the voltage necessary across L to set up the same current through it alone. E is the *actual voltage* across the circuit, which sets up the current I through

both R and L, and which is the resultant of E_R and E_L, as will now be explained.

Curve I in Fig. 262 represents the current through the circuit. Now the current through a non-inductive resistance is at every instant proportional to the voltage producing it. That is to say, curve E_R, representing the voltage necessary if the circuit consisted of R alone, would (as shown) have its zero and maximum values at the same instants as the corresponding values of I. On the other hand, curve E_L, representing the voltage necessary

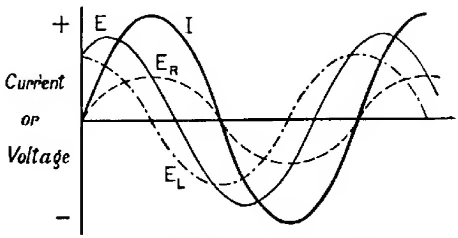

Fig. 262.—Curves relating to Circuit in Fig. 261.

if the circuit consisted of L only, would (as shown) lead 90° in front of the current. This fact has just been proved in the foregoing matter. By adding together the corresponding values of E_R and E_L at different instants, we obtain curve E, which therefore represents the actual voltage necessary to send the current I through the combined circuit. It should now be noted that the phase angle between I and E is less than 90°.

For a given current and frequency, the values of E_R and E_L are directly proportional to the resistance and the inductance respectively of the circuit (*see* § 29).

FURTHER CONSIDERATIONS.

By sketching curves similar to those in Fig. 262, but with different amplitudes for E_R and E_L (representing different values for the resistance and inductance respectively), it can be demonstrated very easily that the larger the resistance and the smaller the inductance, the less is the amount by which the current I lags behind the applied voltage.

Thus in Fig. 260, we have the imaginary case of a circuit with inductance only, and consequently reactance only, and the lag is 90°. Figs. 262 and 263 represent circuits with both resistance and inductance, such as are met with in practice. In Fig. 262, the reactance due to the inductance (and to the frequency) is assumed to be greater than the resistance; and the lag is about 58°. In Fig. 263, the resistance preponderates, and the lag is only about 27°.

It is only by assuming a *fixed frequency* (since the reactance depends upon the frequency as well as upon the inductance) and a *fixed current* that we can compare (as regards the relative effects of inductance and resistance) sets of curves like those in Figs. 262 and 263; for if in either case we alter the frequency, the lag will be affected; and if we alter the current, the amplitudes of all the curves are changed. If we assume this fixed frequency and fixed current, instead of curves, we could draw graphical diagrams similar to that in Fig. 73, but relating to resistance and reactance only.

Thus if, in such diagrams, OA were made equal to the resistance and OB to the reactance for different quantities of resistance and inductance, the diagonal would always be proportional to the impressed e.m.f. necessary for the fixed current; and the angle ϕ would only change when either the resistance or the reactance was changed.

Fig. 73 represents the relationship between the resistance, the reactance, and the impedance of a circuit (like Fig. 261) that is to be operated at a given frequency, but is not necessarily closed for the current to flow. Here we find that the resultant of the reactance and the resistance is impedance, which (like reactance) is expressible in ohms, and which represents the effective opposition of the circuit to the flow of an alternating current at the given frequency.

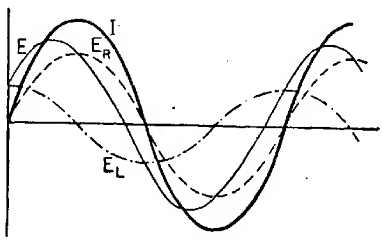

Fig. 263.—Curves similar to those in Fig. 262, but relating to a Circuit with less Lag.

It will be obvious here that with a fixed inductance and resistance, the angle of lag will change if the frequency be changed. But if we keep both the inductance and the frequency (as well as the resistance) constant, the values of the applied or impressed voltage and of the resulting current do not affect the lag. This should be clear from Fig. 72, where it will be seen that the current (I) is a factor of all three quantities.

The whole matter is summed-up in Formulæ 13 and 13A, p. 98, which give in symbols and in words what may be termed the "Ohm's Law" for the alternating-current circuit.

APPENDIX B.

REACTANCE DUE TO CAPACITY.

(Derivation of Formulæ 16 and 17, pp. 104 and 105.)

The unit of capacity is called the *farad. A condenser has a capacity of one farad when it requires a charge of one coulomb to raise the p.d. across its terminals from zero to* 1 *volt.* A condenser having this capacity, however, is much too large for practical work; in fact, a very usual size for use in testing has a capacity of 1 microfarad only, *i.e.*, one-millionth part of a farad.

$$1 \text{ microfarad} = 1 \text{ m.f.d.} = \cdot 000001 \text{ farad.}$$

From the above definition of a farad, it follows that the quantity of electricity required to raise the p.d. across the terminals of a condenser of K farads from zero to V volts, is KV coulombs.

Now consider the quarter-cycle during which an alternating voltage raises the p.d. across a condenser of K farads from zero to the maximum voltage ($E_{max.}$). The quantity of electricity required for this is clearly $KE_{max.}$ coulombs.

The number of coulombs is also given by the *product* of the average current and the time (in seconds) during which the condenser is being charged. In the present case, it is charged in a quarter of a cycle; and if the frequency be f cycles per second, the duration of this quarter-cycle is $\frac{1}{4f}$ second. Also, the average or mean current is the virtual value divided by 1·11 (*see* p. 79).

Hence :—

$$\left.\begin{array}{r}\text{Number of coulombs to}\\ \text{charge condenser}\end{array}\right\} = \text{Average current} \times \frac{1}{4f}$$

$$= \frac{\text{Virtual current}}{1{\cdot}11 \times 4f}$$

$$= \frac{I}{4{\cdot}44f} \text{ coulombs.}$$

On p. 78, it is stated (in other words) that

$$E_{max.} = \frac{E\ (\text{virtual})}{{\cdot}707}$$

Equating the previous two expressions for the charge, we have :—

$$KE_{max.} = \frac{I}{4{\cdot}44f}$$

$$\therefore \frac{E\ (\text{virtual})}{{\cdot}707} = \frac{I}{4{\cdot}44fK}$$

$$\therefore E\ (\text{virtual}) \text{ or } E = \frac{I}{2\pi fK}$$

since E always means the virtual value unless otherwise stated, and since $\frac{{\cdot}707}{4{\cdot}44} = \frac{1}{2\pi}$.

From the above it follows that the reactance due to capacity is $\frac{1}{2\pi fK}$ ohms.

APPENDIX C.

RELATIONSHIP BETWEEN LINE AND PHASE VOLTAGES IN A STAR-CONNECTED SYSTEM.

(Proof of Formula 34, p. 158.)

Let oa, ob, and oc (Fig. 264) be three vectors, spaced 120° apart, and representing the phase voltages of a, b, and c in Fig. 101. Call the voltage positive when it is acting from the neutral point p outwards towards a line conductor, and call it negative when it is acting towards p.

The total voltage acting, say, from B to A in Fig. 101 is the vectorial sum or resultant of the voltages acting from B to p and from p to A. Oa (Fig. 264) represents a voltage whose positive direction is from p to A; but ob indicates a voltage whose positive direction is from p to B; so that to represent the positive direction from B to p, we draw ob_1 equal and opposite to ob. Hence the voltage between B and A in Fig. 101 is the resultant of oa and ob_1, and is given by

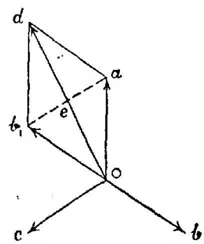

Fig. 264.—Vector Diagram for Voltages in a Star-Connected System.

the diagonal od of the parallelogram $oadb_1$. Join ab_1; then ab_1 bisects od at e. Also angle aob_1 is 60° (*i.e.*, half of aoc), and angle aoe is 30°. Hence :—

$$\begin{aligned} od = 2\ oe &= 2\ oa \cos aoe \quad \text{(See footnote, p. 100.)} \\ &= 2\ oa \cos 30^\circ \\ &= 2\ oa \times \cdot 866 \quad \text{(See Table, p. 271.)} \\ &= 1 \cdot 73\ oa \end{aligned}$$

i.e., Line voltage = 1·73 × phase voltage.